The University of Texas
ARLINGTON

EE5351 Digital Video Coding

INSTRUCTOR: Dr. K.R. Rao

Summer 2008, Final

Thursday, 7 August 2008

6:00 – 7:50 PM (1 hour and 50 minutes)

(OPEN THE TEXTBOOK ONLY, NO INSERTS, CLOSE ALL NOTES)

(NOT ALLOWED CLASSNOTES, NO EXTRA NOTES, NO PREVIOUS EXAM SOLUTIONS)

INSTRUCTIONS:

1. Close notes, open ONLY the textbook.
2. Calculator is allowed.
3. Please show all the steps in your work.
4. You can work problems in any order.
   At the end please rearrange as 6, 7, 8, and 9.
5. Please print your name and student ID.
6. No cheating, no talking.

Name____________________________

Student ID_______________________

PART 1 (Q1) to (Q5) with **[4 Points]** each

(Q1) Which statement(s) verify(s) that DPCM predictor is a causal predictor? Write answer(s) _________________

A. Predictor coefficients are obtained by minimizing the mean squared prediction error.

B. Prediction of a current pixel is linear combination of previous samples.

C. Prediction of a current pixel is based on finite number of pixels.

D. Predictor that is implemented in DPCM with feed-forward prediction.

E. Predictor that is implemented in DPCM with feed-back prediction.

(Q2) Which measurement(s) is (are) used to verify the performance of lossless image codecs? Write answer(s) _________________

A. CR (compression ratio)

B. MSE (mean square error)

C. PSNR (peak signal-to-noise ratio)

D. Bit rate (in bits per pixel)

E. Histogram of pixel difference between original and reconstructed image

(Q3) Which codec(s) can be used for LOSSY image coding? Write answer(s) _________________

A. CALIC

B. JPEG-LS (LOCO)

C. JPEG Baseline

D. JPEG Lossless (Independent)

E. None of the above.

(Q4) Which statement(s) satisfy(s) the conditions for PR (perfect reconstruction) of a two-channel filter bank? Write answer(s) _________________

A. Reconstructed output is the scaled version of the original signal.
B. Reconstructed output is the delayed version of the original signal.
C. Reconstructed output has no aliasing (no phase distortion)
D. The analysis low pass filter is orthogonal to the twice-shifted version of itself.
E. None of the above.

(Q5) Based on the result of minimizing the total reconstruction error $\sigma_R^2 = \alpha \sum_{k=1}^{M} 2^{-2R_k} \sigma_{yk}^2$, where σ_{yk}^2 is the reconstruction error variance for the k-th subband, subject to constraint of total average bit rate $R = \frac{1}{M} \sum_{k=1}^{M} R_k$, where M is the number of subbands, R_k is average number of *bpp* (bit per pixel) for subband k.

What can we conclude about allocating the number of bits to a subband of an image decomposition from the result $R_i = R + \frac{1}{2} \log_2 \sigma_{yi}^2 - \frac{1}{2M} \log_2 \sum_{k=1}^{M} \sigma_{yk}^2$
Write answer(s) _________________

A. Assign more bits to small subband variance.
B. Assign fewer bits to high subband variance.
C. Assign more bits to high subband variance.
D. Assign fewer bits to small subband variance.
E. No conclusion can be drawn from the result.

PART 2 (Q6) to (Q9) with **[20 Points]** each

[20 Points][**Problem 6**] (DPCM)

We want to DPCM-encode images. Your task is to design the 2-D predictor (order 3) of DPCM. Given three tap predictor of the form,

$$\hat{x}_{i,j} = a_1 x_{i,j-1} + a_2 x_{i-1,j} + a_3 x_{i-1,j+1}$$

where, $\hat{x}_{i,j}$ is the predicted pixel at i-row, j-column. Figure 1 shows the neighboring pixels for the predictor. $x_{i,j}$ is the current pixel at i-row, j-column

Row i-1; ———— $x_{i-1,j}$ — $x_{i-1,j+1}$ ————
Row i ; — $x_{i,j-1}$ — $x_{i,j}$

Figure 1.

Find the equations we need to solve to obtain coefficients $\{a_1, a_2, a_3\}$ that minimize the mean squared prediction error.

[6 Points] Write the expression for the mean squared prediction error (MSPE).

