EE5355 DISCRETE TRANSFORMS AND THEIR APPLICATIONS Dr. K. R. Rao

Comparison of various discrete transforms

This will not include the fast algorithms, separability, recursivity, orthogonality and fast algorithms (complexity of implementation). These topics are described elsewhere in detail. The focus is on their various properties. We will consider the random vector $\underline{x}$ is generated by I-order Markov process. Correlation matrix R_{xx} is where $\underline{x} = \left(x_0, x_1, \cdots, x_{N-1}\right)^T$, ($x_0, x_1, \cdots, x_{N-1}$ are the N random variables) generated by the I-order Markov process,

$$\underbrace{R_{xx}}_{(N\times N)} = \underbrace{\left[\rho^{|j-k|}\right]}_{(N\times N)} \quad , \ \rho = \text{adjacent correlation coefficient}, \ j,k = 0,1,\cdots,N-1$$

The covariance matrix $[\Sigma]$ in the data domain is mapped into the transform domain as $[\tilde{\Sigma}]$. *DOT* stands for discrete orthogonal transform.

is $$\underbrace{[\tilde{\Sigma}]}_{(N\times N)} = \underbrace{[DOT]}_{(N\times N)} \underbrace{[\Sigma]}_{(N\times N)} \underbrace{[DOT]^{T*}}_{(N\times N)}$$

Superscript T and * denote transpose and complex conjugate respectively.

When *DOT* is KLT, $[\tilde{\Sigma}]$ is a diagonal matrix as all the transform coefficients in the KLT domain are uncorrelated. For all the other *DOT*, residual correlation (correlation left undone in the *DOT* domain) is defined as [5,6]

$$r = \|\Sigma\|^2 - \frac{1}{N}\sum_{n=0}^{N-1}\|\tilde{\Sigma}_{nn}\|^2$$

where $\|\Sigma\|^2$ is the Hilbert-Schmidt norm defined as $\|\Sigma\|^2 = \frac{1}{N^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}|\Sigma_{mn}|^2$

For a 2D-random signal such as an image assuming row and column statistics are independent of each other the variances of the ($N\times N$) samples can be easily obtained. This concept is extended for computing the variances of the ($N\times N$) transform coefficients.

1) Transform coding gain, G_{TC}

is defined as

$$G_{TC} = \frac{\left[\frac{1}{N}\sum_{i=0}^{N-1}\tilde{\sigma}_{ii}^2\right]}{\left[\prod_{i=0}^{N-1}\tilde{\sigma}_{ii}^2\right]^{\frac{1}{N}}} = \frac{\text{Arithmetic Mean}}{\text{Geometric Mean}}$$

where $\tilde{\sigma}_{ii}^2$ is the variance of the *i*th coefficient in the transform domain. As the sum of the variances in any orthogonal transform domain is invariant (total energy is preserved), G_{TC} can be maximized by minimizing the geometric mean [1].

2) Variance distribution in the transform domain

It is desirable to have few transform coefficients with large variances (this implies the remaining coefficients will have small variances, as the sum of the variances is invariant). This variance distribution can be described graphically or in a tabular form for $N = 8, 16, 32, \cdots$ and $\rho = 0.9, 0.95, \cdots$ etc.

The compaction of the energy in few transform coefficients can be represented by the normalized basis restriction error [2] defined as

$$J_m = \left[\frac{\sum_{k=m}^{N-1}\tilde{\sigma}_{kk}^2}{\sum_{k=0}^{N-1}\tilde{\sigma}_{kk}^2}\right] \quad , \; m = 0, 1, \cdots, N-1$$

where $\tilde{\sigma}_{kk}^2$ are arranged in decreasing order.

3) Rate versus distortion (Rate-distortion) [2]

R_D is the minimum average rate (bits/sample) for coding a signal at a specified distortion *D* (mean square error) defined as

$$D = \frac{1}{N}\sum_{k=0}^{N-1} E\left[\left(x_k - \hat{x}_k\right)^2\right]$$

where $\hat{x}_k$ is the reconstructed sample.

For a fixed average distortion *D*, the rate distortion function R_D is

$$R_D = \frac{1}{N}\sum_{k=0}^{N-1} \max\left[0, \frac{1}{2}\log_2 \frac{\tilde{\sigma}_{kk}^2}{\theta}\right]$$

where θ is determined by solving

$$D = \frac{1}{N}\sum_{k=0}^{N-1} \min\left[\theta, \tilde{\sigma}_{kk}^2\right]$$

Plot R_D vs *D*, for $N = 8, 16, 32, \cdots$ and $\rho = 0.9, 0.95, \cdots$

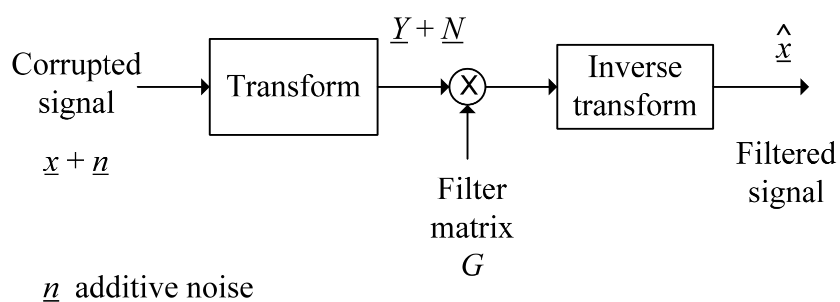
4) Residual correlation

While the KLT completely decorrelates a random vector, other discrete transforms fall short of this. An indication of the extent of decorrelation can be gauged by the correlation left undone by the discrete transform. This can be measured by the absolute sum of the cross covariance in the transform domain i.e.,

$$\sum_{i=0}^{N-1} \sum_{\substack{j=0 \\ (i \neq j)}}^{N-1} \left|\tilde{\sigma}_{ij}^2\right|$$

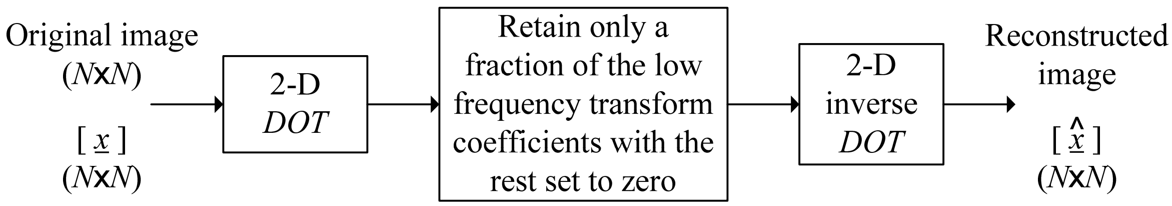
for $N = 8, 16, 32, \cdots$ as a function of ρ.

5) Scalar Wiener filtering [4]

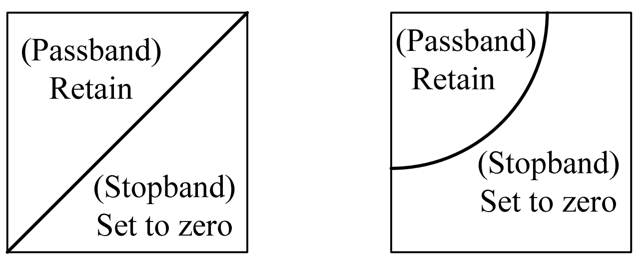


Filter matrix G is optimized for a specific transform, such that the noise can be filtered. Evaluate $MSE = E\left(\left\|\underline{x} - \hat{\underline{x}}\right\|^2\right)$ for the discrete transforms for $N = 4, 8, 16, 32, \cdots$ and $\rho = 0.9$ and 0.95

6) Geometrical Zonal Sampling (GZS)



Geometrical zonal filter can be 2:1, 4:1, 8:1, or 16:1 (sample reduction) see Figure below for 2:1 and 4:1 sample reduction in the 2D-DCT domain



2:1 Sample reduction   4:1 Sample reduction

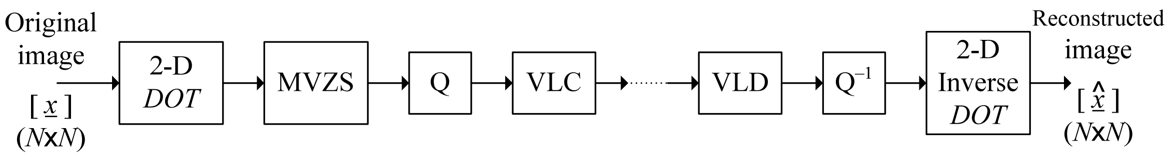
Note that for 2D-DFT, the low frequency zones need to be appropriately identified.

The reconstructed images for various sample reductions can be obtained and a plot of the normalized MSE vs various sample reduction ratios for all the *DOT* can be implemented.

$$\text{Normalized } MSE = \frac{1}{N^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} E\left(\left[x(m,n) - \hat{x}(m,n)\right]^2\right)$$

7) Maximum Variance Zonal Sampling (MVZS)

In MVZS, transform coefficients with large variances can be selected for quantization and coding with the remainder (transform coefficients with small variances) set to zero. At the receiver side inverse operations are carried out resulting in the reconstructed signal or image.



