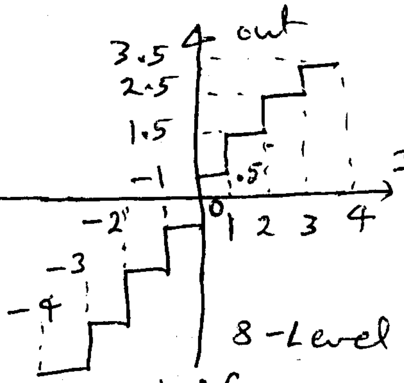

8-Level
Uniform quantizer

EE 5359 Video Coding

K. R. Rao

Test 2 Summer 2001

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3:30 – 5:00 PM

1. Suppose we have a source that can be modeled as a random variable taking values in the interval [-4,4] with more probability mass near the origin than away from it. We want to quantize this using the companding quantizer which composes of compressor and 3 bit uniform quantizer. The compressor characteristic we will use is given by the following equation:

$$c(x) = \begin{cases} 2x & \text{if } -1 \le x \le 1 \\ \dfrac{2x}{3} + \dfrac{4}{3} & x > 1 \\ \dfrac{2x}{3} - \dfrac{4}{3} & x < -1 \end{cases}$$

The mapping is shown graphically in figure 1. The inverse mapping is given by

$$c^{-1}(x) = \begin{cases} \dfrac{x}{2} & \text{if } -2 \le x \le 2 \\ \dfrac{3x}{2} - 2 & x > 2 \\ \dfrac{3x}{2} + 2 & x < -2 \end{cases}$$

The inverse mapping is shown graphically in figure 2. For this companding quantizer, what are the outputs for the following inputs: -0.8, 1.2, 0.5, 0.6, 3.2, -0.3. Compare your results with the case when the input is directly quantized with a uniform quantizer with the same number of levels. Comment on your results.

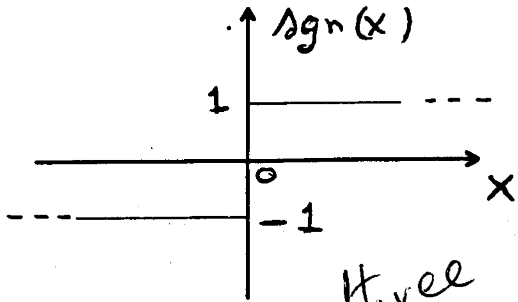
2. Two companding characteristics that are widely used today are μ-law companding and A-law companding. The μ-law companding expander function is given by

$$c^{-1}(x) = \frac{x_{max}}{\mu}\left[(1+\mu)^{\frac{|x|}{x_{max}}} - 1\right]\operatorname{sgn}(x).$$

Show that its compressor function is SHOW ALL THE STEPS

$$c(x) = x_{max}\frac{\ln\left(1+\mu\frac{|x|}{x_{max}}\right)}{\ln(1+\mu)}\operatorname{sgn}(x).$$

where μ is a constant, x is a input value and sgn(x) is the function of the form.



3. We want to DPCM-encode images using a ~~two~~ three-tap predictor of the form

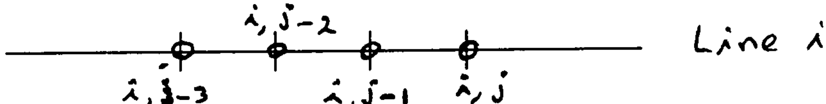
$$\hat{x}_{i,j} = a_1 \times x_{i,j-1} + a_2 \times x_{i,j-2} + a_3 x_{i,j-3}$$

and a four-level quantizer followed by a Huffman coder. Find the equations we need to solve to obtain coefficients a_1, a_2 and a_3 that minimize the mean squared error or