[14 Points] Derive the equations that we need to solve to obtain the predictor coefficients, $\{a_1, a_2, a_3\}$, so that MSPE is minimized.
Hint: show in the form of matrices, inverse matrices.

[20 Points][**Problem 7**] (CALIC)

In CALIC, refinement to initial prediction $\hat{X}$ is as follows. Form the vector $[N, W, NW, NE, NN, WW, 2N - NN, 2W - WW]$. Compare each component of this vector with $\hat{X}$ and set each component to '1' if it is less than $\hat{X}$. Otherwise set to '0'.

Show clearly that because of the dependence of the various components, there are only 144 possible combinations in this vector. Explain clearly how to obtain the 144 combinations.

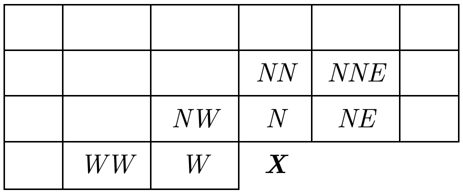


Figure 2. Labeling the neighbors of pixel X

[20 Points][**Problem 8**] (Filter bank and wavelet)

For 2-channel filter bank in Fig. 3, given a two tap low pass filter $h_0(n) = \left[\frac{1}{2}, \frac{1}{2}\right]$.

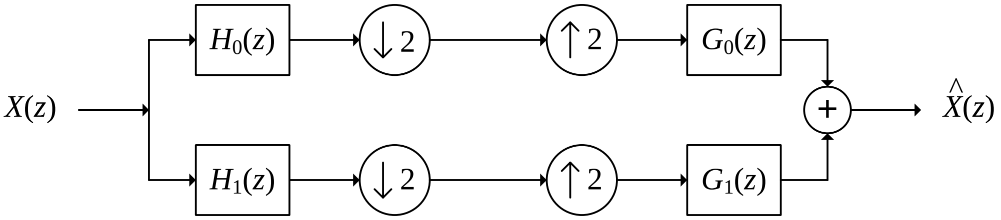


Figure 3. Given 2-channel filter bank

(A)[5 Points] Verify that $h_0(n)$ is LPF and is linear phase. Sketch $|H_0(\omega)|$. Determine the phase of the filter.

(B)[5 Points] Find the QMF analysis and synthesis filter pairs $\{h_0(n),\ h_1(n),\ g_0(n),$ and $g_1(n)\}$. Identify each filter as to which one is low pass or high pass filter.

(C)[5 Points] Write two equations of PR condition (perfect reconstruction) in Z-domain.

(D)[5 Points] Verify that the given pairs of QMF satisfy the PR condition. Hint: verify in either Z-domain or ω-domain.

Hint:

Z-transform: $H(z) = \sum_{n=0}^{N-1} h(n) Z^{-n}$

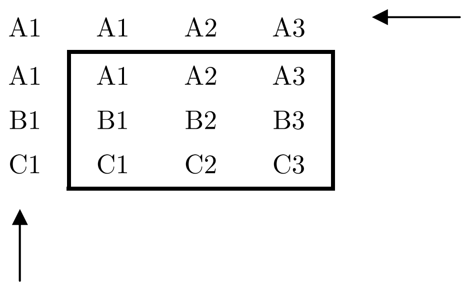
Fourier transform: $H(\omega) = \sum_{n=0}^{N-1} h(n) \exp(-j\omega n)$

[20 Points][**Problem 9**] (Subband coding)

A sample image (4×4 pixels) is shown in Table 1. Use the analysis filter pair $\{h_0(n), h_1(n)\}$ given in [Problem 8], decompose the sample image into 2-D four equal subbands. Identify the LL, LH, HL, and HH subbands. Apply reflection of pixel intensities at the borders in Fig. 4.

Table 1. Sample image (4×4 pixels)

10	10	12	8
12	8	10	10
14	10	6	8
6	6	6	6



A1	A1	A2	A3
A1	A1	A2	A3
B1	B1	B2	B3
C1	C1	C2	C3

Figure 4. Example of row reflection and column reflection at the border of an (3×3) pixel image

Hint:

1-D convolution: $y(n) = h(n) * x(n) = \sum_{m=-\infty}^{n} h(n-m)\,x(m)$

END OF TEST QUESTIONS

EE5351 Final Exam Solution (Summer 08)

