



EE5351 Digital Video Coding

INSTRUCTOR: Dr. K.R. Rao

Summer 2007, Test 1

Tuesday, 19 June 2007

6:00 – 7:45 PM (1 hour and 45 minutes)

(OPEN BOOK, OPEN NOTES)

INSTRUCTIONS:

1. Open books and open notes.
2. Please show all the steps in your works.
4. You can work problems in any order.
At the end please rearrange as 1, 2, 3, 4, and 5.
5. Please print your name and student ID.
6. No cheating, no talking.

Name _____

Student ID _____

[20 Points][**Problem 1**]

Given source alphabets with probabilities.

LETTER	PROBABILITY
a_1	0.8
a_2	0.2

- (a) Calculate the entropy of this source.
- (b) Find Huffman code.
- (c) Calculate its average code length.
- (d) Calculate redundancy.
- (e) Find Extended Huffman code by generate a codeword for every two symbols.
- (f) Calculate extended alphabet size.
- (g) Calculate average codeword length for this extended code (in bit/symbol)
- (h) Calculate average codeword length (in bit/alphabet) (in terms of the original alphabet)

[20 Points][**Problem 2**]

Design a 4-bit Tunstall code. Use the method with repeating the letter “**A**” every iteration. Given source alphabets and associated probabilities.

LETTER	PROBABILITY
A	0.4
B	0.3
C	0.2
D	0.1

[20 Points][**Problem 3**]

Given the probability model in the following table, find the real valued tag (midpoint of the tag interval) for the sequence $S_2 S_2 S_3 S_1 S_1$

LETTER	PROBABILITY
S_1	0.1
S_2	0.3
S_3	0.6

[20 POINTS][**PROBLEM 4**]

A sequence is encoded using LZ77 algorithm. Given that $C(a)=1$, $C(h)=2$, $C(t)=3$, and $C(\phi)=4$. Decode the following sequence of triples:

$\langle 0,0,2 \rangle \langle 0,0,1 \rangle \langle 0,0,3 \rangle \langle 2,3,4 \rangle \langle 7,4,4 \rangle \langle 4,2,4 \rangle \langle 6,2,3 \rangle$

Assume that the size of the window is 20 and the size of the look-ahead buffer is 10.

[20 POINTS][**PROBLEM 5**]

A sequence is encoded using the LZW algorithm and the initial dictionary shown in the table below:

INDEX	ENTRY
1	S
2	ϕ
3	I
4	T
5	H

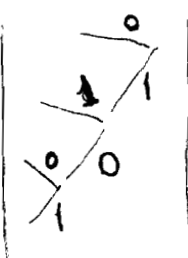
The output of the LZW encoder is the following sequence:

4	5	3	1	2	8	2	7	9	7	4
---	---	---	---	---	---	---	---	---	---	---

Build an LZW dictionary AND decode this sequence.

END OF TEST QUESTIONS

EE5351 Test 1 Solutions

- ①
- (a) Entropy = $-0.8 \log_2 0.8 - 0.2 \log_2 0.2 = 0.7218$ bit/alphabet
- (b) Huffman code is
- | | | |
|-------|-----|---|
| a_1 | 0.8 | 0 |
| a_2 | 0.2 | 1 |
- (c) Average length = $\sum_{i=1}^2 p(a_i) L_{a_i} = 0.8(1) + 0.2(1) = 1$ bit/alphabet
- (d) Redundancy = Average Length - Entropy = $1 - 0.7218 = 0.2782$ bit/alphabet
- (e) Extended Huffman code is
- | | | | |
|-----------|------|--|-----|
| $a_1 a_1$ | 0.64 |  | 0 |
| $a_1 a_2$ | 0.16 | | 11 |
| $a_2 a_1$ | 0.16 | | 100 |
| $a_2 a_2$ | 0.04 | | 101 |
- (f) Extended alphabet size = $2^2 = 4$ ($m=2, n=2$)
- (g) Average codeword length = $0.64(1) + 0.16(2) + 0.16(3) + 0.04(3)$
 $= 1.56$ bit/symbol
- (h) Average codeword length = $\frac{1.56}{2} = 0.78$ bit/alphabet.

