EE5351 Digital Video Coding

Instructor: Dr. K.R. Rao

Summer 2007, Test 2

Tuesday, 10 July 2007

6:00 – 7:45 PM (1 hour and 45 minutes)

(Close book, Close notes)

Instructions:

1. Close books and close notes.
2. Please show all the steps in your works.
4. You can work problems in any order.
   At the end please rearrange as 1, 2, 3, 4, and 5.
5. Please print your name and student ID.
6. No cheating, no talking.

Name__________________________

Student ID______________________

[20 Points][**Problem 1**]

(Uniform quantization of a uniformly distributed source signal)

Given an M-level uniform quantizer with fixed-length code of n bits, $M = 2^n$. Source signal takes values in the interval $[-X_{\max}, X_{\max}]$. Quantization step size is Δ.

(1.1)[4 Points] Find quantization step size Δ in terms of $X_{\max}$ and M.

(1.2)[5 Points] Find signal variance σ_s^2 (signal is uniformly distributed in the interval $[-X_{\max}, X_{\max}]$.

(1.3)[5 Points] Find mean squared quantization error, msqe, σ_q^2 (second moment of quantization error uniformly distributed in the interval $\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$).

(1.4)[6 Points] Show that $\text{SNR}(\text{in dB}) = 6.02n \text{ dB}$. This means, the signal-to-noise ratio increases 6.02 dB for every additional bit in the quantizer.

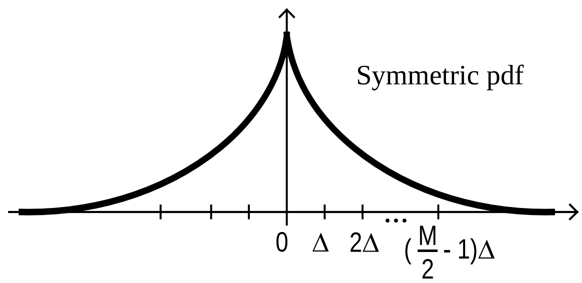
Given $\text{SNR} = \dfrac{\text{Signal variance}}{\text{msqe}}$.

Variance or second moment of a random variable Y with zero mean, is

$E\left[Y^2\right] = \int_{y_1}^{y_2} Y^2 f_Y(Y)\,dY$, where $f_Y(Y)$ is probability density function and $Y \in [y_1, y_2]$.

[20 Points][**Problem 2**]

(Scalar quantization) Forcing a uniform quantizer on a non-uniform symmetric pdf, Δ = step size, M = number of output levels.



Given msqe, $\sigma_q^2 = \left[2\sum_{i=1}^{\frac{M}{2}-1}\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right)^2 f_X(x)\,dx\right] + \left[2\int_{\left(\frac{M}{2}-1\right)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right) f_X(x)\,dx\right]$

(2.1)[6 Points] In these two terms, identify the overload noise and granular noise.

(2.2)[8 Points] To find the optimal value of Δ, we take a derivative of msqe, σ_q^2, with respect to Δ and set it to zero. Show that result is in the expression below,

$$\frac{\partial\sigma_q^2}{\partial\Delta} = -\left[\sum_{i=1}^{\frac{M}{2}-1}(2i-1)\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right) f_X(x)\,dx\right] - \left[(M-1)\int_{\left(\frac{M}{2}-1\right)}^{\infty}\left(x-\frac{M-1}{2}\Delta\right) f_X(x)\,dx\right] = 0$$

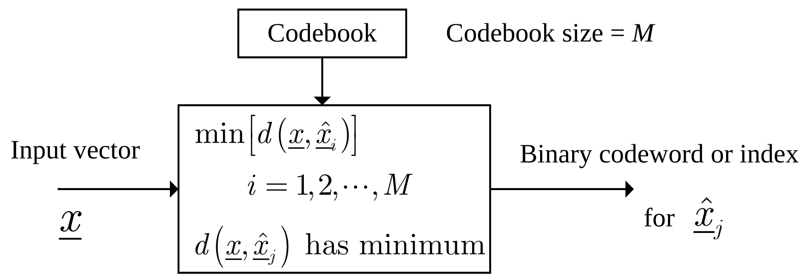
Use Leibnitz's rule, $\frac{\partial}{\partial t}\int_{a(t)}^{b(t)} g(y,t)\,dy = \int_{a(t)}^{b(t)}\frac{\partial g(y,t)}{\partial t}dy + g(b(t),t)\frac{\partial b(t)}{\partial t} - g(a(t),t)\frac{\partial a(t)}{\partial t}$.

