The following material is mainly based on the paper, İ. Avcibaş, B. Sankur and K. Sayood, "Statistical evaluation of image quality measures," *J. Electron. Imaging*, vol. 11(2), pp. 206–223, April 2002. (Go to UTA library and access this paper online.)

Several image quality measures (see Appendix) are discussed, proposed, extended and analyzed. Their properties, characteristics and utility in evaluating image quality and their sensitivity to image coding artifacts are discussed. Specific areas where these measures are particularly useful are described in this paper. These measures are based on

A. Pixel Differences
B. Correlation
C. Edge Quality
D. Spectral Distance
E. Context
F. Human Visual System (HVS)

Each of these categories is further divided into specific measures. The 59 references at the end of this paper elaborate on most of these measures.

$C_k(i,j) \leftarrow (i,j)$ th pel of *k*th band of $\underline{C}(i,j)$, $k = 1,\cdots,K$, # of bands = K

*k*th spectral component at location (i,j)

e.g., Color image (RGB), or YIQ or Y C_R C_B

$$\underbrace{\underline{C}(i,j)}_{(3\times 1)} = \begin{bmatrix} R(i,j) \\ G(i,j) \\ B(i,j) \end{bmatrix}, \quad \underline{C} \text{ Multispectral image}$$

C_k *k*th band of multispectral image $\underline{C}$

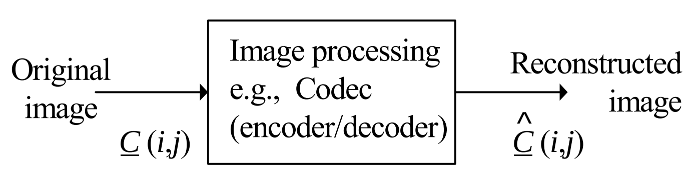
$$\underbrace{\hat{\underline{C}}(i,j)}_{(3\times 1)} = \begin{bmatrix} \hat{R}(i,j) \\ \hat{G}(i,j) \\ \hat{B}(i,j) \end{bmatrix}, \text{ processed or reconstructed multispectral image at location } (i,j)$$

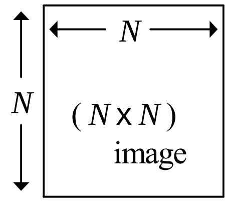
$\varepsilon_k = C_k - \hat{C}_k$, error over all the pels in the *k*th band of $\underline{C}$

Power in *k*th band $\sigma_k^2 = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} C_k^2(i,j)$

$\hat{C}_k(i,j)$, $\hat{\underline{C}}$, $\hat{\underline{C}}(i,j)$ refer to processed or reconstructed (distorted) images.

Note that $\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} = \sum_{i,j=0}^{N-1}$





Sum of the errors in all K bands at pel (i, j)

$$\left\| \underline{C}(i,j) - \hat{\underline{C}}(i,j) \right\|^2 = \sum_{k=1}^{K} \left[C_k(i,j) - \hat{C}_k(i,j) \right]^2$$

(Square of the error in the kth band of pel (i, j))

(K = # of bands, $k = 1, 2, \cdots, K$)

$$\varepsilon_k^2 = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \left[C_k(i,j) - \hat{C}_k(i,j) \right]^2$$

A.1.1 Minkowsky metrics

L_r norm

$$\begin{aligned} \varepsilon^r &= \frac{1}{K} \sum_{k=1}^{K} \left[\frac{1}{N^2} \sum_{i,j=0}^{N-1} \left| C_k(i,j) - \hat{C}_k(i,j) \right|^r \right]^{\frac{1}{r}} \\ &= \frac{1}{N^2} \left(\sum_{i,j=0}^{N-1} \left[\frac{1}{K} \sum_{k=1}^{K} \left| C_k(i,j) - \hat{C}_k(i,j) \right| \right]^r \right)^{\frac{1}{r}} \end{aligned} \qquad \text{(A1)}$$

MSE

$$\begin{aligned} D1 &= \frac{1}{K} \left(\frac{1}{N^2} \sum_{i,j=0}^{N-1} \left\| \underline{C}(i,j) - \hat{\underline{C}}(i,j) \right\|^2 \right) \\ &= \frac{1}{K} \sum_{k=0}^{K} \varepsilon_k^2 \end{aligned} \qquad \text{(A2)}$$

Note here that $\underline{C}$ & $\hat{\underline{C}}$ are vectors whose components are the bands.