REFERENCES

[1] K. R. Rao and P. C. Yip, *The Transform and Data Compression Handbook*. Boca Raton, FL: CRC Press, 2001.

[2] A. K. Jain, *Fundamentals of Digital Image Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[3] P. Yip and D. Hutchinson, “Residual correlation for generalized discrete transforms,” *IEEE Trans. EMC*, vol. 24, pp. 64–68, Feb. 1982.

[4] N. Ahmed and K. R. Rao, *Orthogonal Transforms for Digital Signal Processing*. New York, NY: Springer, 1975.

[5] P.-S. Yeh, “Data compression properties of the Hartley transform,” *IEEE Trans. ASSP*, vol. 37, pp. 450 – 451, March 1989.

[6] M. Hamidi and J. Pearl, “Comparison of the cosine and Fourier transforms of Markov-1 signals,” *IEEE Trans. ASSP*, vol. ASSP-24, pp. 428–429, Oct. 1976.

[7] V. Britanak, P. Yip, and K. R. Rao, *Discrete Cosine and Sine Transforms*. New York, NY: Academic Press, 2007.

[8] J. Pearl, “Basis-restricted transformations and performance measures for spectral representations,” *IEEE Trans. Info. Theory*, vol. IT–17, pp. 751–752, Nov. 1971.

[9] J. Pearl, H. C. Andrews, and W. K. Pratt, “Performance measures for transform data coding,” *IEEE Trans. Commun.*, vol. COM–20, pp. 411–415, June 1972.

[10] P. Yip and K. R. Rao, “Energy packing efficiency for the generalized discrete transforms,” *IEEE Trans. Commun.*, vol. COM–26, pp. 1257–1261, Aug. 1978.

IDFT

$[x] = [x(0), x(1), \ldots, x(N-1)]^T$ is a random vector where $x(0), x(1), \ldots, x(N-1)$ are the N random variables.

Assume $[x]$ is real. Covariance matrix of $[x]$ is $[\Sigma]$.

$$[\Sigma] = E[(x - \bar{x})(x - \bar{x})^T] \tag{4.77a}$$

Superscript T means transpose.

where $\bar{x} = E[x]$ = mean of $[x]$.

$$\underset{(N \times N)}{[\Sigma]} = E\left[\underbrace{\begin{pmatrix} x_0 - \bar{x}_0 \\ x_1 - \bar{x}_1 \\ \vdots \\ x_{N-1} - \bar{x}_{N-1} \end{pmatrix}}_{(N \times 1)} \underbrace{(x_0 - \bar{x}_0, \quad x_1 - \bar{x}_1, \quad \cdots, \quad x_{N-1} - \bar{x}_{N-1})}_{(1 \times N)}\right] \tag{4.77b}$$

The covariance matrix in data domain is

$$[\Sigma] = \begin{bmatrix} \sigma_{00}^2 & \sigma_{01}^2 & \sigma_{02}^2 & \cdots & \sigma_{0,N-1}^2 \\ \sigma_{10}^2 & \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1,N-1}^2 \\ \sigma_{20}^2 & \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2,N-1}^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{N-1,0}^2 & \sigma_{N-1,1}^2 & \sigma_{N-1,2}^2 & \cdots & \sigma_{N-1,N-1}^2 \end{bmatrix} \tag{4.77c}$$

In $[\Sigma]$, the diagonal elements are variances and off diagonal elements are covariances.

$$E[(x_j - \bar{x}_j)(x_k - \bar{x}_k)] = \sigma_{jk}^2 \qquad (j \neq k)$$

= covariance between x_j and x_k.

$$E[(x_j - \bar{x}_j)(x_j - \bar{x}_j)] = \sigma_{jj}^2 = \text{variance of } x_j.$$

The covariance matrix in the DFT domain is (Superscript * means complex conjugate operation)

(EQ.5.65/p.196)

$$\underset{(N \times N)}{[\Sigma^F]} = E[\underbrace{(X^F - \bar{X}^F)}_{(N \times 1)} \underbrace{(X^F - \bar{X}^F)^{T^*}}_{(1 \times N)}]$$

$$= E[[F](x - \bar{x})([F](x - \bar{x}))^{T^*}]$$

$$= [F]E[(x - \bar{x})(x - \bar{x})^T][F^*]$$

$$= [F][\Sigma][F^*]$$

(Similar to 2D-DFT of Σ the covariance matrix in data domain)

$$\underset{(N\times 1)}{\underline{X}^F} = \underset{(N \times N)}{\underline{F}} \underset{(N\times 1)}{\underline{x}} \quad \Big| \quad \underline{x} = \underline{F}^* \underline{X}_F$$

$$[\Sigma^F] = [F][\Sigma][F^*]$$

$$\underset{(N \times N)}{[\Sigma^F]} = \begin{bmatrix} \tilde{\sigma}_{00}^2 & \tilde{\sigma}_{01}^2 & \tilde{\sigma}_{02}^2 & \cdots & \tilde{\sigma}_{0,N-1}^2 \\ \tilde{\sigma}_{10}^2 & \tilde{\sigma}_{11}^2 & \tilde{\sigma}_{12}^2 & \cdots & \tilde{\sigma}_{1,N-1}^2 \\ \tilde{\sigma}_{20}^2 & \tilde{\sigma}_{21}^2 & \tilde{\sigma}_{22}^2 & \cdots & \tilde{\sigma}_{2,N-1}^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{\sigma}_{N-1,0}^2 & \tilde{\sigma}_{N-1,1}^2 & \tilde{\sigma}_{N-1,2}^2 & \cdots & \tilde{\sigma}_{N-1,N-1}^2 \end{bmatrix} \quad (4.78)$$

In $[\Sigma^F]$, the diagonal elements are variances, and the off diagonal elements are covariances in the DFT domain.

For a 2-D ($N \times N$) data array $[x]$ given by, i.e., N^2 random variables

$$\underset{(N \times N)}{[x]} = \begin{bmatrix} x_{00} & x_{01} & x_{02} & \cdots & x_{0,N-1} \\ x_{10} & x_{11} & x_{12} & \cdots & x_{1,N-1} \\ x_{20} & x_{21} & x_{22} & \cdots & x_{2,N-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{N-1,0} & x_{N-1,1} & x_{N-1,2} & \cdots & x_{N-1,N-1} \end{bmatrix} \quad (4.79)$$

has N^4 covariances of which N^2 are variances. Evaluation of the N^2 variances can be simplified by assuming independent row and column statistics.

Assume row and column statistics of the 2-D data are independent of each other. Let the variances of elements of any row be (each row has the same statistics.)

$\{\sigma_{00R}^2 \quad \sigma_{11R}^2 \quad \sigma_{22R}^2 \quad \cdots \quad \sigma_{N-1,N-1,R}^2\}$ in the data domain.
Similarly each column has the same statistics (not necessarily the same statistics of any row.). $\{\sigma_{00C}^2 \quad \sigma_{11C}^2 \quad \sigma_{22C}^2 \quad \cdots \quad \sigma_{N-1,N-1,C}^2\}$ in the data domain.

Then the variances of $[x]$ are

$$\left[\underset{(N \times 1)}{\begin{pmatrix} \sigma_{00R}^2 \\ \sigma_{11R}^2 \\ \sigma_{22R}^2 \\ \vdots \\ \sigma_{N-1,N-1,R} \end{pmatrix}} \underset{(1 \times N)}{\{\sigma_{00C}^2 \quad \sigma_{11C}^2 \quad \sigma_{22C}^2 \quad \cdots \quad \sigma_{N-1,N-1,C}^2\}} \right] = A \quad (4.80)$$

$\left(\dfrac{5.38}{p.140}\right)$

Let the 2-D DFT of $[x]$ be $[X^F]$,

$$[X^F(k_1, k_2)] = [F]\,[x(n_1, n_2)]\,[F^*]$$

$$\left[X^F(k_1,k_2)\right] = \begin{bmatrix} X^F_{00} & X^F_{01} & X^F_{02} & \cdots & X^F_{0,N-1} \\ X^F_{10} & X^F_{11} & X^F_{12} & \cdots & X^F_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ X^F_{N-1,0} & X^F_{N-1,1} & X^F_{N-1,2} & \cdots & X^F_{N-1,N-1} \end{bmatrix} \tag{4.81}$$

Let the variances of any row of $[X^F]$ be $\left\{\tilde{\sigma}^2_{00R} \quad \tilde{\sigma}^2_{11R} \quad \tilde{\sigma}^2_{22R} \quad \cdots \quad \tilde{\sigma}^2_{N-1,N-1,R}\right\}$. Similarly let the variances of any column of $[X^F]$ be $\left\{\tilde{\sigma}^2_{00C} \quad \tilde{\sigma}^2_{11C} \quad \tilde{\sigma}^2_{22C} \quad \cdots \quad \tilde{\sigma}^2_{N-1,N-1,C}\right\}$.

