EE 5359 Solutions to Test 1 (2-17-97)

Spring 1997

1. $A = \{a_1, a_2, a_3, a_4, a_5\}$

$P(a_1) = 0.15$, $P(a_2) = 0.04$, $P(a_3) = 0.26$, $P(a_4) = 0.05$

$P(a_5) = 0.50$

a.) $H = -\sum_{i=1}^{5} P(a_i) \log_2(P(a_i))$

$= -[0.15 \log_2 0.15 + 0.04 \log_2 0.04 + 0.26 \log_2 0.26$
$+ 0.05 \log_2 0.05 + 0.5 \log_2 0.5]$

$= 1.817$ bits/symbol

b.)

$0.50 \leftarrow a_5$	$0.50 \leftarrow a_5$	$0.50 \leftarrow a_5$	$0.50 \leftarrow a_5$ 0
$0.26 \leftarrow a_3$	$0.26 \leftarrow a_3$	$0.26 \leftarrow a_3$ 0	$0.50 \leftarrow a_3'$ 1
$0.15 \leftarrow a_1$	$0.15 \leftarrow a_1$ 0	$0.24 \leftarrow a_1'$ 1	
$0.05 \leftarrow a_4$ 0	$0.09 \leftarrow a_4'$ 1		
$0.04 \leftarrow a_2$ 1			

HUFFMAN CODE

a_5	0
a_3	10
a_1	110
a_4	1110
a_2	1111

c.) $L = \sum_{i=1}^{5} P(a_i) n(a_i)$

$= 0.50(1) + 0.26(2) + 0.15(3) + 0.05(4) + 0.04(4)$

$= 1.83$ bits/symbol

REDUNDANCY $= L - H = 0.013$ bits/symbol

2. $A = \{a_1, a_2, a_3, a_4\}$

$P(a_1) = 0.1$, $P(a_2) = 0.3$, $P(a_3) = 0.25$, $P(a_4) = 0.35$

a.)

$0.35 \leftarrow a_4$	$0.35 \leftarrow a_4$	$0.65 \leftarrow a_3''$ 0
$0.3 \leftarrow a_2$	$0.35 \leftarrow a_3'$ 0	$0.35 \leftarrow a_4$ 1
$0.25 \leftarrow a_3$ 0	$0.3 \leftarrow a_2$ 1	
$0.1 \leftarrow a_1$ 1		

HUFFMAN CODE

a_4	1
a_2	01
a_3	000
a_1	001

b.)

$0.35 \leftarrow a_4$	$0.35 \leftarrow a_3'$	$0.65 \leftarrow a_4'$ 0
$0.3 \leftarrow a_2$	$0.35 \leftarrow a_4$ 0	$0.35 \leftarrow a_3'$ 1
$0.25 \leftarrow a_3$ 0	$0.3 \leftarrow a_2$ 1	
$0.1 \leftarrow a_1$ 1		

HUFFMAN CODE

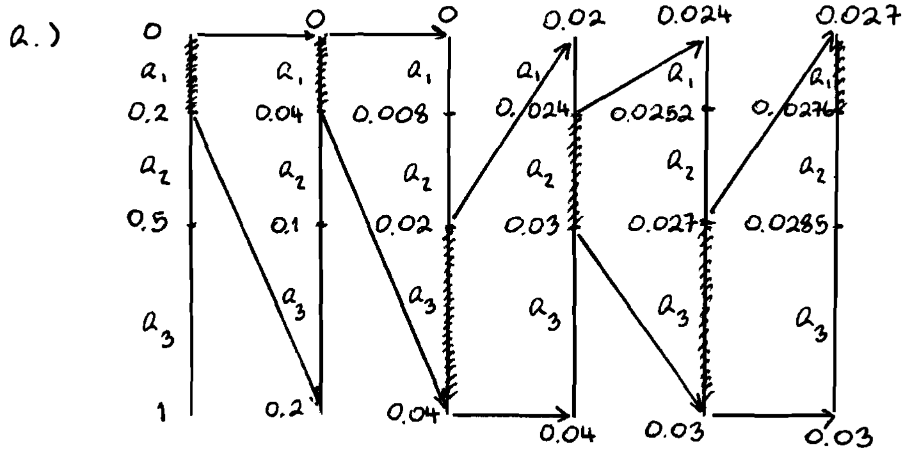
a_4	00
a_2	01
a_3	10
a_4	11

ALTHOUGH METHOD A AND B HAVE THE SAME AVERAGE LENGTH OF THE CODE, CODE OF METHOD B HAS A LOWER VARIANCE (CONSTANT LENGTH).

3. $A : \{a_1, a_2, a_3\}$

$P(a_1) = 0.2$, $P(a_2) = 0.3$, $P(a_3) = 0.5$

a.)



$$\text{TAG} = \frac{0.0276 - 0.027}{2} = 0.0273_{10} \quad \text{(DECIMAL)}$$

$$= (0.0000011\ldots)_2 \quad \text{(BINARY)}$$

THE FIRST 5 BITS BINARY CODE '00000'

b.) $F(a_0) = 0$, $F(a_1) = 0.2$, $F(a_2) = 0.5$, $F(a_3) = 1$

TAG = 0.63215699

$$l^{(1)} = 0 + (1-0)F(x_1 - 1) = F(x_1 - 1)$$

$$u^{(1)} = 0 + (1-0)F(x_1) = F(x_1)$$

TAG LIES IN $[0.5, 1)$, THE FIRST SYMBOL IS a_3

$$l^{(2)} = 0.5 + (1-0.5)F(x_2 - 1) = 0.5 + 0.5F(x_2 - 1)$$

$$u^{(2)} = 0.5 + (1-0.5)F(x_2) = 0.5 + 0.5F(x_2)$$

AFTER SCALING, TAG IS 0.26431398

TAG LIES IN $[0.2, 0.5)$, THE SECOND SYMBOL IS a_2

CONTINUE THE PROCESS.