① B or B, C

② A, D

③ B, C

④ A, B, C, D or A, B, C

⑤ C, D

⑥ $$\hat{x}_{i,j} = a_1 x_{i,j-1} + a_2 x_{i-1,j} + a_3 x_{i-1,j+1}$$

$$\text{MSPE} = E\left[\left(x_{ij} - \hat{x}_{i,j}\right)^2\right]$$

$$= E\left[\left(x_{i,j} - a_1 x_{i,j-1} - a_2 x_{i-1,j} - a_3 x_{i-1,j+1}\right)^2\right]$$

$$\frac{\partial \text{MSPE}}{\partial a_1} = -2E\left[\left(x_{i,j} - a_1 x_{i,j-1} - a_2 x_{i-1,j} - a_3 x_{i-1,j+1}\right) x_{i,j-1}\right] = 0$$

① $$\longrightarrow \underbrace{E[x_{i,j} . x_{i,j-1}]}_{D} = a_1 \underbrace{E[x_{i,j-1} x_{i,j-1}]}_{A} + a_2 \underbrace{E[x_{i-1,j} x_{i,j-1}]}_{B} + a_3 \underbrace{E[x_{i-1,j-1} x_{i,j-1}]}_{C}$$

$$\frac{\partial \text{MSPE}}{\partial a_2} = -2E\left[\left(x_{i,j-1} - a_1 x_{i,j-1} - a_2 x_{i-1,j} - a_3 x_{i-1,j+1}\right) x_{i-1,j}\right] = 0$$

② $$\longrightarrow \underbrace{E[x_{i,j}\ x_{i-1,j}]}_{H} = a_1 \underbrace{E[x_{i,j-1} x_{i-1,j}]}_{E} + a_2 \underbrace{E[x_{i-1,j} x_{i-1,j}]}_{F} + a_3 \underbrace{E[x_{i-1,j+1} x_{i-1,j}]}_{G}$$

$$\frac{\partial \text{MSPE}}{\partial a_3} = -2E\left[\left(x_{i,j-1} - a_1 x_{i,j-1} - a_2 x_{i-1,j} - a_3 x_{i-1,j+1}\right) x_{i-1,j+1}\right] = 0$$

③ $$\longrightarrow \underbrace{E[x_{i,j}\ x_{i-1,j+1}]}_{N} = a_1 \underbrace{E[x_{i,j-1} x_{i-1,j+1}]}_{K} + a_2 \underbrace{E[x_{i-1,j} x_{i-1,j+1}]}_{L} + a_3 \underbrace{E[x_{i-1,j+1} x_{i-1,j+1}]}_{M}$$

solve for a_1, a_2, a_3,

① ; $Aa_1 + Ba_2 + Ca_3 = D$

② ; $Ea_1 + Fa_2 + Ga_3 = H$

③ ; $Ka_1 + La_2 + Ma_3 = N$

$$\begin{bmatrix} A & B & C \\ E & F & G \\ K & L & M \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} D \\ H \\ N \end{bmatrix}$$

$$\underline{R} \cdot \underline{a} = \underline{r}$$

$$\underline{a} = \underline{R}^{-1} \underline{r}$$

//

⑦ From binary vector $[N, W, NW, NE, NN, WW, 2N-NN, 2W-WW]$

Each component is set to 1 or 0

(1) Consider sub-component $[N, NN, (2N-NN)]$ → because of dependencies

There are 6 combinations

1	1	0/1
0	0	0/1
1	0	1
0	1	0

$$\begin{aligned} N<\hat{x} &\rightarrow 0<2N<2\hat{x} \\ NN<\hat{x} &\rightarrow -NN>-\hat{x} \\ & \quad -\hat{x}<-NN<0 \end{aligned} \quad +$$

$$\boxed{-\hat{x}<2N-NN<2\hat{x}}$$

$$\begin{aligned} N>\hat{x} &\rightarrow 2\hat{x}<2N<+\infty \\ NN>\hat{x} &\rightarrow -\infty<-NN<-\hat{x} \end{aligned} \quad +$$

$$\boxed{-\infty<2N-NN<+\infty}$$

$$\begin{aligned} N>\hat{x} &\rightarrow 2N>2\hat{x} \\ NN<\hat{x} &\rightarrow -NN>-\hat{x} \end{aligned} \quad +$$

$$\boxed{2N-NN>\hat{x}}$$

$$\begin{aligned} N<\hat{x} &\rightarrow 2N<2\hat{x} \\ NN>\hat{x} &\rightarrow -NN<-\hat{x} \end{aligned} \quad +$$

$$\boxed{2N-NN<\hat{x}}$$

(2) There are 6 combinations for sub-component $[W, WW, \underline{2W-WW}]$ ↓ component is not independent