Show all steps.

(2.3)[6 Points] How does Δ affect the granular noise and overload noise.

[20 Points][**Problem 3**]

(Vector quantization) VQ codebook contains M codewords (codebook size). Each codeword is fixed length binary coded.



(3.1)[4 Points] How many codebook indices are used for this quantizer?

(3.2)[4 Pionts] Calculate the number of bits needed to transmit each codebook index.

(3.3)[4 Points] Given each input vector has L pixels (dimension of input vector), find the compression rate of the encoder in bpp (bits per pixel).

(3.4)[8 Points] Explain in detail the splitting algorithm for designing a codebook. What is an empty cell problem and how can this be solved.

[20 Points][**Problem 4**]

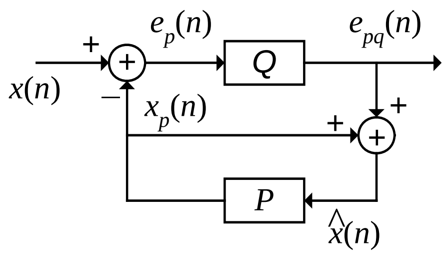
(4.1)[10 Points] Write the mathematical expressions (definitions) of two distortion measurements, MSE (mean squared error) and MAE (mean absolute error). Define all variables and parameters that appear in the expressions.

(4.2)[10 Points] Suppose the L_6 distortion metric, $\frac{1}{K}\sum_{m=1}^{K}\left[x(m) - \hat{x}(m)\right]^6$, is used in the VQ process. What are the implications of this?

[20 Points][**Problem 5**]

(DPCM with linear-causal K-th order predictor)

(5.1)[7 Points] Given $\sigma_{pe}^2 = E\left[e_p^2(n)\right]$, prediction error $e_p(n)$ and predicted value $x_p(n)$ are uncorrelated, $x_p(n) = \sum_{i=1}^{K} a_i x(n-i)$, where $x(n)$ is input signal, and a_i are predictor weights. Also $e_p(n)$ has zero mean.



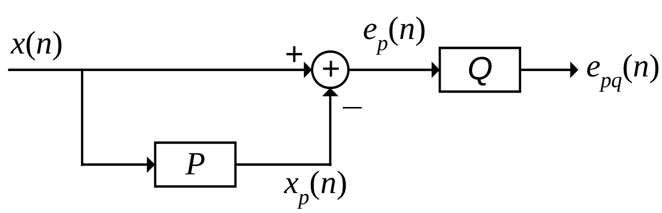
Show that prediction error variance, $\sigma_{pe}^2 = \sigma_x^2 - \underline{a}^T_{(1\times K)} \; \underline{r}_{(K\times 1)}$

where, $\underline{a}_{(K\times 1)} = \left[a_1, a_2, \cdots, a_K\right]^T$,

$\underline{r}_{(K\times 1)} = \left[r_1, r_2, \cdots, r_K\right]^T$

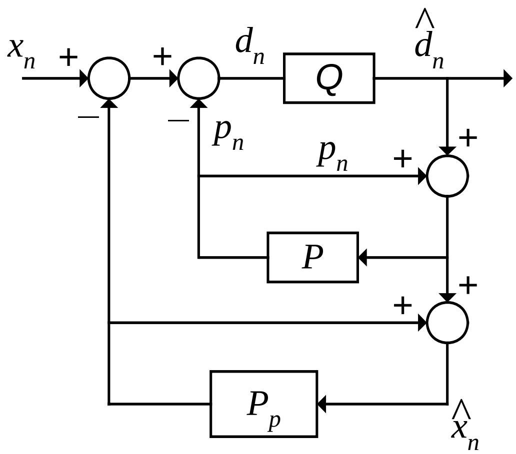
$$\left[E\left[x(n)\,x(n-1)\right], E\left[x(n)\,x(n-2)\right], \cdots, E\left[x(n)\,x(n-K)\right]\right]^T$$

(5.2)[6 Points] Given a DPCM with feed forward predictor.



