6:00 – 7:30 PM

DIGITAL VIDEO CODING
TEST # 1
DATE : - 6/2/04
INSTRUCTOR: - Prof. K.R.RAO

All problems are equally weighted.
(NO solution Manual is allowed.)
closed Book

1 a) Determine whether the following codes are uniquely decodable. Justify your answers.

1) { 0,01,11,111 }
2) { 0,01,110,111 }
3) { 0,10,110,111 }
4) { 1,10,110,111 }

b) Of the following codes, find which are Prefix codes. Justify your answers.

Symbol	Code 1	Code 2	Code 3	Code 4
S_0	0	0	0	00
S_1	10	01	01	01
S_2	110	001	011	10
S_3	1110	0010	110	110
S_4	1111	0010	111	111

2

Symbol	Probability
S_0	0.4
S_1	0.2
S_2	0.2
S_3	0.1
S_4	0.1

a) Find Huffman Code and get its average code length.
b) Find Minimum Variance Huffman Code and get its average code length.
c) Find the Redundancy.
d) Encode the sequence: - $S_2S_1S_3S_2S_1S_2$
   Using Huffman and Minimum Variance Huffman Code.
   Now, suppose the first bit is received in error at the receiver.
   Decode using both Huffman and Minimum Variance Huffman Codes.
   How many characters are received in error before the first correctly decoded character in each case?

3 Given the probability model in the following table, find the real valued tag for the Sequence

$S_1 S_1 S_3 S_2 S_3 S_1$

Letter	Probability
S_1	0.2
S_2	0.3
S_3	0.5

4 Given an initial dictionary consisting of letters " *a b r y b̸* ", encode the following message using LZW algorithm: - *a b̸ b a r b̸ a r r a y b̸ b y b̸ b a r r a y a r b̸ b a y*

Index	Entry
1	a
2	b
3	r
4	y
5	b̸

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(Key to Test 1)

100

1(a)

1) {0, 01, 11, 111}

By using test of unique decodability, we proceed as follows.

{0, 01, 11, 111, 1} ∵ 0 is prefix of 01 with suffix 1
11 is " " 111 with suffix 1

Now 1 is prefix of 111 with dangling suffix 11 which is a codeword.

Hence NOT Uniquely Decodable

1(a) 2) {0, 01, 110, 111}

By test

{0, 01, 110, 111, 1} 0 is prefix of 01 with dangling suffix

{0, 01, 110, 111, 1, 10, 11} 1 is prefix of 110 & 111 with dangling suffix 10 & 11

11 is a prefix of 110 with dangling suffix of 0 which is a codeword.

Hence, NOT uniquely decodable.

1(a) 3) {0, 10, 110, 111}

By test, there is no codeword as prefix to another codeword

Hence, Uniquely decodable code

1(a) 4) {1, 10, 110, 111}

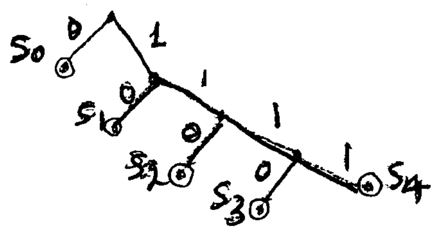
By test,

1 is prefix of 110 with dangling suffix 10 which is a codeword.

Hence, <u>NOT a Uniquely</u> decodable code.

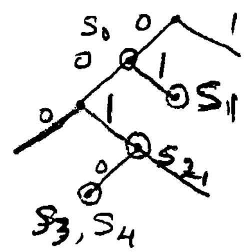
1(b)

<u>Code 1</u>



Since all the codewords are associated with External nodes, it is a Prefix Code

<u>Code 2</u>

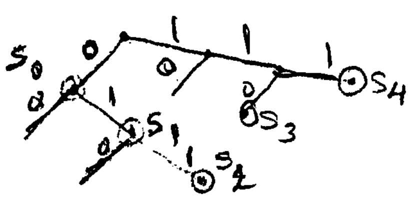


Not a Prefix Code because

1. S_3, S_4 have same codeword
2. Not all codes are external nodes.

Actually, point 1 rules out the code being a Prefix code.

