The University of Texas
ARLINGTON

EE5351 Digital Video Coding

INSTRUCTOR: Dr. K.R. Rao

Summer 2008, Test 2

Thursday, 10 July 2008

6:00 – 7:50 PM (1 hour and 50 minutes)

(OPEN THE TEXTBOOK ONLY, NO INSERTS, CLOSE ALL NOTES)

(NOT ALLOWED CLASSNOTES, NO EXTRA NOTES, NO PREVIOUS EXAM SOLUTIONS)

INSTRUCTIONS:

1. Close notes, open ONLY the textbook.
2. Calculator is allowed.
3. Please show all the steps in your work.
4. You can work problems in any order.
   At the end please rearrange as 6, 7, 8, and 9.
5. Please print your name and student ID.
6. No cheating, no talking.

Name__________________________

Student ID______________________

PART 1 (Q1) to (Q5) with **[4 Points]** each

(Q1) For a uniform quantizer on a uniform pdf of input sequence, if the number of quantization levels increases, what conclusion can be drawn? (Pick one answer.)

A. SNR increases
B. Bit rate increases
C. MSQE decreases
D. Both B and C are correct
E. A, B, and C are correct

(Q2) In forcing a uniform quantization on a non-uniform pdf of the input sequence, and given a fixed number of quantization levels, what conclusion(s) is (are) TRUE about changing the value of step size? Write the answer(s) _____________

A. If step size increases, then granular noise increases.
B. If step size decreases, then overload noise increases.
C. If step size increases, then granular noise decreases.
D. If step size decreases, then overload noise decreases.
E. None of the above.

(Q3) Which conclusion(s) about a scalar quantizer is (are) INCORRECT ?
Write the answer(s) _____________

A. Quantization is a nonlinear operation.
B. Quantization error cannot be recovered.
C. Quantization is a lossy process.
D. Quantization error can be minimized during the design of a quantizer
E. None of the above

(Q4) Output of the quantizer is required to contain zero '0'. Which uniform quantizer(s) can be used? Write answer(s) _____________

A. Midrise quantizer
B. Midtread quantizer
C. Quantizer with deadzone
D. None of the above

(Q5) What is (are) advantage(s) of LBG algorithm?
Write answer(s) _____________

A. The algorithm guarantees a desirable final codebook from an initial codebook.
B. The algorithm's computations grow linearly with vector dimension and desired bit rate.
C. The algorithm is independent of the distortion measure.
D. The algorithm does not require explicit probabilistic models of input vectors.
E. The algorithm is well-designed and is practical at high rate and high dimensionality vector quantizers.

PART 2 (Q6) to (Q9) with **[20 Points]** each

[20 Points][**Problem 6**] (SQ)

Suppose we have a source that can be modeled as a random variable taking values in the interval $[-4,4]$ with more probability mass near the origin than away from it. We want to quantize this using the compander which composes of compressor, 3-bit uniform quantizer, and an expander. The compressor characteristic we will use, is given by the following equation,

$$c(x)=\begin{cases} 2x & , -1 \le x \le 1 \\ \dfrac{2x}{3}+\dfrac{4}{3} & , x>1 \\ \dfrac{2x}{3}-\dfrac{4}{3} & , x<-1 \end{cases}$$

The mapping is shown graphically in Fig. 1. The inverse mapping is given by,

$$c^{-1}(x)=\begin{cases} \dfrac{x}{2} & , -2 \le x \le 2 \\ \dfrac{3x}{2}-2 & , x>2 \\ \dfrac{3x}{2}+2 & , x<-2 \end{cases}$$

The inverse mapping is shown in Fig. 2. For this companding quantizer, what are the outputs for the following inputs: $-0.8,\ 1.2,\ 0.5,\ 0.6,\ 3.2,\ -0.3$. Compare your results with the case when the input is directly quantized with a uniform quantizer with the same number of levels.

(Question Continues Next Page.)

([Problem 6] Continues ...)

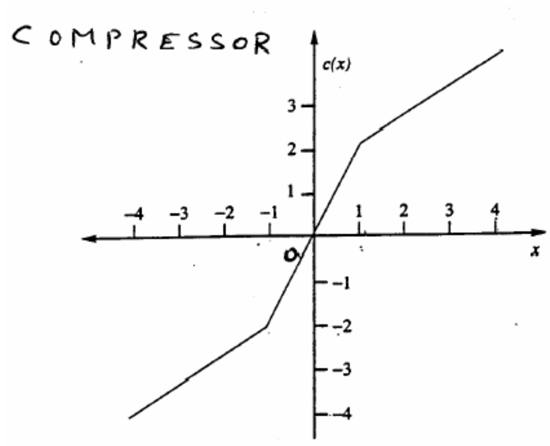


Figure 1.

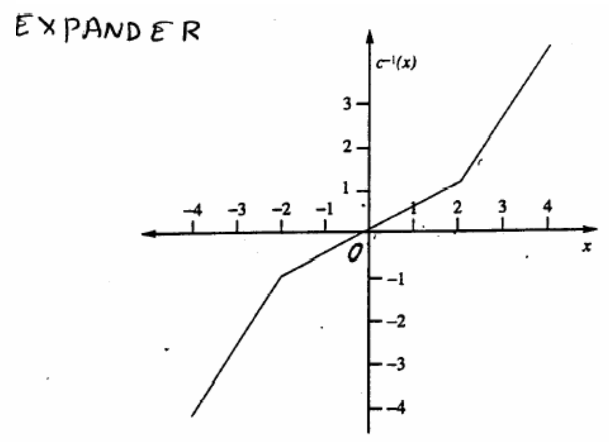


Figure 2.