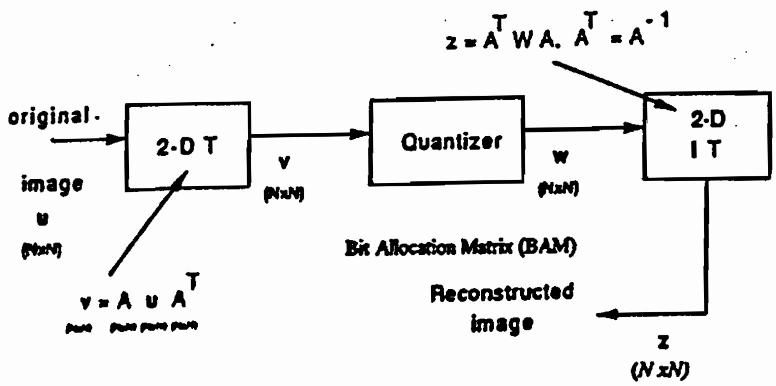
In section 3.4, it was stated that efficient data compression can be achieved by adopting a bit allocation based on the variances of the transform coefficients. This concept is described here in detail for the DFT. It is, of course, valid for any orthogonal transform.

Then the variances of $[X^F]$ are

$$\left[\underset{(N\times 1)}{\begin{pmatrix} \tilde{\sigma}^2_{00R} \\ \tilde{\sigma}^2_{11R} \\ \tilde{\sigma}^2_{22R} \\ \vdots \\ \tilde{\sigma}_{N-1,N-1,R} \end{pmatrix}} \underset{(1\times N)}{\left\{\tilde{\sigma}^2_{00C} \quad \tilde{\sigma}^2_{11C} \quad \tilde{\sigma}^2_{22C} \quad \cdots \quad \tilde{\sigma}^2_{N-1,N-1,C}\right\}}\right]$$

$$= \underset{(N\times N)}{\begin{bmatrix} (\tilde{\sigma}^2_{00R}\ \tilde{\sigma}^2_{00C}) & (\tilde{\sigma}^2_{00R}\ \tilde{\sigma}^2_{11C}) & \cdots & (\tilde{\sigma}^2_{00R}\ \tilde{\sigma}^2_{N-1,N-1,C}) \\ (\tilde{\sigma}^2_{11R}\ \tilde{\sigma}^2_{00C}) & (\tilde{\sigma}^2_{11R}\ \tilde{\sigma}^2_{11C}) & \cdots & (\tilde{\sigma}^2_{11R}\ \tilde{\sigma}_{N-1,N-1,C}) \\ \cdots & \cdots & \cdots & \cdots \\ (\tilde{\sigma}^2_{N-1,N-1,R}\ \tilde{\sigma}^2_{00C}) & (\tilde{\sigma}^2_{N-1,N-1,R}\ \tilde{\sigma}^2_{11C}) & \cdots & (\tilde{\sigma}^2_{N-1,N-1,R}\ \tilde{\sigma}^2_{N-1,N-1,C}) \end{bmatrix}} \tag{4.82}$$

Quantization of transform coefficients can be based on their variances.



Separable transform

Transform is implemented on a block – by – block (N x N) basis.

See Fig. 11.18 p. 506

B_{ij} = # of bits allocated to transform coefficient i, j, (v_{ij})
$i, j = 0, 1, \ldots, N-1$

$$B_{ij} = B + \frac{1}{2}\log_2 \tilde{\sigma}^2_{i,j} - \left[\frac{1}{2N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} \log_2 \tilde{\sigma}^2_{k,l}\right] \quad (4.83)$$

$$B = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} B_{kl} = \text{average bit rate.}$$

$B_{ij} \sim \log_2 \tilde{\sigma}^2_{i,j}$
B_{ij} is proportional to variance of transform coefficienct v_{ij}

# of quantization levels = $2^{B_{ij}}$

If some transform coefficients have very low variances, then they can be replaced by zeros.

MVZS: Maximum variance zonal sampling

The covariance matrix in transform (orthogonal) domain $\tilde{\tilde{\Sigma}}$ and in the data domain Σ are related by

$$\tilde{\tilde{\Sigma}} = [A]\, E\left[\left(x - \bar{x}\right)\left(x - \bar{x}\right)^T\right][A]^T = [A]\,[\Sigma]\,[A]^T \quad (4.84)$$

Assume $[A]$ is real and unitary (normalized) $[A]^T = [A]^{-1}$

$$\left[\left(x\right)\right] = [x_0\, x_1\, x_2 \ldots x_{N-1}]^T$$

$$\tilde{\tilde{\Sigma}} = \begin{bmatrix} \tilde{\sigma}^2_{00} & \tilde{\sigma}^2_{01} & \cdots & \tilde{\sigma}^2_{0,N-1} \\ \tilde{\sigma}^2_{10} & \tilde{\sigma}^2_{11} & \cdots & \tilde{\sigma}^2_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{\sigma}^2_{N-1,0} & \tilde{\sigma}^2_{N-1,1} & \cdots & \tilde{\sigma}^2_{N-1,N-1} \end{bmatrix} \quad (4.85)$$

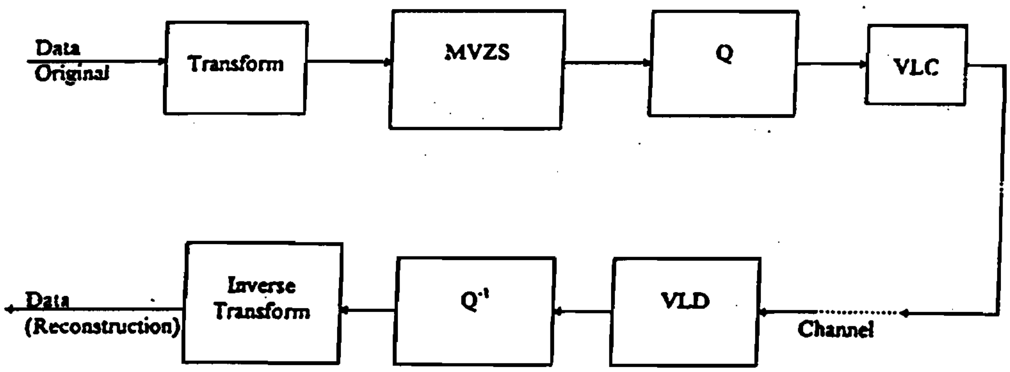
In any orthogonal transform domain the variances of a 2D data can be obtained based on independence of row and column statistics (similar to the DFT) case. For the 1D data, the variance distribution in the transform domain for I order Markov process is shown in Table 4.1. Sum of the variances along any column is the same (energy invariance). The normalized basis restriction error is:

$$J_m = \frac{\sum_{k=m}^{N-1} \tilde{\sigma}_{kR}^2}{\sum_{k=0}^{N-1} \tilde{\sigma}_{kR}^2}, \quad m = 0, 1, \ldots, N-1 \tag{4.86}$$

$\left(\frac{5.179}{p\,171}\right)$

Where $\tilde{\sigma}_{kR}^2$ are rearranged in nonincreasing order. See Fig. 4.22

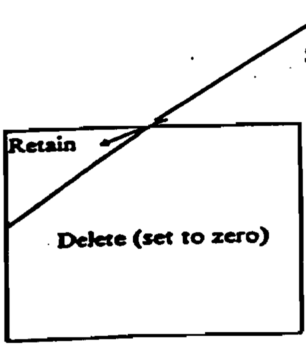
In MVZS transform coefficients with large variances can be selected for quantization and coding with remainder (those with small variances) set to zero. The bitstream representing these coefficients are transmitted to the receiver and inverse operations i.e., decoding, inverse quantization, inverse transform etc., lead to reconstruction of the signal or image.



MVZS Transform Coding

(Threshold Sampling) Geometrical zonal sampling (GZS)
Replace MVZS by GZS

Keep the transform coefficients in this zone and set the rest to zero. See Figs. 4.23, 4.24, and 4.26



2D Transform Domain

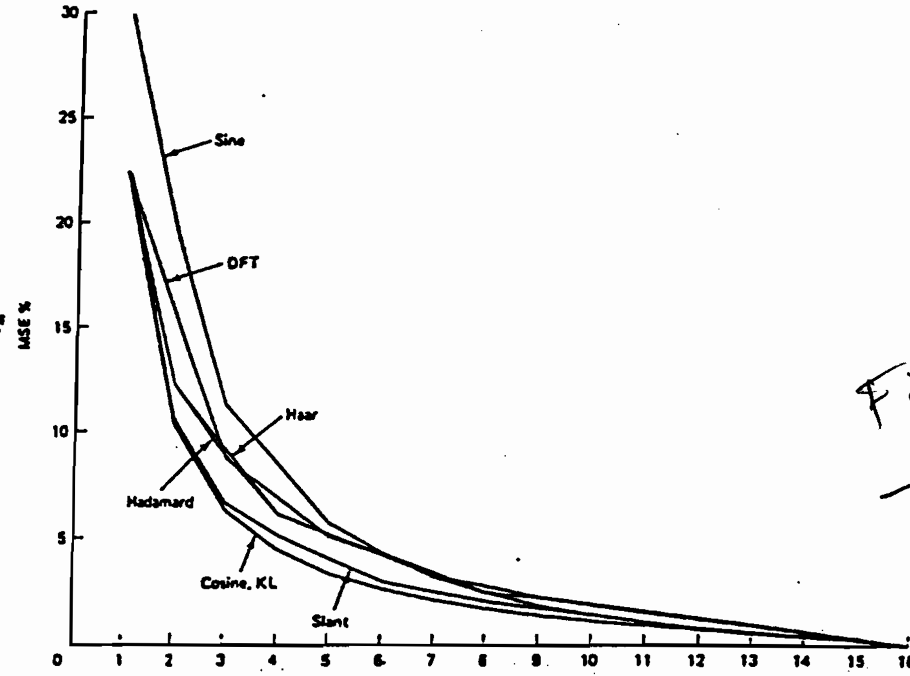


Fig.5.19
P.172

Fig 4.22 Performance of different unitary transforms with respect to basis restriction errors (J_m) versus the number of basis (m) for a stationary Markov sequence with $N = 16, \rho = 0.95$.