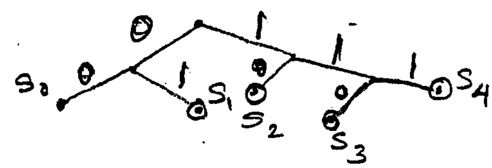
<u>Code 3</u>



S_0, S_1 are internal nodes

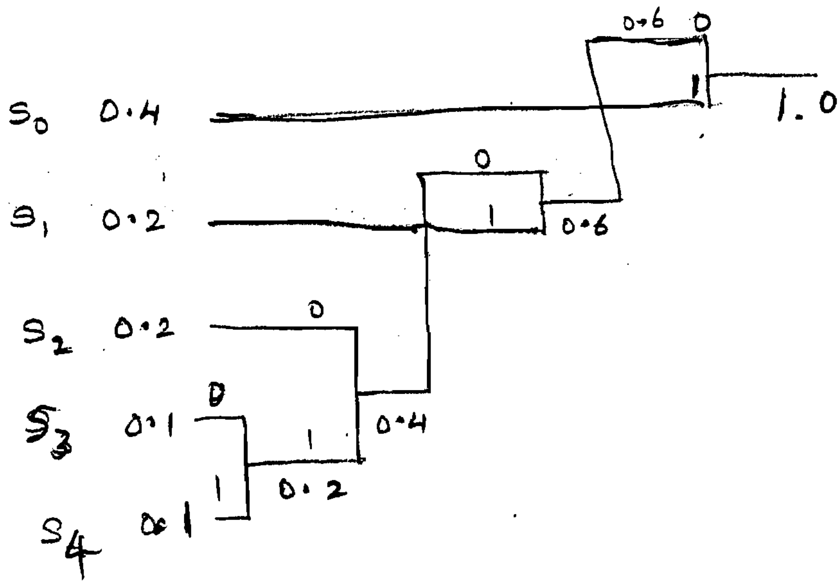
Hence Code 3 is not a Prefix code.

<u>Code 4</u>



All the codewords are External nodes. Hence Code 4 is a Prefix Code

2.(a)

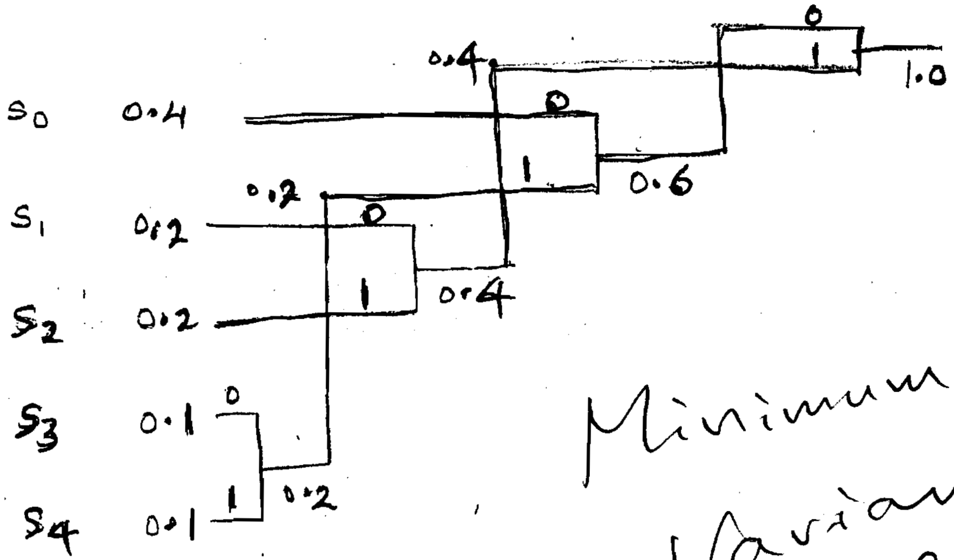


S_0	1
S_1	01
S_2	000
S_3	0010
S_4	0011

Average code length $= \bar{l} = \sum_{i=0}^{4} P(s_i)\, n(s_i)$

$= \underline{2.2 \text{ bits/symbol}}.$

(b)



4.

Minimum Variance code

s_0	00
s_1	10
s_2	11
s_3	010
s_4	011

Average code Length $= \bar{l} = \sum_{i=0}^{4} P(s_i)\, n(s_i)$

$= 2.2$ bits/symbol

2(c)

Entropy of the code,

$H(s) = -\sum p(s_i) \log P(s_i)$

$= \frac{-1}{\log_{10} 2}\left[0.4\log 0.4 + 0.2\log 0.2 + 0.2\log 0.2 + 0.1\log 0.1 + 0.1\log 0.1\right]$

$= \frac{-1}{\log_{10} 2}\left[-0.63875\right]$

$H(s) = 2.12$ bits/symbol

Redundancy = $\bar{l} - H(s)$
$= 2.2 - 2.12$
$= \underline{0.08 \text{ bits/symbol}}$

	Huffman code	Min Variance
s_0	1	00
s_1	01	10
s_2	000	11
s_3	0010	010
s_4	0011	011

2(d)

sequence : $s_2\, s_1\, s_3\, s_2\, s_1\, s_2$

Huffman code: 0000100010 00001000

Min. var.
Huffman code : 111001011 1011

<u>Huffman Decoder</u>

The received sequence is

100010010 00001000

The Decoded sequence is

1|000|1|0010|000|01|000

$s_0\, s_2\, s_0\, s_3\, s_2\, s_1\, s_2$

Three symbols are received in error before the first correctly decoded symbol which happens after first 5 bits.

11|00|011 1011

<u>Minimum Variance Huffman Decoder</u>

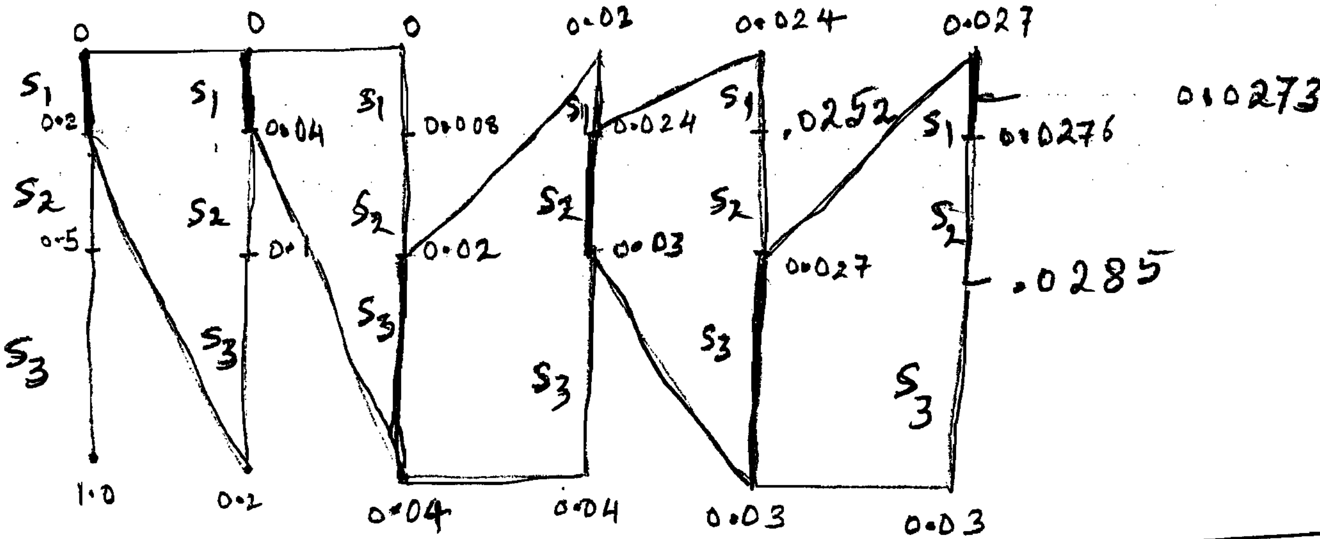
011|00|10|11|10|11

The Decoded sequence is

$s_4\, s_0\, s_1\, s_2\, s_1\, s_2$

Three symbols are received in error before the first correctly decoded symbol which is after first 7 bits

3. $s_1\ s_1\ s_3\ s_2\ s_3\ s_1$



s_1	0.2
s_2	0.3
s_3	0.5

$l^{(1)} = 0 + (1-0)\,0 = 0$

$u^{(1)} = 0 + (1-0)\,\underset{.2}{F_X(s_1)} = 0.2$

$l^{(2)} = 0 + (0.2-0)\,0 = 0$

$u^{(2)} = 0 + (0.2-0)\,\underset{.2}{F_X(s_1)} = 0.2 \times 0.2 = 0.04$

$l^{(3)} = 0 + (0.04-0)\,F_X(s_2) = 0.04 \times 0.5 = 0.02$

$u^{(3)} = 0 + (0.04-0)\,F_X(s_3) = 0.04 \times 1 = 0.04$

$l^{(4)} = 0.02 + (0.04-0.02)\,F_X(s_1) = 0.02 + 0.02 \times 0.2 = 0.024$

$u^{(4)} = 0.02 + (0.04-0.02)\,F_X(s_2) = 0.02 + 0.02 \times 0.5 = 0.03$

$l^{(5)} = 0.024 + (0.03-0.024)\,F_X(s_2) = 0.027$

$u^{(5)} = 0.024 + (0.03-0.024)\,F_X(s_3) = 0.03$

$l^{(6)} = 0.027 + (0.03-0.027)\,0 = 0.027$

$u^{(6)} = 0.027 + (0.03-0.027)\,\underset{.2}{F_X(s_1)} = 0.0276$

The mid-point $\frac{0.027 + 0.0276}{2} = 0.0273$ can be sent as tag ~~for~~ the given sequence.

4.

a b̸ b a r b̸ a r r a y b̸ b y b̸ b a r r a y a r b̸ b a y

Index	Entry
1	a
2	b
3	r
4	y
5	b̸
6	a b̸
7	b̸ b
8	b a
9	a r
10	r b̸
11	b̸ a
12	a r r
13	r a
14	a y
15	y b̸
16	b̸ b y
17	y b̸ b
18	b a r
19	r r
20	r a y
21	y a
22	a r b̸
23	b̸ b a

LZW code for the sequence:

1 5 2 1 3 5 9 3 1 4 7 (15) 8 3 (13) 4 9 7 (14)