2

Keep repeating 'A'

Source Alphabet

A	0.4
B	0.3
C	0.2
D	0.1

1st iteration

B	0.3
C	0.2
D	0.1
AA	0.16
AB	0.12
AC	0.08
AD	0.04

2nd iteration

B	0.3
C	0.2
D	0.1
AB	0.12
AC	0.08
AD	0.04
AAA	0.064
AAB	0.048
AAC	0.032
AAD	0.016

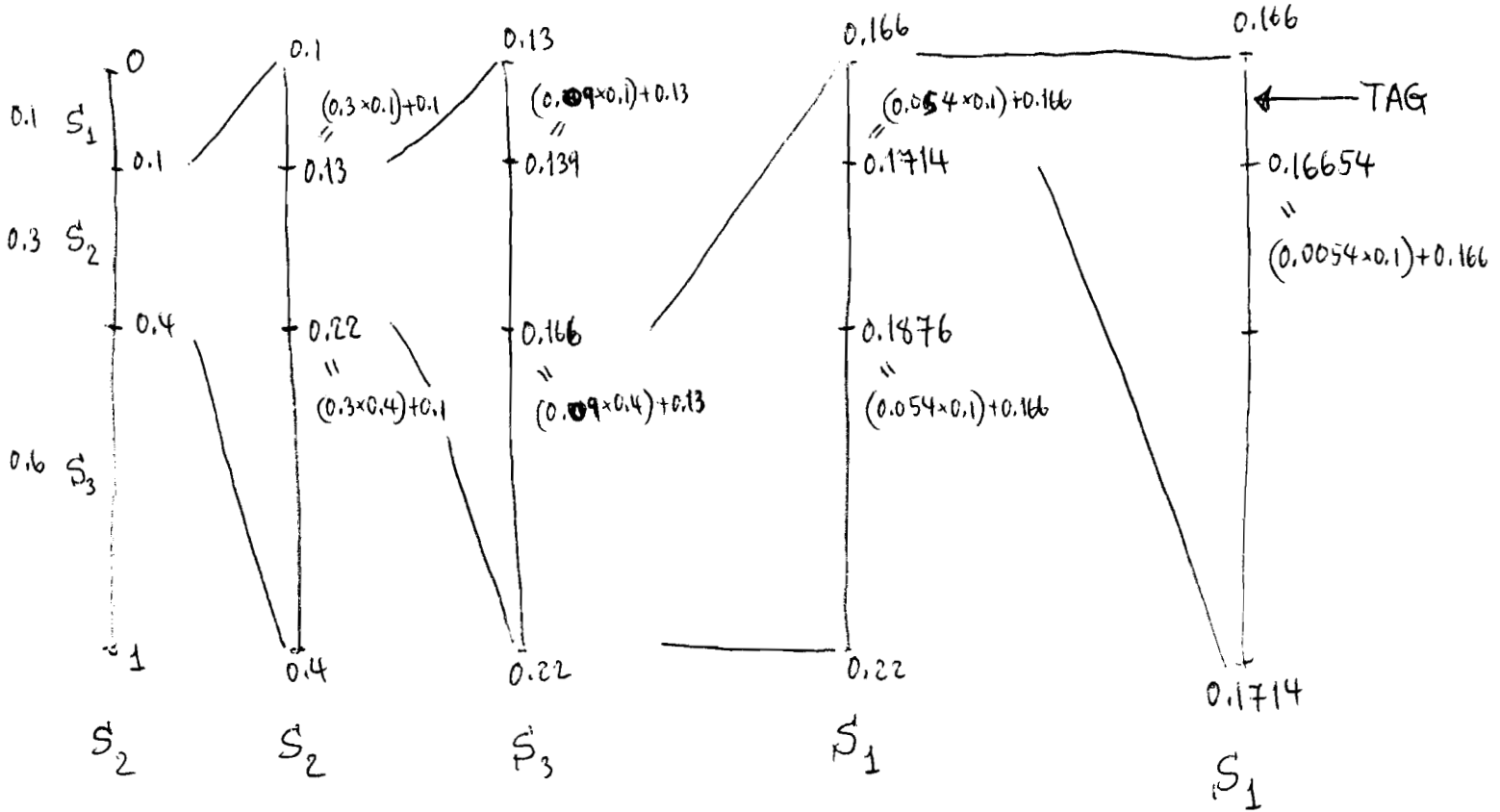
3rd iteration

B	0.3
C	0.2
D	0.1
AB	0.12
AC	0.08
AD	0.04
AAB	0.048
AAC	0.032
AAD	0.016
AAAA	0.0256
AAAB	0.0192
AAAC	0.0128
AAAD	0.0064

4th iteration

B	0000
C	0001
D	0010
AB	0011
AC	0100
AD	0101
AAB	0110
AAC	0111
AAD	1000
AAAB	1001
AAAC	1010
AAAD	1011
AAAA	1100
AAAB	1101
AAAC	1110
AAAD	1111

3



Real value tag at midpoint of $S_2 S_2 S_3 S_1 S_1 = \frac{0.166 + 0.16654}{2} = 0.16627$

4 LZ77 decoded sequence is

h a t a t a ϕ h a t a ϕ a t ϕ t a t

\uparrow \uparrow \uparrow \uparrow

- $\langle 0, 0, 2 \rangle \equiv h$
- $\langle 0, 0, 17 \rangle \equiv a$
- $\langle 0, 0, 37 \rangle \equiv t$
- $\langle 2, 3, 4 \rangle \equiv a t a \phi$
- $\langle 7, 4, 4 \rangle \equiv h a t a \phi$