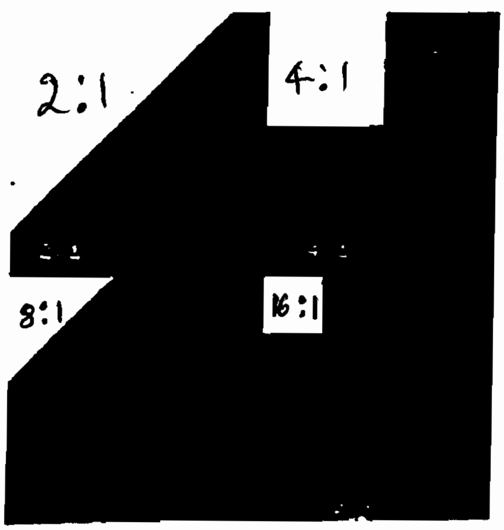


Fig. 5.20
P.172

Fig. 4.23 Zonal filters for 2:1, 4:1, 8:1, 16:1 sample reduction. White areas are passbands, dark areas are stopbands.

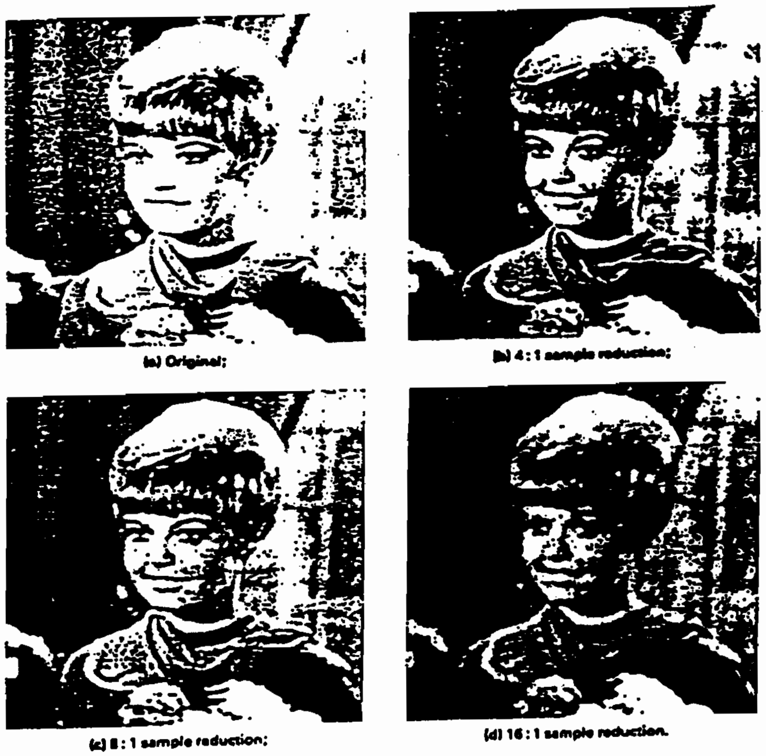
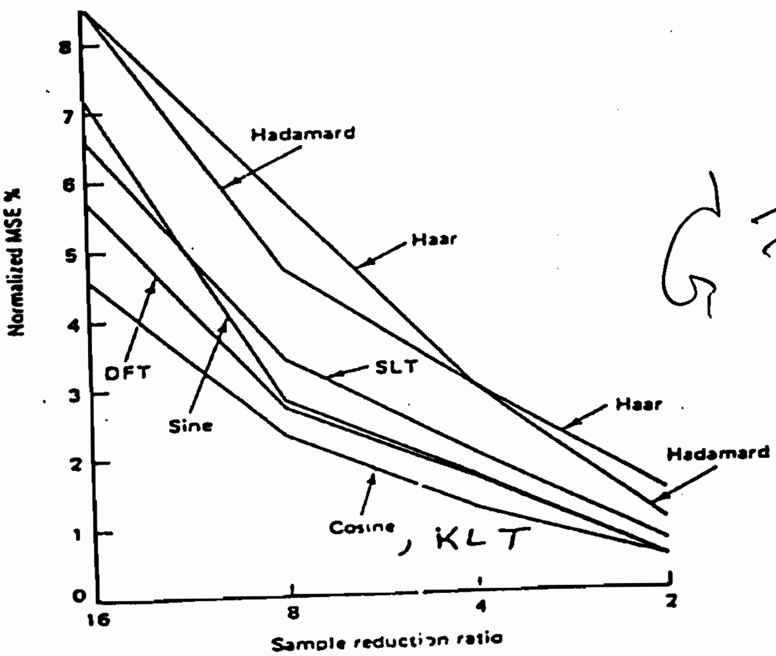


Fig. 4.24 Basis restriction zonal filtered images in cosine transform domain

(GZS
Fig. 5.21
p. 173)



GZS

(Fig. 5.23
p. 175)

Fig. 4.25 Performance comparison of different transforms with respect to basis restriction zonal filtering for 256 x 256 images.

GZS : Geometrical zonal sampling

(a) Cosine; (b) sine; (c) unitary DFT; (d) Hadamard; (e) Haar; (f) Slant.

GZS

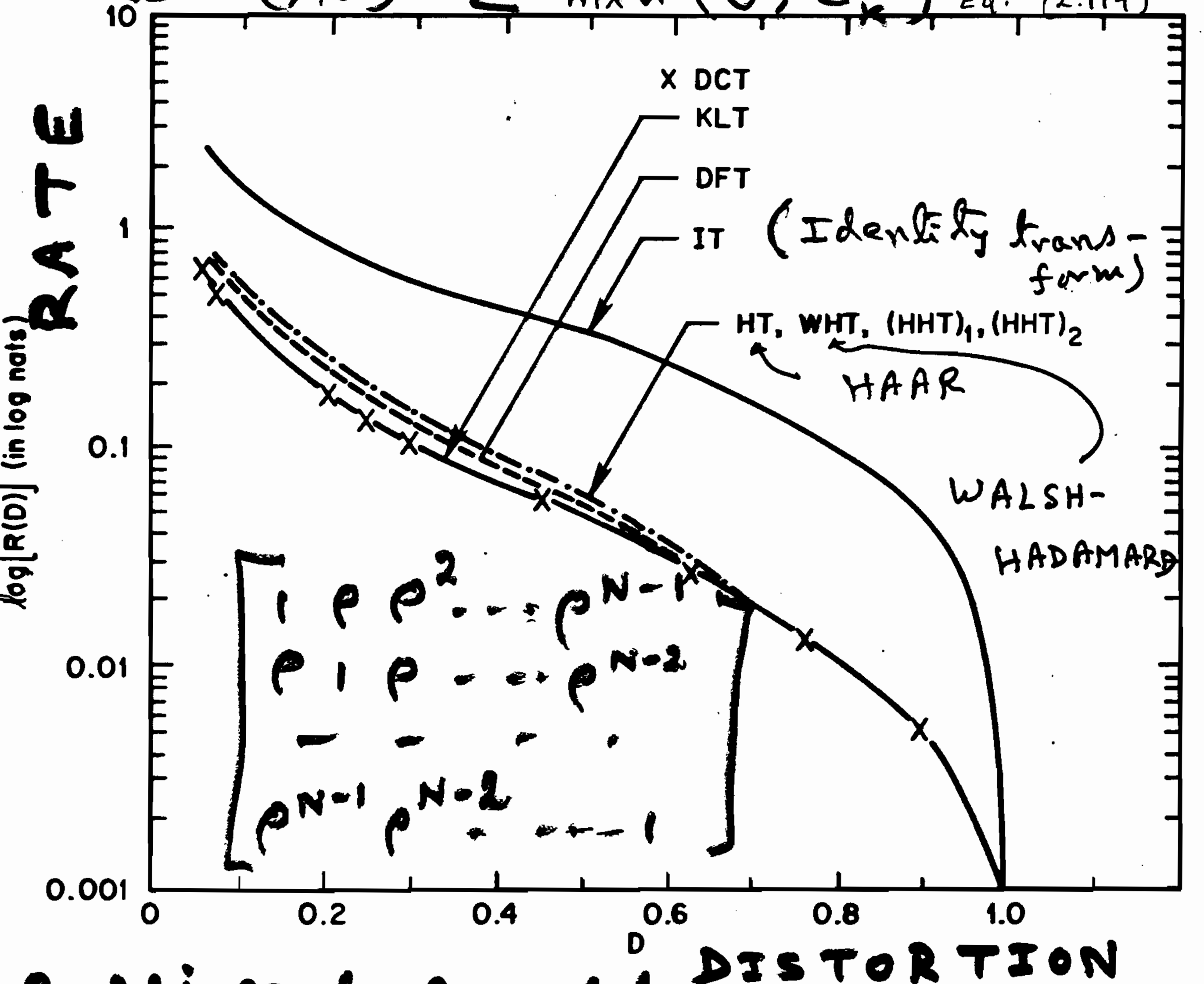
Fig. 4.26 basis restriction zonal filtering using different transforms with 4:1

