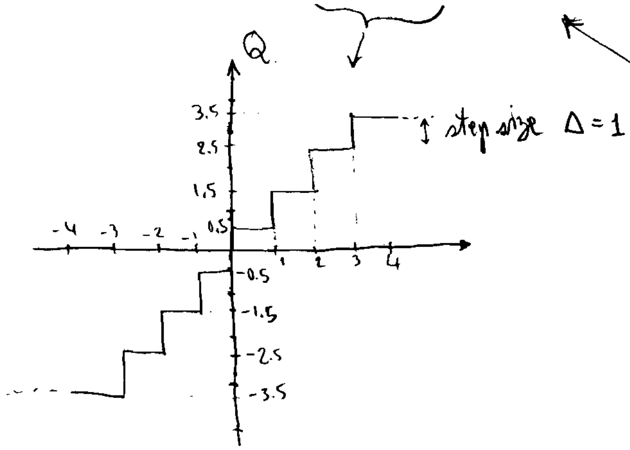
1. uniform step size $\Delta = \frac{4 \times 2}{2^3} = 1$.

input x	$C(x)$	$Q(C(x))$	output $C^{-1}(x)$	$x - C^{-1}(x)$, Compander error	quantizer directly $Q(x)$	$x - Q(x)$ quantizer error
−0.8	−1.6	−1.5	−0.75	−0.05	−0.5	−0.3
1.2	2.133	2.5	1.75	−0.55	1.5	−0.3
0.5	1	1.5	0.75	−0.25	0.5	0
0.6	1.2	1.5	0.75	−0.15	0.5	0.1
3.2	3.467	3.5	3.25	−0.05	3.5	−0.3
−0.3	−0.6	−0.5	−0.25	−0.05	−0.5	0.2



output Ans

$$\text{MSE with compander} = \frac{(0.05)^2 + (0.55)^2 + (0.25)^2 + (0.15)^2 + (0.05)^2 + (0.05)^2}{6}$$

$$= \frac{0.395}{6} = 0.06583$$

$$\text{MSE without compander} = \frac{(0.3)^2 + (0.3)^2 + (0.1)^2 + (0.3)^2 + (0.2)^2 + 0}{6}$$

$$= \frac{0.32}{6} = 0.0533$$

Without using compander, input data are mostly quantized in few level. When we apply compander system, all input data equally quantized (after Compresser C(x)) then then are mapped back by expander.

-1

(2)

From $C^{-1}(x) = \frac{x_{max}}{\mu}\left[(1+\mu)^{\frac{|x|}{x_{max}}} - 1\right] sgn(x).$

Let $C^{-1}(x) = y$

$$sgn(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

For $x > 0$; $y = \frac{x_{max}}{\mu}\left[(1+\mu)^{\frac{x}{x_{max}}} - 1\right]$

$$1 + \frac{y\mu}{x_{max}} = (1+\mu)^{\frac{x}{x_{max}}}$$

Take ln →; $\ln\left(1 + \frac{y\mu}{x_{max}}\right) = \frac{x}{x_{max}} \ln(1+\mu)$

$$x = x_{max} \frac{\ln\left(1 + \frac{\mu y}{x_{max}}\right)}{\ln(1+\mu)}$$ (1)

For $x < 0$; $y = \frac{x_{max}}{\mu}\left[(1+\mu)^{\frac{-x}{x_{max}}} - 1\right](-1)$

$$1 - \frac{\mu y}{x_{max}} = (1+\mu)^{-\frac{x}{x_{max}}}$$

Take ln; $\ln\left(1 - \frac{\mu y}{x_{max}}\right) = -\frac{x}{x_{max}} \ln(1+\mu)$

$$x = -x_{max} \frac{\ln\left(1 + \frac{\mu(-y)}{x_{max}}\right)}{\ln(1+\mu)}$$ (2)

Combine case (1) and (2) for all x;

$$x = x_{max} \frac{\ln\left(1 + \frac{\mu|y|}{x_{max}}\right)}{\ln(1+\mu)} sgn(y)$$

Let change variable;

$y \to x$

$x \to C(x)$.

$$C(x) = x_{max} \frac{\ln\left(1 + \frac{\mu|x|}{x_{max}}\right)}{\ln(1+\mu)} sgn(x)$$

Ans

(3) $\hat{x}_{i,j} = a_1 x_{i,j-1} + a_2 x_{i-1,j}$

$$MSQE = E\left[(x_{ij} - \hat{x}_{ij})^2\right]$$

$$= E\left[(x_{ij} - a_1 x_{i,j-1} - a_2 x_{i-1,j})^2\right]$$

$$\frac{\partial MSQE}{\partial a_1} = -2E\left[(x_{ij} - a_1 x_{i,j-1} - a_2 x_{i-1,j})\, x_{i,j-1}\right] = 0$$

$$\underbrace{E[x_{ij} x_{i,j-1}]}_{D} = a_1 \underbrace{E[x_{i,j-1} x_{i,j-1}]}_{A} + a_2 \underbrace{E[x_{i-1,j} x_{i,j-1}]}_{B} \quad \text{— (1)}$$

$$\frac{\partial MSQE}{\partial a_2} = -2E\left[(x_{ij} - a_1 x_{i,j-1} - a_2 x_{i-1,j})\, x_{i-1,j}\right] = 0$$

$$\underbrace{E[x_{ij} x_{i-1,j}]}_{E} = a_1 \underbrace{E[x_{i,j-1} x_{i-1,j}]}_{B} + a_2 \underbrace{E[x_{i-1,j} x_{i-1,j}]}_{C} \quad \text{— (2)}$$

solve for a_1, a_2

(1) $\rightarrow$ $a_1 A + a_2 B = D$

(2) $\rightarrow$ $a_1 B + a_2 C = E$

$$\begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} D \\ E \end{bmatrix}$$

$$\underline{R}\ \underline{a} = \underline{r}$$

$$\underline{a} = \underline{R}^{-1} \underline{r}$$

Ans

(4.)

Find Partition $P(\hat{A}_0)$

given uniform quantizer. $S = \{ \underbrace{\overset{S_1}{(2,2)}, \overset{S_2}{(2,-2)}, \overset{S_3}{(-2,2)}, \overset{S_4}{(-2,-2)}}_{\hat{x}} \}$

$S_1 = \{ x_6, x_8, x_{10} \}$

$S_2 = \{ x_2, x_9 \}$

$S_3 = \{ x_1, x_3, x_7 \}$

$S_4 = \{ x_4, x_5 \}$

set $m = 0$

Distortion $D_0 = \frac{1}{10} \sum_{i=1}^{10} \min \underbrace{d(\underline{x}_i, \underline{\hat{x}})}_{(\underline{x}_i - \underline{\hat{x}})^2} = 4.6270$

$\min d(\underline{x}_i - \underline{\hat{x}}) = (2-1.1)^2 + (2-0.5)^2 + (2-0.1)^2 + (2-1.7)^2 + (2-0.3)^2 + (2-4.8)^2$
$+ (2-0.6)^2 + (-2+0.1)^2 + (2-0.7)^2 + (-2+0.3)^2$
$+ (-2+0.5)^2 + (2-1)^2 + (-2+0.8)^2 + (2-0.6)^2 + (-2+0.6)^2 + (2-0.2)^2$
$+ (-2+0.7)^2 + (-2+1.2)^2 + (-2+0.2)^2 + (-2+0.9)^2 = 4.6270$

Let $D_{-1} = \infty$

$\frac{D_{-1} - D_0}{D_0} = \frac{\infty - 4.6270}{4.6270} > 0.001$ ε continue. Find $P(\hat{A}_1)$.

next page.

④ continue

Partition $P(\hat{A}_1)$ such that $d(x_i; \hat{x}_j) < d(x_i; \hat{x}_k)$; $j \neq k$,

we get. $\hat{x}(S_1) = \left(\frac{x_6 + x_8 + x_{10}}{3}\right) = \left(\frac{1.1 + 0.1 + 0.3}{3}, \frac{0.5 + 1.7 + 4.8}{3}\right) = (0.5, 2.333)$

$\hat{x}(S_2) = \left(\frac{x_2 + x_9}{2}\right) = \left(\frac{0.6 + 0.7}{2}, \frac{-0.1 - 0.3}{2}\right) = (0.65, 0.2)$

$\hat{x}(S_3) = \left(\frac{x_1 + x_3 + x_7}{3}\right) = \left(\frac{-0.5 - 0.8 - 0.6}{3}, \frac{1 + 0.6 + 0.2}{3}\right) = (0.633, 0.6)$

$\hat{x}(S_4) = \left(\frac{x_4 + x_5}{2}\right) = \left(-\frac{0.7 - 0.2}{2}, -\frac{1.2 - 0.9}{2}\right) = (-0.45, -1.05)$

set $m = 1$

Distortion $D_1 = \frac{1}{10} \sum_{i=1}^{10} \min d(x_i, \hat{x}) = 1.2697$

$\frac{D_0 - D_1}{D_1} = \frac{4.627 - 1.2697}{1.2697} = 2.644 > 0.001$ continue Find $P(\hat{A}_2)$

$P(\hat{A}_2) = \hat{x} P(\hat{A}_1) = \hat{A}_1$

then $D_2 = D_1$

so $\frac{D_1 - D_2}{D_2} = 0 < 0.001$ Halt (stop).

Therefore, the codebook is

$(0.5, 2.333)$, $(0.65, 2)$, $(0.633, 0.6)$, $(-0.45, 1.05)$

Ans