- $\langle 4, 2, 4 \rangle \equiv a t \phi$
- $\langle 6, 2, 3 \rangle \equiv t a t$

Received sequence: 4, 5, 3, 1, 2, 8, 2, 7, 9, 7, 4

Received	Decode	Update Dictionary	
4	<i>T</i>	Entry 6: <i>T...</i>	
5	<i>H</i>	Entry 6: <i>TH</i>	Entry 7: <i>H...</i>
3	<i>I</i>	Entry 7: <i>HI</i>	Entry 8: <i>I...</i>
1	<i>S</i>	Entry 8: <i>IS</i>	Entry 9: <i>S...</i>
2	<i>ß</i>	Entry 9: <i>Sß</i>	Entry 10: <i>ß...</i>
8	<i>IS</i>	Entry 10: <i>ßI</i>	Entry 11: <i>IS...</i>
2	<i>ß</i>	Entry 11: <i>ISß</i>	Entry 12: <i>ß...</i>
7	<i>HI</i>	Entry 12: <i>ßH</i>	Entry 13: <i>HI...</i>
9	<i>Sß</i>	Entry 13: <i>HIS</i>	Entry 14: <i>Sß</i>
7	<i>HI</i>	Entry 14: <i>SßH</i>	Entry 15: <i>HI...</i>
4	<i>T</i>	Entry 15: <i>HIT</i>	Entry 16: <i>T...</i>

Final Dictionary:

Index	Entry
1	<i>S</i>
2	<i>ß</i>
3	<i>I</i>
4	<i>T</i>
5	<i>H</i>
6	<i>TH</i>
7	<i>HI</i>
8	<i>IS</i>
9	<i>Sß</i>
10	<i>ßI</i>
11	<i>ISß</i>
12	<i>ßH</i>
13	<i>HIS</i>
14	<i>SßH</i>
15	<i>HIT</i>
16	<i>T...</i>