GZS: Geometrical Zonal Sampling

Fig. 5.22/p. 77 174

$$R_D = (1/N)\sum_{k=0}^{N-1} \max\left[0, \frac{1}{2}\log_2\left(\sigma_k^2/\theta\right)\right] \quad \text{Eq. (2.118)}$$

$$D = (1/N)\sum \min\left(\theta, \sigma_k^2\right) \quad \text{Eq. (2.119)}$$



ρ = Adjacent Correlation coefficient

Rate versus distortion of a first-order Markov process for ρ = 0.9 and N = 16 (1 nat = 1.44 bits ; i.e. , $\log_2 e = 1.44$)

$(N = 32,\ \log_2 N = 5 \text{ bits})$, $\left[\log_e N = \dfrac{(\log_2 N)}{(\log_2 e)} \text{ NATS}\right]$

$\log_2 N = (\log_e N)(\log_2 e)$

$$R_D = \left(\frac{1}{2}\right)\log_2\left(\tilde{\sigma}^2/D\right), \quad D \le \tilde{\sigma}^2$$

$$\boxed{\text{Eq. (2.116)}} \quad = 0 \qquad D > \tilde{\sigma}^2$$

$$R_D = \max\left[0, \left(\frac{1}{2}\right)\log_2\left(\tilde{\sigma}^2/D\right)\right]$$

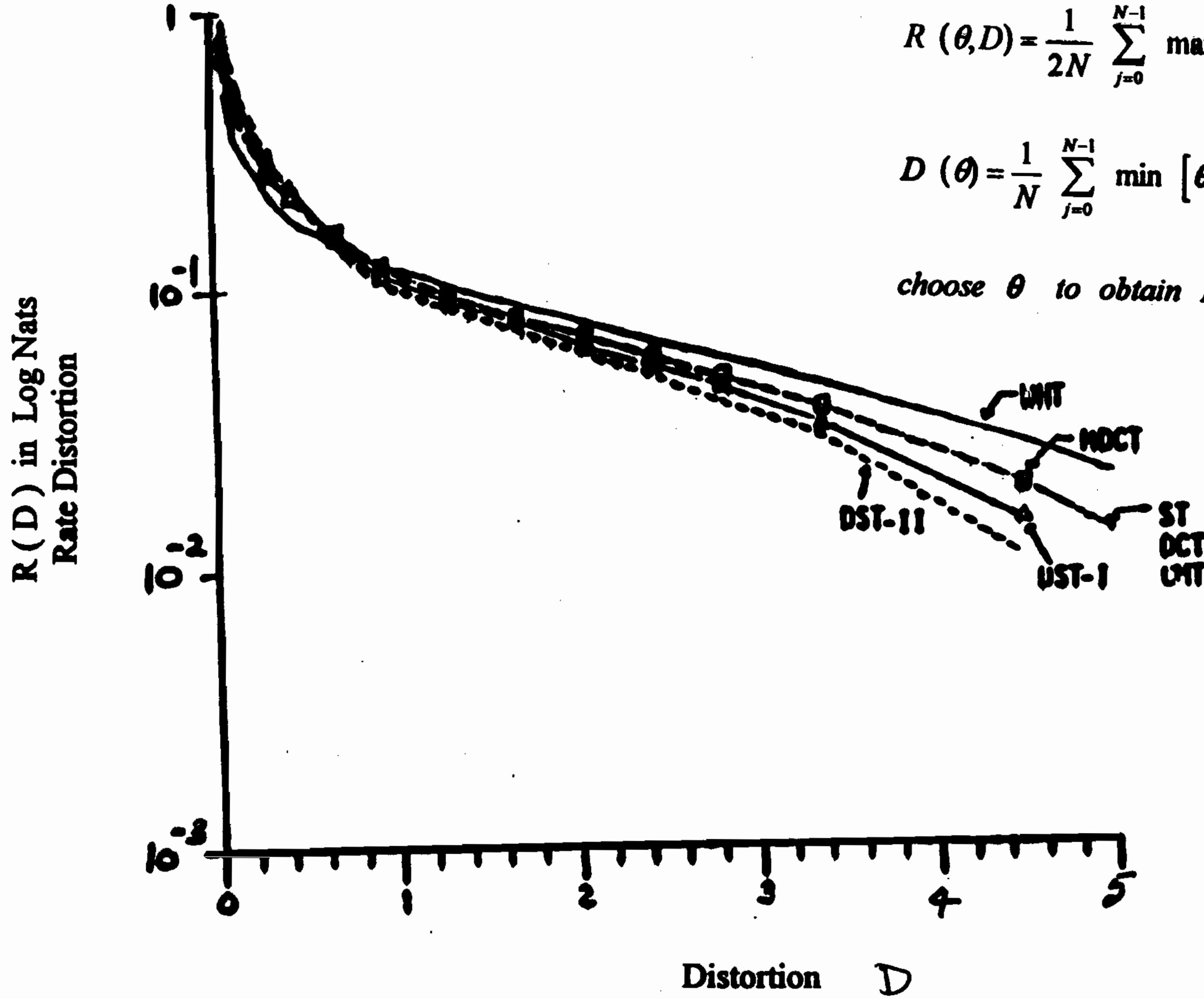
Minimum rate required to transmit a signal (bits/transform coefficient) when a maximum distortion D is allowed.

$$R\ (\theta, D) = \frac{1}{2N} \sum_{j=0}^{N-1} \max \left(0, \log_2 \frac{X^{c2}(j,j)}{\theta} \right)$$

$$D\ (\theta) = \frac{1}{N} \sum_{j=0}^{N-1} \min \left[\theta, X^{c2}(j,j) \right]$$

choose θ *to obtain* $D(\theta)$

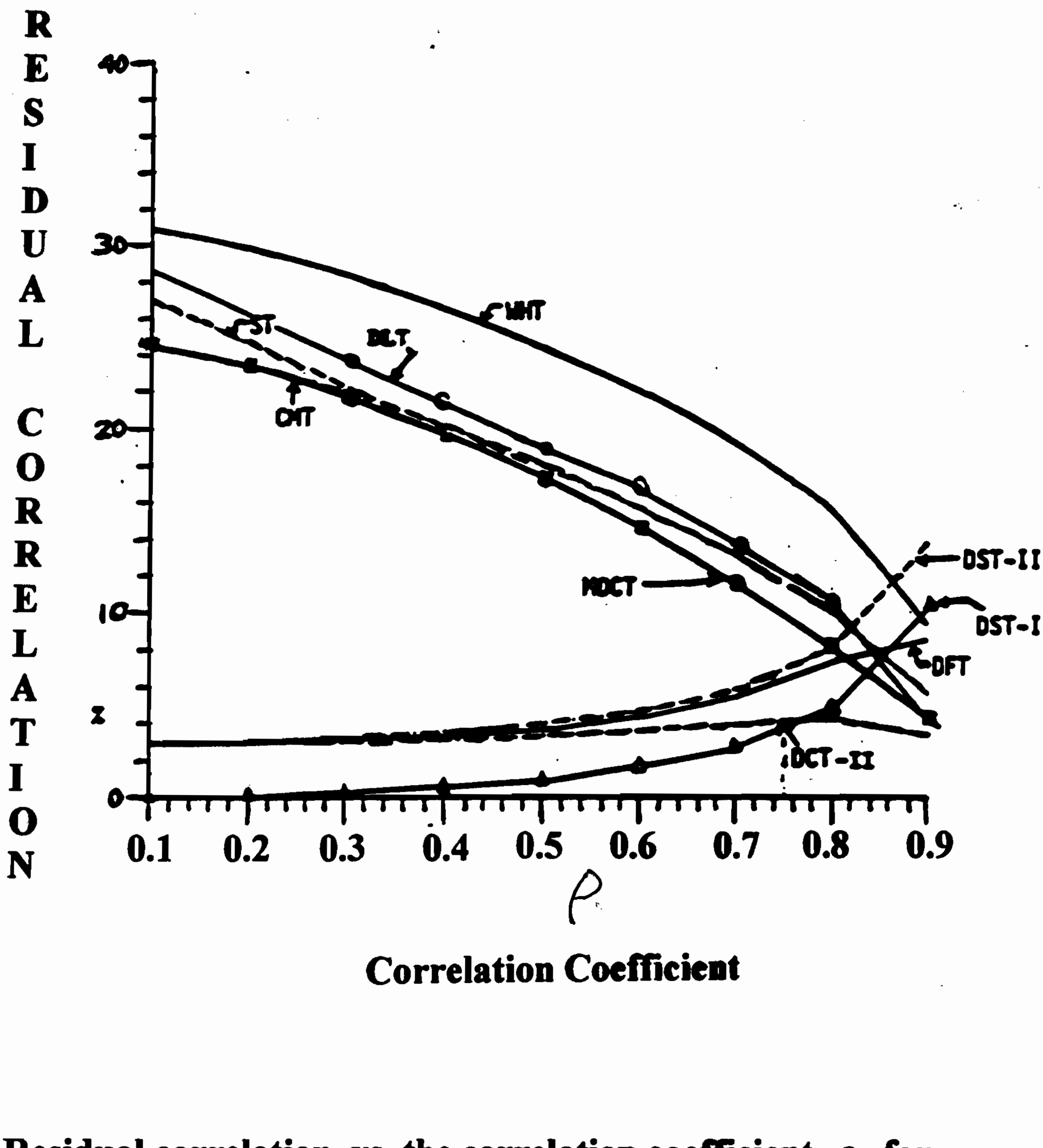
diagonal elements of $\left[X^{c2} \right]$