(3) Sub-component $[NW, NE]$ has 4 combinations.
$\quad$ 0/1 $\quad$ 0/1

Therefore, total # of combinations for the 8-component vector

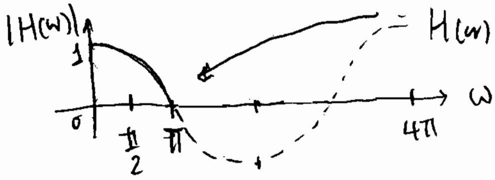
is $6 \times 6 \times 4 = 144$

In general, there are 256 possible vectors for 8-component binary vector //

⑧

(A) $h_0(n) = [\frac{1}{2} \ \ \frac{1}{2}]$

$$H(\omega) = \sum_{n=0}^{1} h_0(n) e^{-j\omega n} = \frac{1}{2} + \frac{1}{2} e^{-j\omega} = e^{-j\frac{\omega}{2}} \left(\frac{e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}}{2} \right) = e^{-j\frac{\omega}{2}} \cos\left(\frac{\omega}{2}\right)$$



$H(\omega)$ is 0 at $\omega = \pi$, $H(\omega)$ is LPF

$$\begin{cases} \frac{\omega}{2} = 0 \rightarrow \omega = 0 \\ \frac{\omega}{2} = 2\pi \rightarrow \omega = 4\pi \end{cases}$$

$\angle H(\omega) = -\frac{\omega}{2}$ → linear phase //

(B) For QMF, given $h_0(n)$

$h_1(n) = (-1)^n h_0(n)$, $H_1(\omega) = H_0(\omega + \pi)$, $H_1(z) = H_0(-z)$

$g_0(n) = c(-1)^n h_1(n)$, $G_0(\omega) = c H_1(\omega + \pi)$, $G_0(z) = c H_1(-z)$

$g_1(n) = -c(-1)^n h_0(n)$, $G_1(\omega) = -c H_0(\omega + \pi)$, $G_1(z) = -c H_0(-z)$

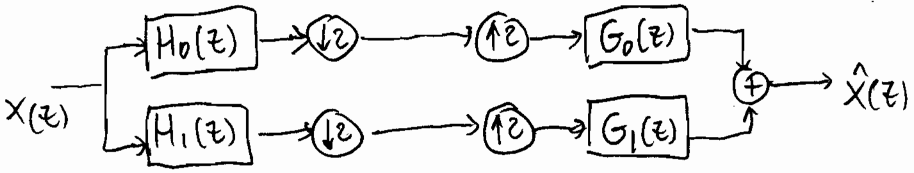
$h_1(n) = [\frac{1}{2}, -\frac{1}{2}]$

$g_0(n) = [\frac{1}{2}, \frac{1}{2}]$

$g_1(n) = [-\frac{1}{2}, \frac{1}{2}]$

for $c = 1$

(C)



$$PR \begin{cases} G_0(z) H_0(z) + G_1(z) H_1(z) = c z^{-n_0} & \text{—①} \\ G_0(z) H_0(-z) + G_1(z) H_1(-z) = 0 & \text{—②} \end{cases}$$

only scaling & delay

(No amplitude distortion)

(No aliasing distortion)

⑧

(D) $h_0(n) = [\frac{1}{2} \ \ \frac{1}{2}] \rightarrow H_0(z) = \frac{1}{2} + \frac{1}{2}z^{-1}$

$h_1(n) = [\frac{1}{2} \ \ -\frac{1}{2}] \rightarrow H_1(z) = \frac{1}{2} - \frac{1}{2}z^{-1}$

$g_0(n) = c[\frac{1}{2} \ \ \frac{1}{2}] \rightarrow G_0(z) = \frac{c}{2} + \frac{c}{2}z^{-1}$

$g_1(n) = c[-\frac{1}{2} \ \ \frac{1}{2}] \rightarrow G_1(z) = -\frac{c}{2} + \frac{c}{2}z^{-1}$