Describe its disadvantages. Sketch the corresponding inverse DPCM.

(5.3)[7 Points] Given a DPCM structure with pitch predictor P_p (used in speech coding).



What are its <u>advantages</u>? Sketch the inverse DPCM.

End of Test Questions

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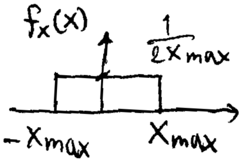
1

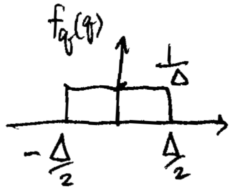
(1.1) $\Delta = \dfrac{2X_{max}}{M}$

(1.2)



$$\sigma_x^2 = \int_{-X_{max}}^{X_{max}} x^2 \cdot \frac{1}{2X_{max}}\, dx = \frac{X_{max}^2}{3}$$

(1.3)



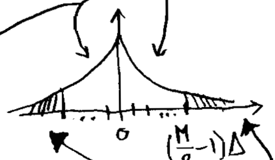
$$\sigma_q^2 = \int_{-\Delta/2}^{\Delta/2} q^2 \cdot \frac{1}{\Delta}\, dq = \frac{\Delta^2}{12} = \frac{X_{max}^2}{3M^2} = \frac{X_{max}^2}{3(2^{2n})}$$

(1.4)

$$\begin{aligned} SNR\,(dB) &= 10 \log_{10}\left(\frac{\sigma_x^2}{\sigma_q^2}\right) \\ &= 10 \log_{10} M^2 \\ &= 10 \log_{10} 2^{2n} \\ &= 6.02n \text{ dB.} \end{aligned}$$

(2)

(2.1)
$$\sigma_q^2 = \underbrace{2\sum_{i=1}^{\frac{M}{2}-1}\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right)^2 f_X(x)\,dx}_{\text{Granular noise}} + \underbrace{2\int_{(\frac{M}{2}-1)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right)^2 f_X(x)\,dx}_{\text{Overload noise}}$$



(2.2) Apply Leibnitz's rule,

$$\frac{\partial \sigma_q^2}{\partial \Delta} = \cancel{2}\sum_{i=1}^{\frac{M}{2}-1}\left[2\left(-\left(\frac{2i-1}{2}\right)\right)\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right) f_X(x)\,dx + \frac{\Delta^2}{4} f_X(i\Delta)\,i - \frac{\Delta^2}{4} f_X((i-1)\Delta)\,(i-1)\right]$$

$$+\ \cancel{2}\left[(2)\left(\frac{-(M-1)}{2}\right)\int_{(\frac{M}{2}-1)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right) f_X(x)\,dx + 0 - \frac{\Delta^2}{4} f_X\left(\left(\frac{M}{2}-1\right)\Delta\right)\cdot\left(\frac{M}{2}-1\right)\right] = 0$$

Consider summation of these two terms,

$$\sum_{i=1}^{\frac{M}{2}-1}\left[i\frac{\Delta^2}{4} f_X(i\Delta) - \frac{(i-1)\Delta^2}{4} f_X((i-1)\Delta)\right]$$

$$= \cancel{\frac{\Delta^2}{4} f_X(\Delta)} + \cancel{\frac{2\Delta^2}{4} f_X(2\Delta)} + \dots + \cancel{\frac{(\frac{M}{2}-2)\Delta^2}{4} f_X((\tfrac{M}{2}-2)\Delta)} + \frac{(\frac{M}{2}-1)\Delta^2}{4} f_X((\tfrac{M}{2}-1)\Delta)$$

$$- 0 - \cancel{\frac{\Delta^2}{4} f_X(\Delta)} - \cancel{\frac{2\Delta^2}{4} f_X(2\Delta)} - \dots - \cancel{\frac{(\frac{M}{2}-2)\Delta^2}{4} f_X((\tfrac{M}{2}-2)\Delta)}.$$

$$= \frac{(\frac{M}{2}-1)}{4}\Delta^2 f_X\left(\left(\frac{M}{2}-1\right)\Delta\right)$$