Rate distortion versus distortion for various discrete transforms for N = 8 and ρ = 0.9, I - order Markov Process

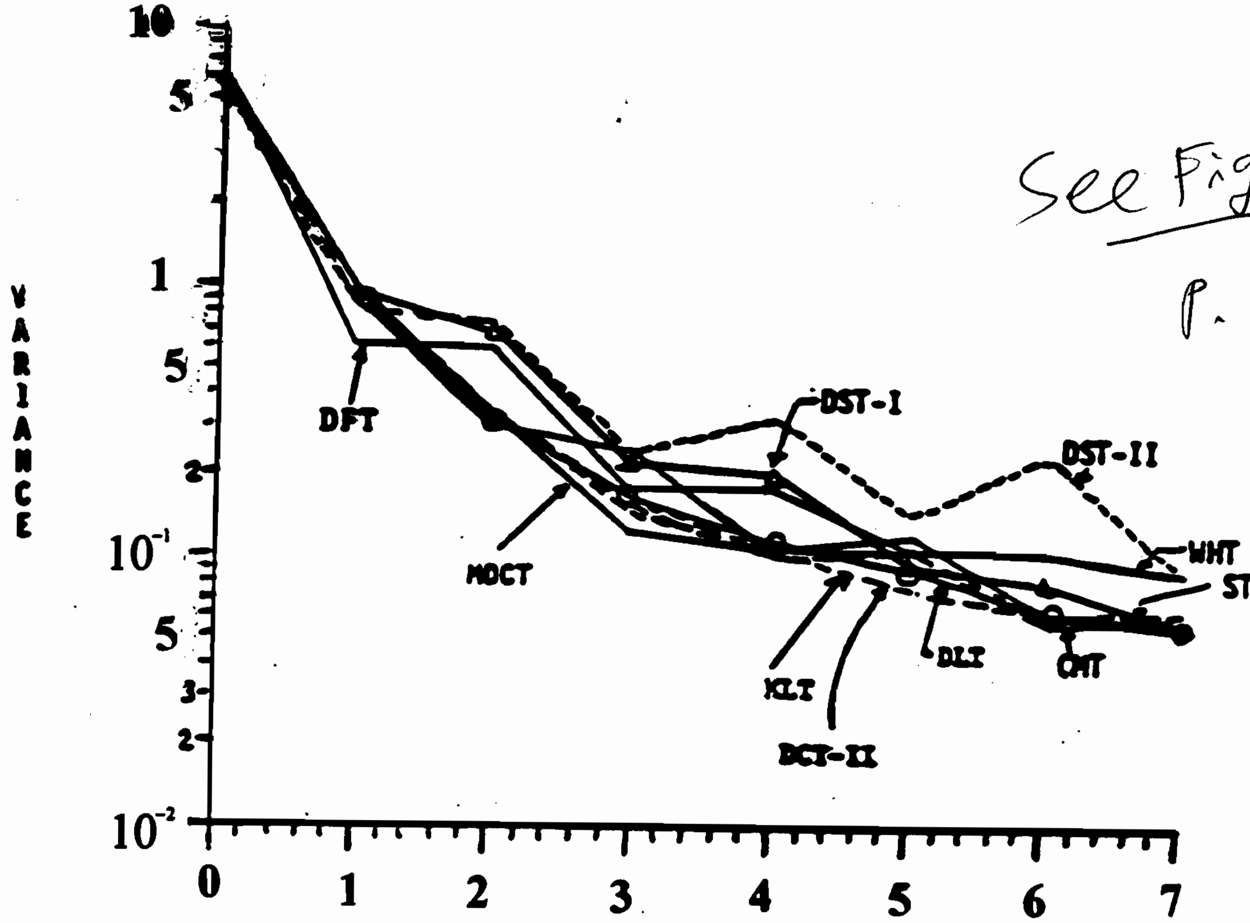
$\longrightarrow \Sigma$ = correlation matrix in data domain

$$\underset{(N \times N)}{\left[X^{c2} \right]} = \underset{(N \times N)}{[DCT]} \underset{(N \times N)}{\Sigma} \underset{(N \times N)}{[DCT]^T}$$ = correlation matrix in 2D-DCT domain



Residual correlation vs the correlation coefficient ρ for $N = 32$, I order Markov process

See Fig. 5.18
p. 170

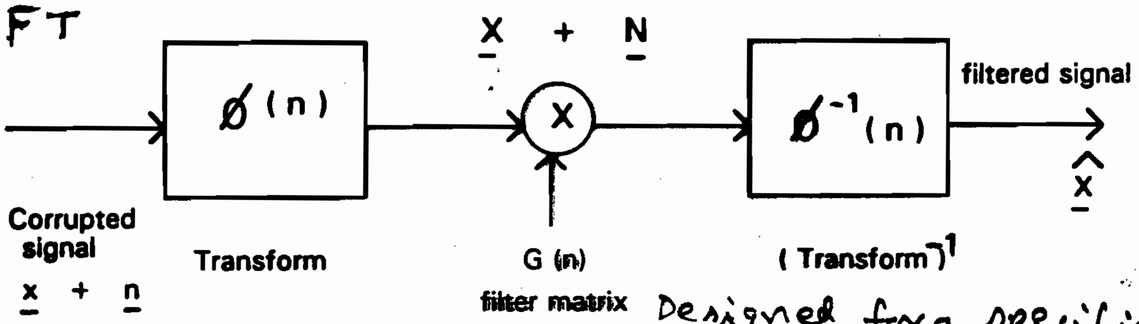


Transform Coefficient

The variance distribution for various discrete transforms for N = 8 and ρ = 0.9 , I order Markov process.