$$MSE = E[(x_{i,j} - \hat{x}_{i,j})^2]$$

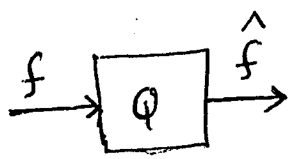
and autocorrelation is

$$R_{xx}(k) = E[x_n x_{n+k}] = E[x_n x_{n-k}]$$

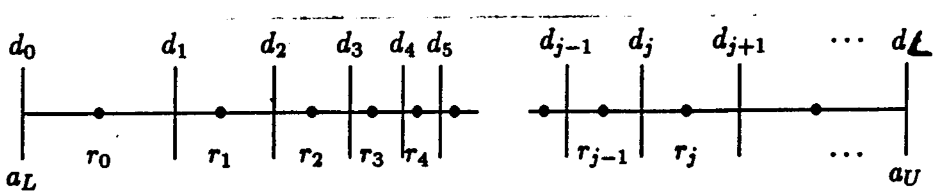


4. Lloyd-Max quantizer is a quantizer that is designed to minimize mean square quantization error (MSQE) or

$$MSQE = E[(f - \hat{f})^2] = \int_{a_L}^{a_U} (f - \hat{f})^2 p(f) df,$$



where $p(f)$ is probability density function of random varible f. The boundary of inputs are specified as a_L and a_U, number of levels is L.
Derive the constraints for decision level (input level), d_i and reconstruction level (output level), r_i of the Lloyd-Max quantizer.



d_i = decision level (input level)
r_i = reconstruction level (output level)

COMPRESSOR

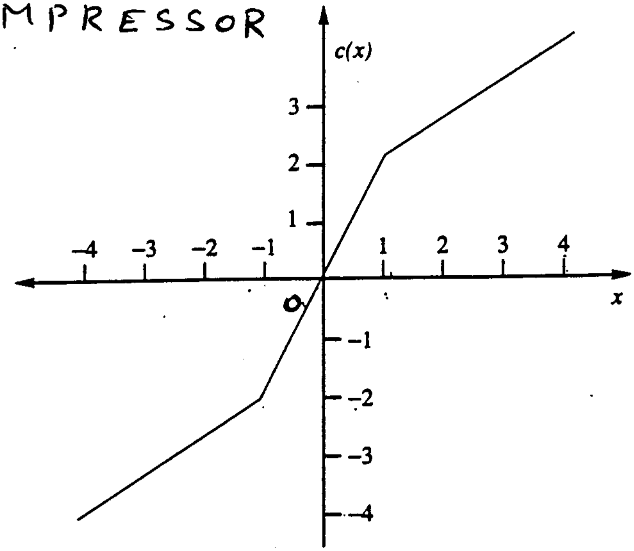


FIGURE 1.

EXPANDER

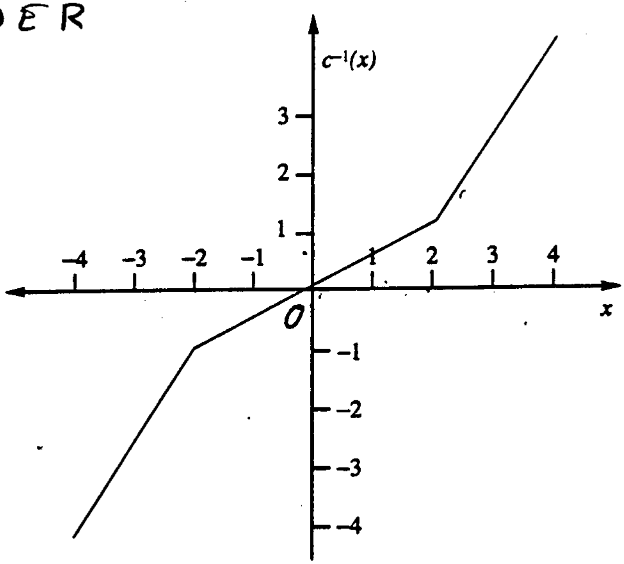


FIGURE 2.