$$D1 = \frac{1}{N^2} \sum_{i,j=0}^{N-1} \frac{1}{K} \sum_{k=1}^{K} \left[C_k(i,j) - \hat{C}_k(i,j) \right]^2 \qquad \text{(A2)}$$

SNR & PSNR (Signal-to-noise ratio and peak-to-peak SNR)

For $r = 1$,

$$\varepsilon^1 = \frac{1}{K} \sum_{k=1}^{K} \left(\frac{1}{N^2} \sum_{i,j=0}^{N-1} \left| C_k(i,j) - \hat{C}_k(i,j) \right| \right)$$

Maximum difference or infinity norm

$$\varepsilon^{\infty} = \max_{i,j}\left(\sum_{k=1}^{K}\frac{1}{K}\left|C_k(i,j) - \hat{C}_k(i,j)\right|\right)$$
$$= \max_{i,j}\left(\left\|\underline{C}(i,j) - \hat{\underline{C}}(i,j)\right\|\right)$$

Ranked list of pel difference

$\Delta_l\left(\underline{C} - \hat{\underline{C}}\right)$, $\quad l = 1, 2, \cdots, N^2$, N^2 = # of pels in an $(N \times N)$ image

$\Delta_l\left(\underline{C} - \hat{\underline{C}}\right)$ = lth largest deviation among all pels.

Define

$$D3 = \sqrt{\frac{1}{r}\sum_{m=1}^{r}\Delta_m^2\left(\underline{C} - \hat{\underline{C}}\right)} \qquad \text{(A3)}$$

RMS value of the ranked largest differences $\Delta_1, \Delta_2, \cdots, \Delta_r$
(Root Mean Square)

Several other measures are described in A (Measures based on pel differences)

B. Correlation-based measures

B.1 Image Correlation Measures

Structural content

$$C1 = \frac{1}{K}\sum_{k=1}^{K}\left[\frac{\sum_{i,j=0}^{N-1} C_k^2(i,j)}{\sum_{i,j=0}^{N-1} \hat{C}_k^2(i,j)}\right] \qquad \text{(A7)}$$

Normalized cross correlation measure

$$C2 = \frac{1}{K}\sum_{k=1}^{K}\left[\frac{\sum_{i,j=0}^{N-1} C_k(i,j)\hat{C}_k(i,j)}{\sum_{i,j=0}^{N-1} C_k^2(i,j)}\right] \qquad \text{(A8)}$$

Czenakowski (coefficient) distance for non-negative components

$$C3 = \frac{1}{N^2} \sum_{i,j=0}^{N-1} \left(1 - \frac{2\sum_{k=1}^{K} \min\left[C_k(i,j), \hat{C}_k(i,j) \right]}{\sum_{k=1}^{K} \left[C_k(i,j) + \hat{C}_k(i,j) \right]} \right) \qquad \text{(A9)}$$

This coefficient measures the similarity among different samples, components and quadrates. Several other measures and their effectiveness as outlined on page 1 are described in detail.

B.1.2 Moments of the angles

Combined angular correlation and magnitude difference between two vectors $\underline{C}$ and $\hat{\underline{C}}$

$$\chi_{ij} = 1 - \left(1 - \left(\frac{2}{\pi}\right) \cos^{-1} \left[\frac{\left\langle \underline{C}(i,j), \hat{\underline{C}}(i,j) \right\rangle}{\left(\left\| \underline{C}(i,j) \right\| \right) \left(\left\| \hat{\underline{C}}(i,j) \right\| \right)} \right] \right) \times \left[1 - \frac{\left\| \underline{C}(i,j) - \hat{\underline{C}}(i,j) \right\|}{\sqrt{(3) \times (255)^2}} \right]$$

$\frac{2}{\pi}$ is the normalization factor.

3: For three components (R,G,B)

255: (255 for 8 bit PCM)

As both $\underline{C}$ and $\hat{\underline{C}}$ are positive vectors, we are constrained to I quadrant of Cartesian space.

Max difference attained is $\frac{\pi}{2}$.

$$\left\langle \underline{C}(i,j), \hat{\underline{C}}(i,j) \right\rangle = \sum_{k=1}^{K} C_k(i,j) \hat{C}_k(i,j)$$

Inner or dot or scalar product

$$\left\| \underline{C}(i,j) \right\| = \left[\sum_{k=1}^{K} C_k^2(i,j) \right]^{\frac{1}{2}}$$

Moments of the spectral (Chromatic) vector difference as distortion measures

$$C4 = \mu_\chi = 1 - \left(\frac{1}{N^2} \right) \sum_{i,j=0}^{N-1} \left[\frac{2}{\pi} \cos^{-1} \left(\frac{\left\langle \underline{C}(i,j), \hat{\underline{C}}(i,j) \right\rangle}{\left\| \underline{C}(i,j) \right\| \left\| \hat{\underline{C}}(i,j) \right\|} \right) \right]$$

= Mean of the angle difference

$$C5 = \frac{1}{N^2} \sum_{i,j=0}^{N-1} \chi_{ij}$$

mean of the combined angle-magnitude difference. These moments have been used to assess the directional correlation among color vectors.