KLT
DCT
DHT
DFT

Performance Criteria

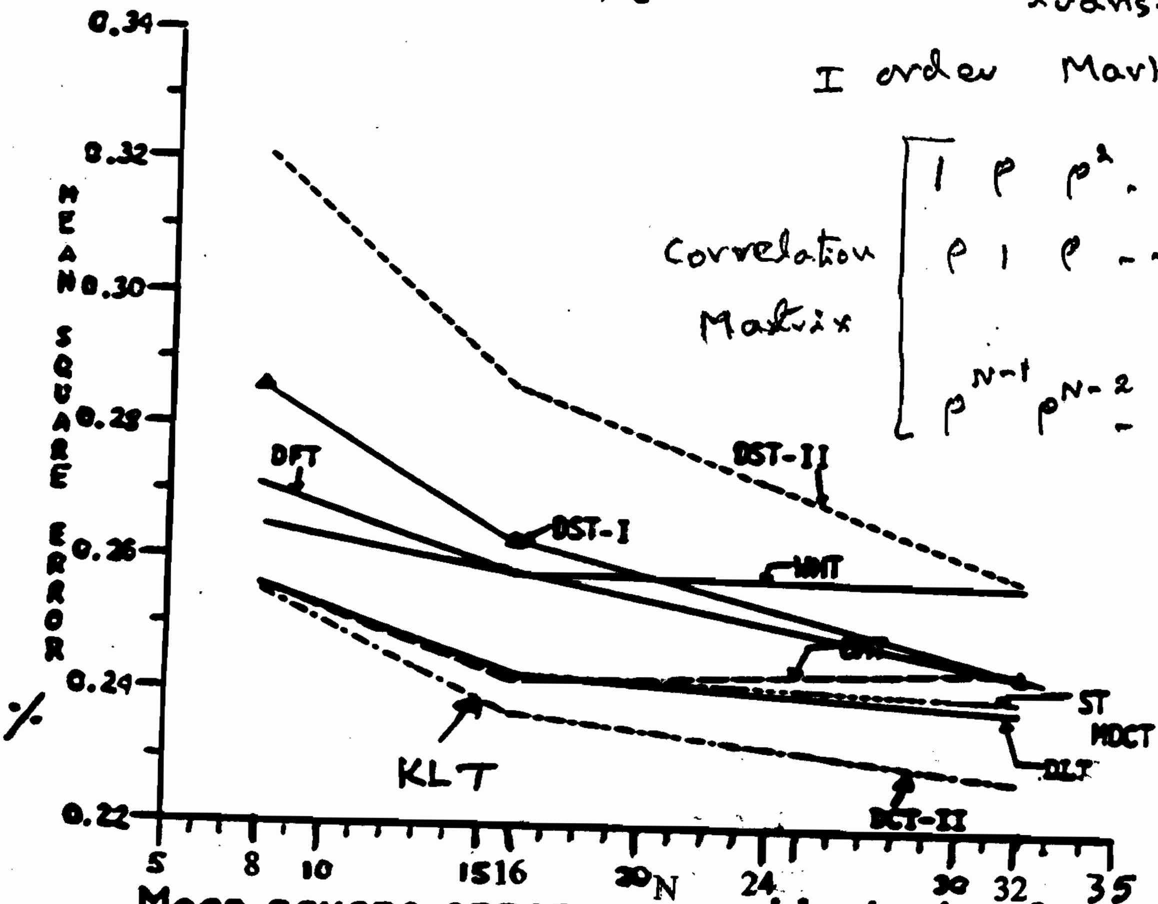


additive random noise

Designed for a specific transform

I order Markov process

Correlation Matrix
$$\begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{N-1} \\ \rho & 1 & \rho & \cdots & \rho^{N-2} \\ & & & & \\ \rho^{N-1} & \rho^{N-2} & & \cdots & 1 \end{bmatrix}$$



Mean square error versus block size for various discrete transform for $\rho = 0.9$ and SNR, $k_0 = 1$

Scalar Wiener Filtering

$\underline{x} = (x_1, x_2, x_3, \ldots x_N)^T$

Data sequence of length N

$$MSE = \| \hat{\underline{x}} - \underline{x} \|^2$$

L_2 - Norm

$$\|\underline{x}\|^2 = \sum_{i=1}^{N} x_i^2$$

L1 - NORM

$$\sum_{i=1}^{N} |x_i|$$

$$MAE = | \hat{\underline{x}} - \underline{x} | = \sum_{i=1}^{N} | \hat{x}_i - x_i |$$

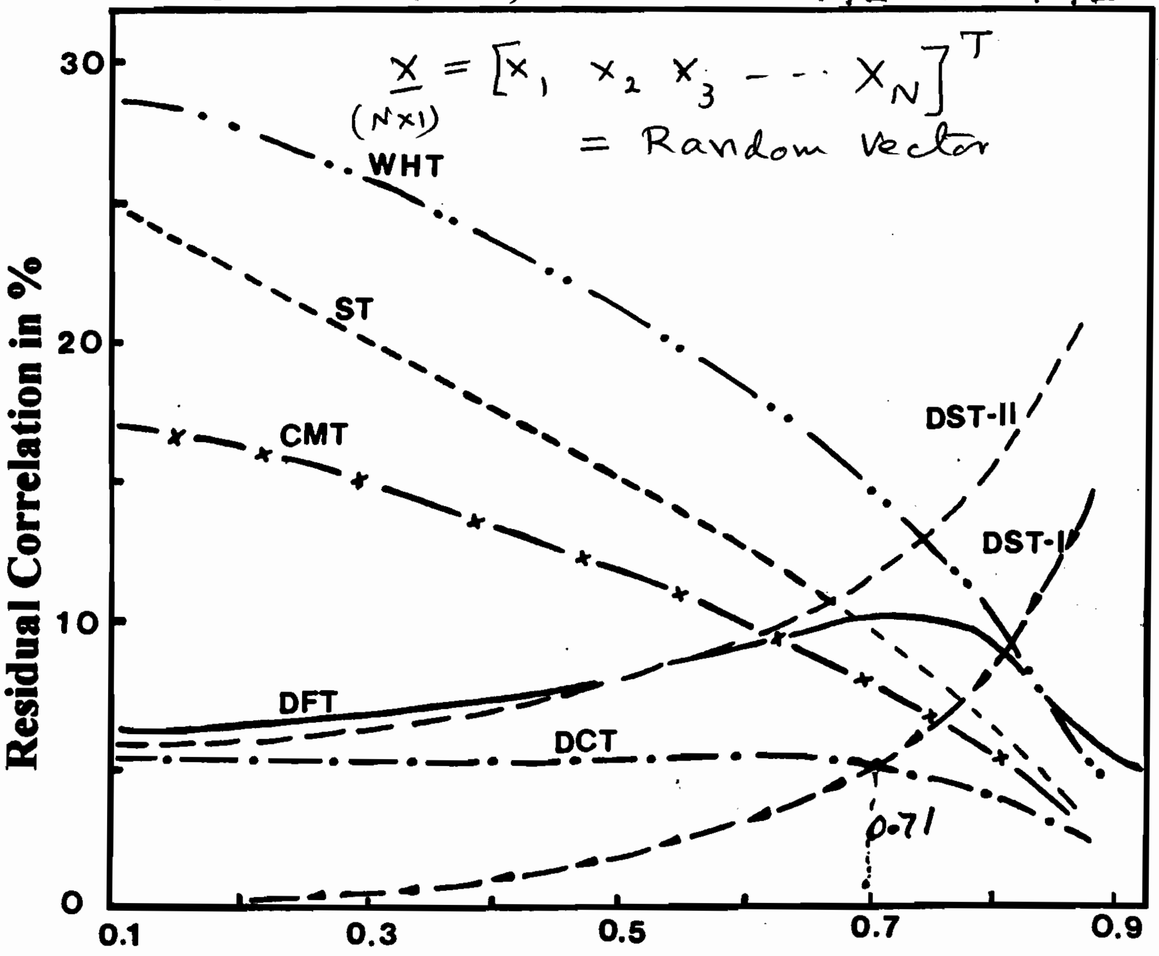
$$\left[\underline{\Phi}(n)\right]^{-1} = \left[\underline{\Phi}(n)\right]^{T}$$

$$\underset{(N\times N)}{\underline{A}} = E\left[\underset{(N\times 1)(1\times N)}{\underline{X}\,\underline{X}^{T}}\right]$$ correlation matrix (CM) in data domain

$$\underset{(N\times N)}{\underline{A}_{TR}} = \underset{(N\times N)}{\left[\underline{\Phi}(n)\right]}\left(E\left[\underline{x}\,\underline{x}^{T}\right]\right)\underset{(N\times N)}{\left[\underline{\Phi}(n)\right]^{T}},$$ CM in transform domain

$$\underset{(N\times N)}{\hat{\underline{A}}} = \underset{(N\times N)}{\left[\underline{\Phi}(n)\right]^{T}}\underset{(N\times N)}{\left[\underline{A}_{TR}\right]_{jj}}\underset{(N\times N)}{\left[\underline{\Phi}(n)\right]}$$

Fractional correlation $\left|\underline{A} - \hat{\underline{A}}\right|^{2} / \left|\underline{A} - \underline{I}_N\right|^{2}$



See P-S. Yeh, "Data compression properties of Hartley Transform", IEEE Tran. ASSP, vol. 37, pp. 450-451, March 1989.

Correlation Coefficient ρ

Diagonal matrix of variances in the transform domain $= \left[\underline{A}_{TR}\right]_{jj} = \begin{bmatrix} \tilde{\sigma}_{11}^{2} & & O \\ & \tilde{\sigma}_{22}^{2} & \\ O & & \ddots \ \tilde{\sigma}_{NN}^{2} \end{bmatrix}$

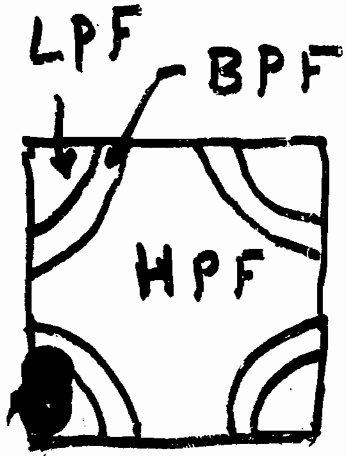
Residual Correlation vs Correlation Coefficient ρ for