From ①; $(\frac{c}{2} + \frac{c}{2}z^{-1})(\frac{1}{2} + \frac{1}{2}z^{-1}) + (-\frac{c}{2} + \frac{c}{2}z^{-1})(\frac{1}{2} - \frac{1}{2}z^{-1})$

$= \cancel{\frac{c}{4}} + \underline{\frac{c}{4}z^{-1} + \frac{c}{4}z^{-1}} + \cancel{\frac{c}{4}z^{-2}} - \cancel{\frac{c}{4}} + \underline{\frac{c}{4}z^{-1} + \frac{c}{4}z^{-1}} - \cancel{\frac{c}{4}z^{-2}}$

$= cz^{-1}$ is in the form cz^{-n_0} //

From ②; $(\frac{c}{2} + \frac{c}{2}z^{-1})(\frac{1}{2} - \frac{1}{2}z^{-1}) + (-\frac{c}{2} + \frac{c}{2}z^{-1})(\frac{1}{2} + \frac{1}{2}z^{-1})$

$= \cancel{\frac{c}{4}} - \cancel{\frac{c}{4}z^{-1}} + \cancel{\frac{c}{4}z^{-1}} - \cancel{\frac{c}{4}z^{-2}} - \cancel{\frac{c}{4}} - \cancel{\frac{c}{4}z^{-1}} + \cancel{\frac{c}{4}z^{-1}} + \cancel{\frac{c}{4}z^{-2}}$

$= 0$ //

∴ satisfy both conditions of PR //

(E) c can be any constant.

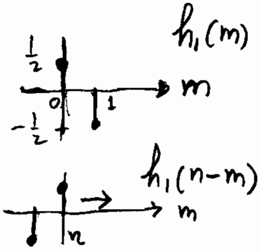
$n_0 = 1$ //

⑨ Given an image

$$\begin{array}{c|cccc} 10 & 10 & 10 & 12 & 8 \\ \hline 10 & 10 & 10 & 12 & 8 \\ 12 & 12 & 8 & 10 & 10 \\ 14 & 14 & 10 & 6 & 8 \\ 6 & 6 & 6 & 6 & 6 \end{array}$$

Boundary reflection

$h_1(n) = [\frac{1}{2} \ -\frac{1}{2}]$



$$y(n) = \sum_m h_1(n-m)\, x(m)$$

Apply h_0 (LPF) along row ⇉

$$\begin{array}{cccc} 10 & 10 & 11 & 10 \\ \hline 10 & 10 & 11 & 10 \\ 12 & 10 & 9 & 10 \\ 14 & 12 & 8 & 7 \\ 6 & 6 & 6 & 6 \\ & \times & & \times \end{array} \quad ⓶↓$$

Apply $h_1(n)$ (HPF) along row ⇉

$$\begin{array}{cccc} 0 & 0 & 1 & -2 \\ \hline 0 & 0 & 1 & -2 \\ 0 & -2 & 1 & 0 \\ 0 & -2 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ & \times & & \times \end{array} \quad \boxed{\downarrow 2}$$

Apply h_0 along column ↓↓

$$\begin{array}{ccc} & 10 & 11 \\ \times & 11 & 5 \\ & 13 & 8.5 \\ \times & 10 & 7 \end{array}$$

$\underbrace{\qquad}_{LL}$

Apply h_1 along column ↓↓

$$\begin{array}{ccc} & 0 & 0 \\ \times & 1 & -1 \\ & 1 & -0.5 \\ \times & -4 & -1 \end{array}$$

$\underbrace{\qquad}_{LH}$

$\boxed{\downarrow 2}$

Apply h_0 along column ↓↓

$$\begin{array}{ccc} & 0 & 1 \\ \times & 0 & 1 \\ & 0 & -0.5 \\ \times & 0 & -1 \end{array}$$

$\underbrace{\qquad}_{HL}$

Apply h_1 along column ↓↓

$$\begin{array}{ccc} & 0 & 0 \\ \times & 0 & 0 \\ & 0 & -1.5 \\ \times & 0 & 1 \end{array}$$

$\underbrace{\qquad}_{HH}$

$$\therefore \begin{array}{cc|cc} 10 & 11 & 0 & 0 \\ 13 & 8.5 & 1 & -0.5 \\ \hline 0 & 1 & 0 & 0 \\ 0 & -0.5 & 0 & -1.5 \end{array} \equiv \begin{array}{c|c} LL & LH \\ \hline HL & HH \end{array}$$

//