(A)[2 Points] Determine the step size of the uniform quantizer.

(B)[2 Points] Show the result from the compressor.

(C)[2 Points] Show the result from the expander.

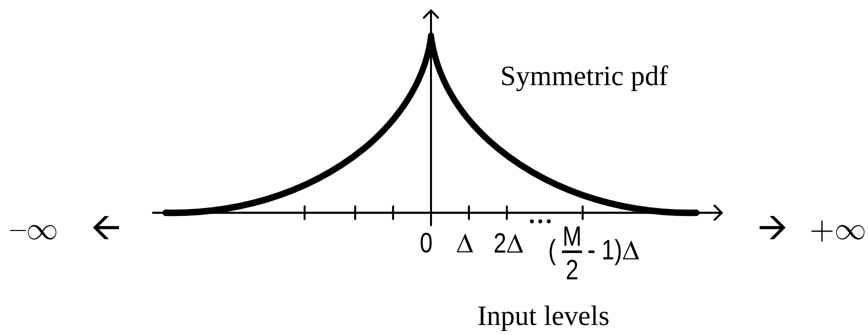
(D)[6 Points] Calculate the compander error of each input, and MSE.

(E)[2 Points] Show the result for the case that input is directly quantized by the uniform quantizer.

(F)[6 Points] Calculate the uniform quantizer error of each input of [Problem 6]**(E)**, and MSE.

[20 Points][**Problem 7**] (Scalar quantization)

Forcing a uniform quantizer on a non-uniform symmetric pdf, Δ = step size, M = number of output levels.



Given MSQE, $$\sigma_q^2 = \left[2\sum_{i=1}^{\frac{M}{2}-1}\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right)^2 f_X(x)\,dx\right] + \left[2\int_{\left(\frac{M}{2}-1\right)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right) f_X(x)\,dx\right]$$

(A)[6 Points] In these two terms, identify the overload noise and granular noise.

(B)[8 Points] To find the optimal value of Δ that minimizes the MSQE, we take a derivative of MSQE, σ_q^2, with respect to Δ and set it to zero. Show that result is as shown below,

$$\frac{\partial\sigma_q^2}{\partial\Delta} = -\left[\sum_{i=1}^{\frac{M}{2}-1}(2i-1)\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right) f_X(x)\,dx\right] - \left[(M-1)\int_{\left(\frac{M}{2}-1\right)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right) f_X(x)\,dx\right] = 0$$

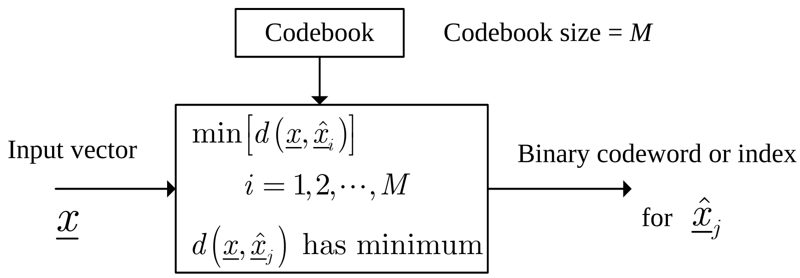
Use Leibnitz's rule, $\frac{\partial}{\partial t}\int_{a(t)}^{b(t)} g(y,t)\,dy = \int_{a(t)}^{b(t)}\frac{\partial g(y,t)}{\partial t}\,dy + g\left(b(t),t\right)\frac{\partial b(t)}{\partial t} - g\left(a(t),t\right)\frac{\partial a(t)}{\partial t}$.

Show all steps.

(C)[6 Points] How does Δ affect the granular noise and overload noise.

[20 Points][**Problem 8**] (Vector quantization)

VQ codebook contains M codewords (codebook size). Each codeword is fixed length binary coded.



(A)[4 Points] How many codebook indices are used for this quantizer?

(B)[4 Pionts] Calculate the number of bits needed to transmit each codebook index, for FLC (fixed-length code).

(C)[4 Points] Given each input vector has L components (dimension of input vector), find the compression rate of the encoder in bpp (bits per component).

(D)[8 Points] Explain in detail the splitting algorithm for designing a codebook. What is an empty cell problem and how can this be solved.

[20 Points][**Problem 9**] (LBG)

Using LBG algorithm, design the codebook for these training vectors (Choose the threshold $\varepsilon = 0.001$

$\underline{x}_1 = [-0.5,\ 1];$ $\quad \underline{x}_6 = [1.1, 0.5];$

$\underline{x}_2 = [0.6, -0.1];$ $\quad \underline{x}_7 = [-0.6, 0.2];$

$\underline{x}_3 = [-0.8, 0.6];$ $\quad \underline{x}_8 = [0.1, 1.7];$

$\underline{x}_4 = [-0.7, -1.2];$ $\quad \underline{x}_9 = [0.7, -0.3];$

$\underline{x}_5 = [-0.2, -0.9];$ $\quad \underline{x}_{10} = [0.3, 4.8];$

Start with a uniform quantizer (codebook). Show all steps.

$$(\hat{\underline{x}}_1, \hat{\underline{x}}_2, \hat{\underline{x}}_3, \hat{\underline{x}}_4) = \left(\frac{S_1}{2,2}\right), \left(\frac{S_2}{2,-2}\right), \left(\frac{S_3}{-2,2}\right), \left(\frac{S_4}{-2,-2}\right)$$

END OF TEST QUESTIONS