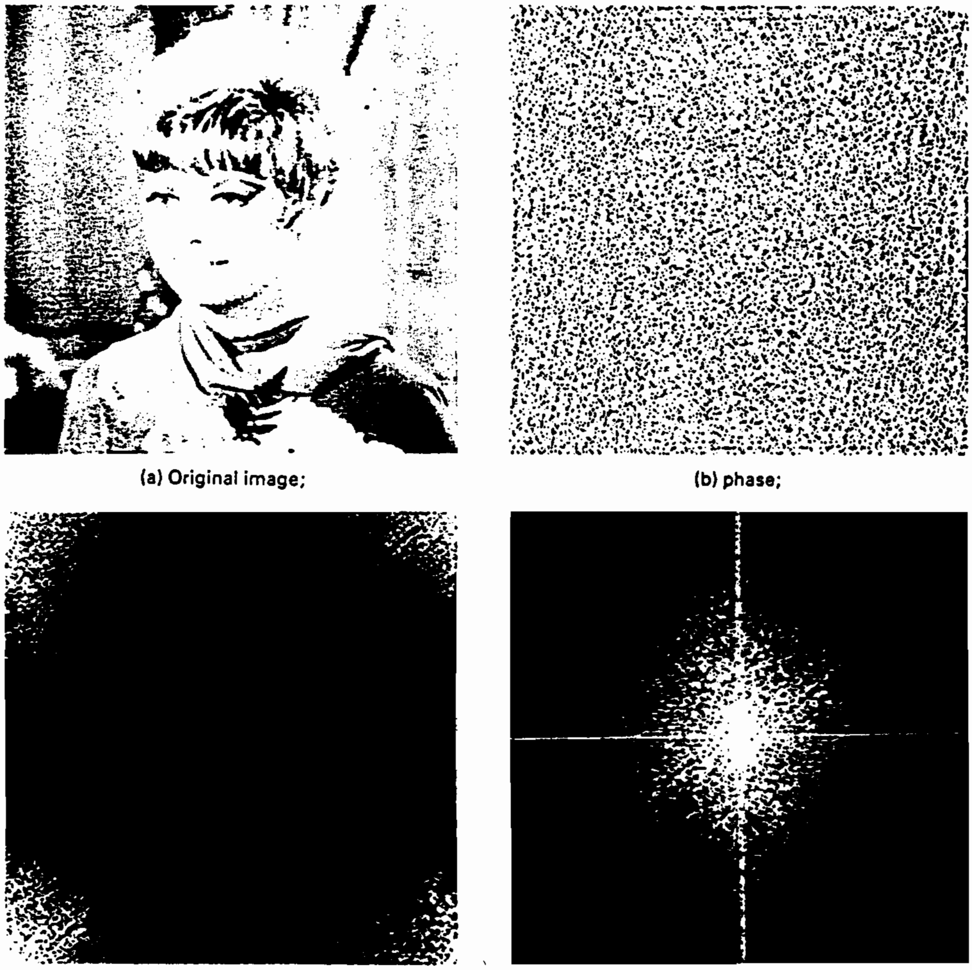
N = 16 , (I order Markov Process)

M. Hamidi and J. Pearl, "Performance Comparison of Cosine & Fourier ..." IEEE Trans. ASSP-24, vol. ASSP-24, pp. 428-429, oct. 1976.

See Z. Wang and B. R. Hunt, "The discrete cosine transform - A new version", pp. 1256-1259, ICASSP 83, Boston, MA, 1983.

(256 × 256)
8 bpp





(a) Original image; (b) phase;

(c) magnitude; (d) magnitude centered.

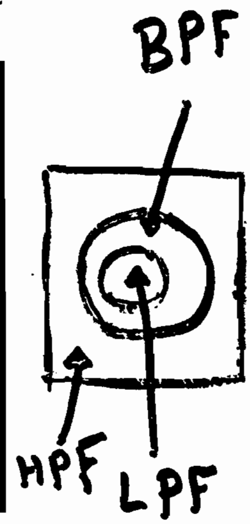
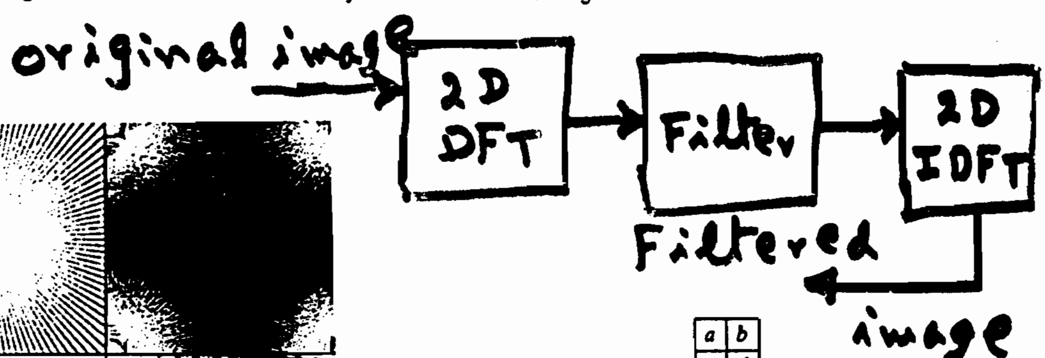


Figure 5.6 Two-dimensional unitary DFT of a 256 × 256 image.

(Fig. 5.6 / P. 146)



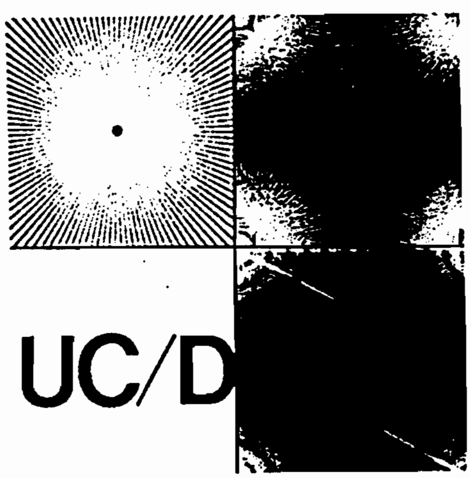


Figure 5.7 Unitary DFT of images

(a) Resolution chart;
(b) its DFT;
(c) binary image;
(d) its DFT. The two parallel lines are due to the '/' sign in the binary image.

(Fig. 5.7 / P. 147)

Table 4.1 Variances σ_k^2 of transform coefficients of a stationary Markov sequence with $\rho = 0.95$ and $N = 16$.

Transform / $\downarrow k$	KL	Cosine	Sine	Unitary DFT	Hadamard	Haar	Slant
0	12.442	12.406	11.169	12.406	12.406	12.406	12.406
1	1.946	1.943	1.688	1.100	1.644	1.644	1.904
2	0.615	0.648	1.352	0.292	0.544	0.487	0.641
3	0.292	0.295	0.421	0.139	0.431	0.487	0.233
4	0.171	0.174	0.463	0.086	0.153	0.144	0.173
5	0.114	0.114	0.181	0.062	0.152	0.144	0.172
6	0.082	0.083	0.216	0.051	0.149	0.144	0.072
7	0.063	0.063	0.098	0.045	0.121	0.144	0.072
8	0.051	0.051	0.116	0.043	0.051	0.050	0.051
9	0.043	0.043	0.060	0.045	0.051	0.050	0.051
10	0.037	0.037	0.067	0.051	0.051	0.050	0.051
11	0.033	0.033	0.040	0.062	0.051	0.050	0.051
12	0.030	0.030	0.042	0.086	0.051	0.050	0.031
13	0.028	0.028	0.031	0.139	0.051	0.050	0.031
14	0.027	0.027	0.029	0.292	0.050	0.050	0.031
15	0.026	0.026	0.026	1.100	0.043	0.050	0.031

Table 5.2/p.171

10^2

10

1

10^{-1}

10^{-2}

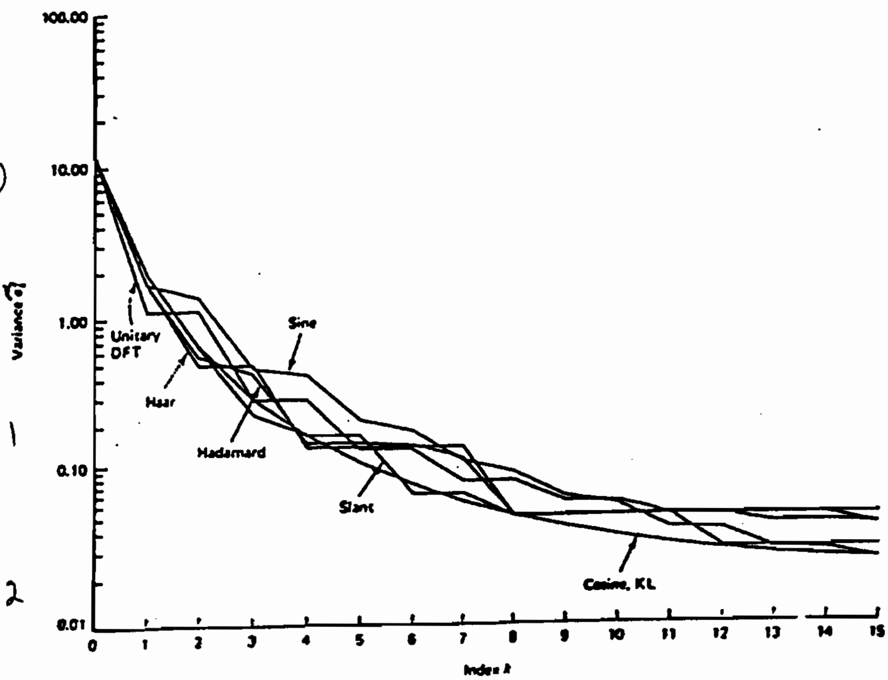


Fig. 5.18 / p. 170

Fig. 4.24 Distribution of variances of the transform coefficients (in decreasing order) a stationary Markov sequence with $N = 16, \rho = 0.95$.