D. Spectral Distance Measures

D.1 Define $W_N = \exp\left(\frac{-j2\pi}{N}\right)$ = Nth root of unity

$$\Gamma_k(u,v) = \sum_{m,n=0}^{N-1} C_k(m,n) W_N^{mu} \; W_N^{nv}, \quad k = 1,2,\cdots,K \quad , (u,v = 0,1,\cdots,N-1)$$

$= $ 2D-DFT of $C_k(m,n)$

$$\hat{\Gamma}_k(u,v) = \sum_{m,n=0}^{N-1} \hat{C}_k(m,n) W_N^{mu} \; W_N^{nv}, \quad (u,v = 0,1,\cdots,N-1) \;, \quad k = 1,2,\cdots,K$$

$= $ 2D-DFT of $\hat{C}_k(m,n)$

<u>Phase spectra</u>

$\phi(u,v) = \arctan\left[\Gamma(u,v)\right]$ | $\hat{\phi}(u,v) = \arctan\left[\hat{\Gamma}(u,v)\right]$

Magnitude spectra

$$M(u,v) = \left|\left[\Gamma(u,v)\right]\right|, \quad \hat{M}(u,v) = \left|\left[\hat{\Gamma}(u,v)\right]\right|$$

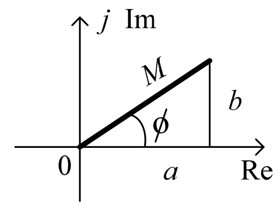
Spectral magnitude distortion

$$S = \frac{1}{N^2} \sum_{u,v=0}^{N-1} \left(\left|M(u,v) - \hat{M}(u,v)\right|\right)^2$$

Spectral phase distortion

$$S1 = \frac{1}{N^2} \sum_{u,v=0}^{N-1} \left[\left|\phi(u,v) - \hat{\phi}(u,v)\right|^2\right]$$

$a + jb$



Magnitude

$$M = \sqrt{a^2 + b^2} = |a + jb|$$

Phase

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

Weighted spectral distortion

$$S2 = \frac{1}{N^2}\left[\lambda\left(\sum_{u,v=0}^{N-1}\left|\phi(u,v) - \hat{\phi}(u,v)\right|^2\right) + (1-\lambda)\left(\sum_{u,v=0}^{N-1}\left|M(u,v) - \hat{M}(u,v)\right|^2\right)\right]$$

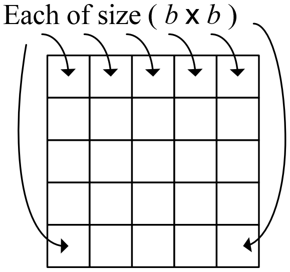
λ is chosen to match commensurate weights to the phase and magnitude terms.

Let 2D-DFT of lth block of kth band image $C_k^l(m,n)$ be

$$\Gamma_k^l(u,v) = \sum_{m,n=0}^{b-1} \left(C_k^l(m,n) W_b^{mu} \; W_b^{nv}\right)$$

$$W_b = \exp\left(\frac{-j2\pi}{b}\right)$$

Each of size (b x b)



$$\Gamma_k^l(u,v) = \left|\Gamma_k^l(u,v)\right| e^{j\phi_k^l(u,v)}$$
$$= m_k^l(u,v) e^{j\phi_k^l(u,v)}$$

($u,v = 0,1,\cdots,b-1$, $l = 1,2,\cdots,L$)

L is # of $(b\times b)$ size overlapping or non-overlapping blocks

$$J_M^l = \frac{1}{K}\left(\sum_{k=1}^{K}\left\{\sum_{u,v=0}^{b-1}\left[\left|\Gamma_k^l(u,v)\right| - \left|\hat{\Gamma}_k^l(u,v)\right|\right]^r\right\}^{\frac{1}{r}}\right)$$

$$J_\varphi^l = \frac{1}{K}\sum_{k=1}^{K}\left(\sum_{u,v=0}^{b-1}\left[\left|\phi_k^l(u,v)\right| - \left|\hat{\phi}_k^l(u,v)\right|\right]^r\right)^{\frac{1}{r}}$$

$$J^l = \lambda J_M^l + (1-\lambda) J_\varphi^l$$

λ relative weighting factor of the magnitude and phase spectra

Various rank order operations of the block spectral difference J_M and/or J_φ can be useful.

Let $J^{(1)}, J^{(2)}, \cdots, J^{(L)}$ be the rank ordered block distortions such that, $J^{(L)} = \max_l\left(J^l\right)$

Consider rank order averages,

Median block distortion,

Average block distortion, $\frac{1}{2}\left[J^{\frac{L}{2}} + J^{\left(L+\frac{1}{2}\right)}\right]$

Maximum block distortion, $J^{(L)}$

Average block distortion, $\frac{1}{L}\left(\sum_{i=1}^{L} J^{(i)}\right)$

$S3 = \text{median}_l\, J_m^l$ ← (Most effective average of rank ordered block spectral distortion)

$S4 = \text{median}_l\, J_\phi^l$

$S5 = \text{median}_l\, J^l$

(use $r = 2$), block sizes (32×32) & (64×64) ← yield better results

F. HVS Based Measures (Human Visual System)

F.1 HVS Modified Spectral Distortion

Both the original and coded images can be preprocessed that simulate the HVS. e.g., BPF with a transfer function in polar form [54]

[54] N. B. Nill "A visual model weighted cosine transform for image compression and quality assessment," *IEEE Trans. Commun*. vol. 33, pp. 551–557, June 1985.

One of the models for the HVS is given as bandpass filter (BPF) with a transfer function in polar coordinates as

$$H(\rho) = \begin{cases} 0.05 e^{\rho^{0.554}} & , \ \rho < 7 \\ e^{-9\left[\left|\log_{10}\rho - \log_{10} 9\right|\right]^{2.3}} & , \ \rho \geq 7 \end{cases}$$

$$\rho = \left(u^2 + v^2\right)^{\frac{1}{2}}$$

Image processed through spectral mask and 2D-IDCT is applied

$$U\left[C(i,j)\right] = \mathrm{DCT}^{-1}\left[H\left(\sqrt{u^2+v^2}\right)\Omega(u,v)\right]$$

$\Omega(u,v)$ = 2D-DCT of image

Possible HVS based measures
(Normalized absolute error for *K* component multispectral image)

$$H1 = \frac{1}{K}\sum_{k=1}^{K} \frac{\sum_{i,j=0}^{N-1}\left|U\left[C_k(i,j)\right] - U\left[\hat{C}_k(i,j)\right]\right|}{\sum_{i,j=0}^{N-1}\left|U\left[C_k(i,j)\right]\right|}$$

Normalized absolute error (A23)

<u>L2 norm</u>

$$H2 = \frac{1}{K}\sum_{k=1}^{K}\left[\frac{1}{N^2}\sum_{i,j=0}^{N-1}\left|U\left[C_k(i,j)\right] - U\left[\hat{C}_k(i,j)\right]\right|^2\right]^{\frac{1}{2}} \qquad \text{(A24)}$$

See also DCTune [56]
(Technique for optimizing JPEG still image compression)

[56] A. B. Watson, "DCTune: A technique for visual optimization of DCT quantization matrices for individual images," *Society for Image Display (SID) Digest XXIV*, pp. 946–949, 1993. (http://vision.arc.nasa.gov/dctune/dctune2.0.html)

Video quality metrics and subjective quality assessment

[1] Z. Wang and A. C. Bovik, *Modern Image Quality Assessment*. Morgan & Claypool, 2006 (154 References)

[2] H. Wu and K. R. Rao (Ed.), *Digital Video Image Quality and Perceptual Coding*. Boca Raton, FL: CRC Press, 2006. (Several references at the end of each chapter)

[3] ITU-R Rec. BT 500–11, "Methodology for the Subjective Assessment of Quality for Television Pictures," June 2002.

[4] VQEG (Video Quality Experts Group) (March 2000) [Online]. Available: `http://www.vqeg.org`

[5] SSIM (Structural SIMilarity) index [Online]. Available: `http://www.cns.nyu.edu/~lcv/ssim`

[6] `http://live.ece.utexas.edu/research/quality`

Z. Wang *et al*., "Image quality assessment: from error visibility to structural similarity," *IEEE Trans. Image Processing*, vol. 13, pp. 600–612, April 2004.

A. C. Bovik (Ed.), The Handbook of Image and Video Processing. II Edition, Burlington MA: Elsevier, 2005.