Therefore,

$$\frac{\partial \sigma_q^2}{\partial \Delta} = -\left[\sum_{i=1}^{\frac{M}{2}-1}(2i-1)\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right) f_X(x)\,dx\right] - \left[(M-1)\int_{(\frac{M}{2}-1)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right) f_X(x)\,dx\right] = 0$$

(2.3) With fixed number of quantization level M,

step size Δ $\downarrow$ (decreases), granular noise $\downarrow$, overload noise $\uparrow$ (increases)

step size Δ $\uparrow$ (increases), granular noise $\uparrow$, Overload noise $\downarrow$ (decreases)

3

(3.1) M indices are used for M codewords.

(3.2) Smallest integer that is greater or equal to $\log_2 M$.

(3.3) Compression rate = $\dfrac{\log_2 M}{L}$ bit/pixel.

(3.4) VQ splitting algorithm, see class notes pages. 14-20.

Empty cell problem, see class notes page 23.

④

(4.1) $$\text{MSE} = \frac{1}{K}\sum_{m=1}^{K}\left[x(m) - \hat{x}(m)\right]^2$$

$$\text{MAE} = \frac{1}{K}\sum_{m=1}^{K}\left|x(m) - \hat{x}(m)\right|$$

K = Dimension of input vector

$x(m)$ = $m^{\underline{th}}$ components of input vector , $m = 1, 2, \dots, K$

$\hat{x}(m)$ = $m^{\underline{th}}$ components of codebook vector.

(4.2) Consider error (distortion)

	Error from L_1	Eerror from L_6
(Small error)	2	2^6
(large error)	4	4^6
Ratio of $\frac{\text{large error}}{\text{small error}}$ =	$\frac{4}{2} = 2$	$\frac{4^6}{2^6} = 64$

Hence L_6 gives more weight on large error.

5

(5.1)

$$\sigma_{pe}^2 = E[e_p^2(n)] = E[e_p(n)\, e_p(n)]$$

$$= E[e_p(n)[x(n) - x_p(n)]]$$

= 0 Both are uncorrelated

$$= E[e_p(n)\, x(n)] - \cancel{E[e_p(n)\, x_p(n)]}$$

$$= E[[x(n) - x_p(n)]\, x(n)]$$

$$= E[x^2(n)] - E\left[\left[\sum_{i=1}^{k} a_i x(n-i)\right] x(n)\right]$$

$$= \sigma_x^2 - \sum_{i=1}^{k} a_i E[x(n-i)\, x(n)]$$

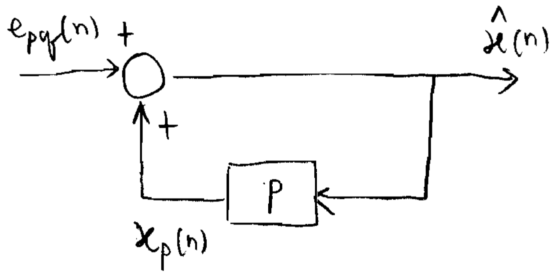
$$= \sigma_x^2 - \sum_{i=1}^{k} a_i r_i$$

$$= \sigma_x^2 - \underline{a}^T_{(1\times k)}\, \underline{r}_{(k\times 1)}$$

//

(5.2) Feed forward DPCM disadvantage : Error accumulates in the process.

Inverse DPCM :



(5.3) Advantage of DPCM with pitch predictor:

Outer prediction loop can separate pitch information (periodic) from the input sequence (speech signal). see pages 345-346.

Inverse DPCM: