5359 Final exam 5/14/97 Solution
(Intro to DC' K. Sayood)

3

LPF: 8-tap Johnston LPF
see Table 11.1 / p. 298

$\{ h_0 \; h_1 \; h_2 \; h_3 \; h_4 \; h_5 \; h_6 \; h_7 \}$ $(h_n = h_{N-1-n})$

Symmetric filter $h_0 \; h_1 \; h_2 \; h_3 \; h_3 \; h_2 \; h_1 \; h_0$

frequency response is $\sum_{n=0}^{7} h_n e^{-jn\omega}$

K.R. RAO

For symmetric filter, $H(e^{j\omega})$ is

$$\left[h_0 + h_7 e^{-j7\omega} + h_1 e^{-j\omega} + h_6 e^{-j6\omega} + h_2 e^{-j2\omega} + h_5 e^{-j5\omega} + h_3 e^{-j3\omega} + h_4 e^{-j4\omega}\right]$$

$$= \Big[(h_0)\left(e^{j\frac{7}{2}\omega} + e^{-j\frac{7}{2}\omega}\right) + (h_1)\left(e^{j\frac{5}{2}\omega} + e^{-j\frac{5}{2}\omega}\right) + (h_2)\left(e^{j\frac{3}{2}\omega} + e^{-j\frac{3}{2}\omega}\right) + (h_3)\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)\Big] e^{-j\frac{7}{2}\omega}$$

(For symmetric filter $h_0 = h_7$, $h_1 = h_6$, $h_2 = h_5$ and $h_3 = h_4$ $(h_n = h_{N-1-n})$)

LPF

$$H(e^{j\omega}) = 2\left[h_0 \cos\tfrac{7}{2}\omega + h_1 \cos\tfrac{5}{2}\omega + h_2 \cos\tfrac{3}{2}\omega + h_3 \cos\tfrac{\omega}{2}\right] e^{-j\frac{7}{2}\omega}$$

Linear phase filter

Linear phase FIR filter

HPF: $H_{HP,n} = H_{LP,n}(-1)^n$

(antisymmetric)

$$H(e^{j\omega}) = 2\left[h_0 \sin\tfrac{7}{2}\omega - h_1 \sin\tfrac{5}{2}\omega + h_2 \sin\tfrac{3}{2}\omega - h_3 \sin\tfrac{\omega}{2}\right] j e^{-j\frac{7}{2}\omega}$$

prob. 2 :

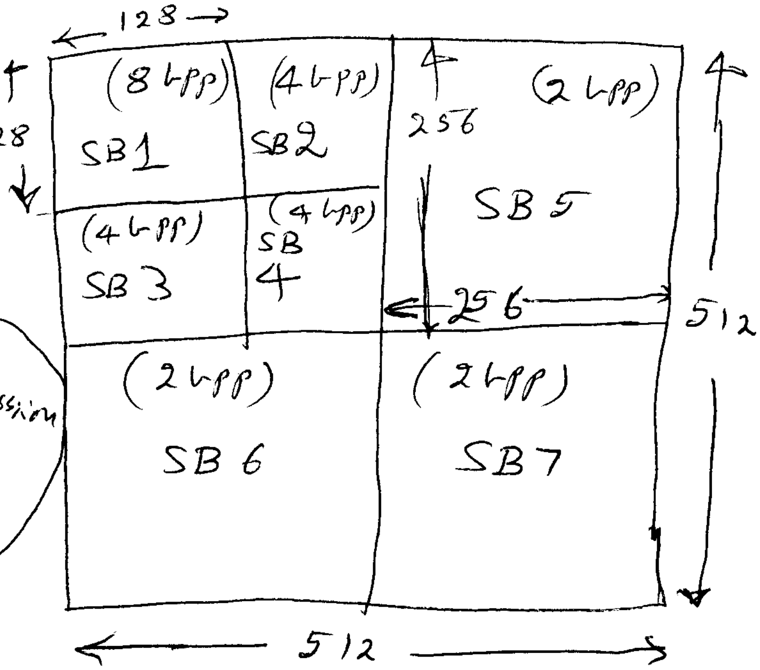
channel

(9.6 kbps)

original image transmission

$$\frac{(512 \times 512) \times 8}{9.6 \times 10^3} \text{ sec}$$

$= 218.45$ sec



stage 1

$$13.653 \text{ sec} = \frac{(128 \times 128) \times 8}{9.6 \times 10^3} \text{ sec}$$

stage 2

$$20.48 \text{ sec} = \frac{(128 \times 128) \times 4 \times 3}{9.6 \times 10^3} \text{ sec}$$

stage 3

$$40.96 \text{ sec} = \frac{(256 \times 256) \times 2 \times 3}{9.6 \times 10^3} \text{ sec}$$

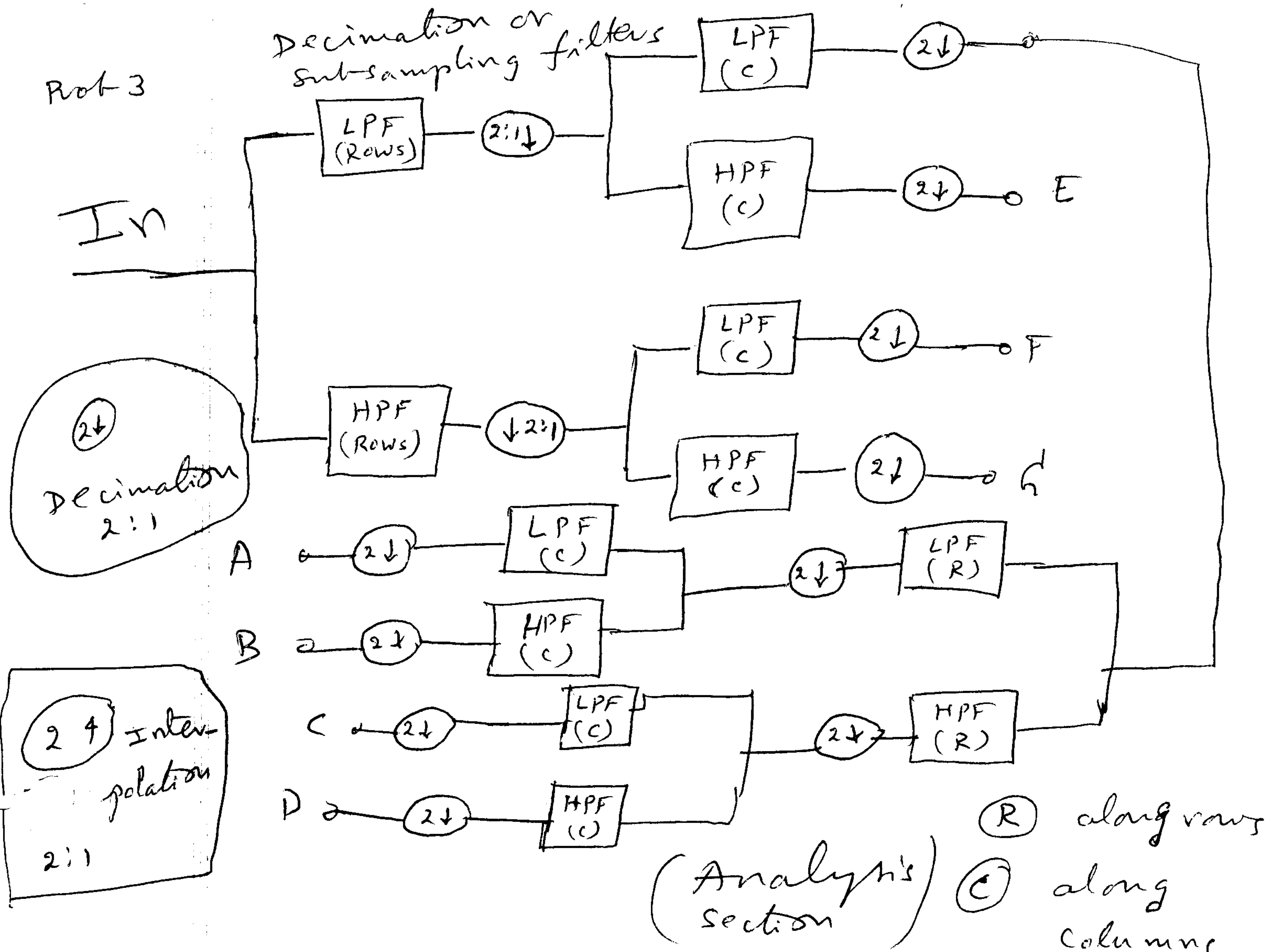
Fraction of image sent during I stage $\left(\frac{13.653}{218.45}\right)$
(F O I SD) $= 0.0624$

$$\text{FOISD II stage} = \frac{20.48}{218.45} = 0.09375$$

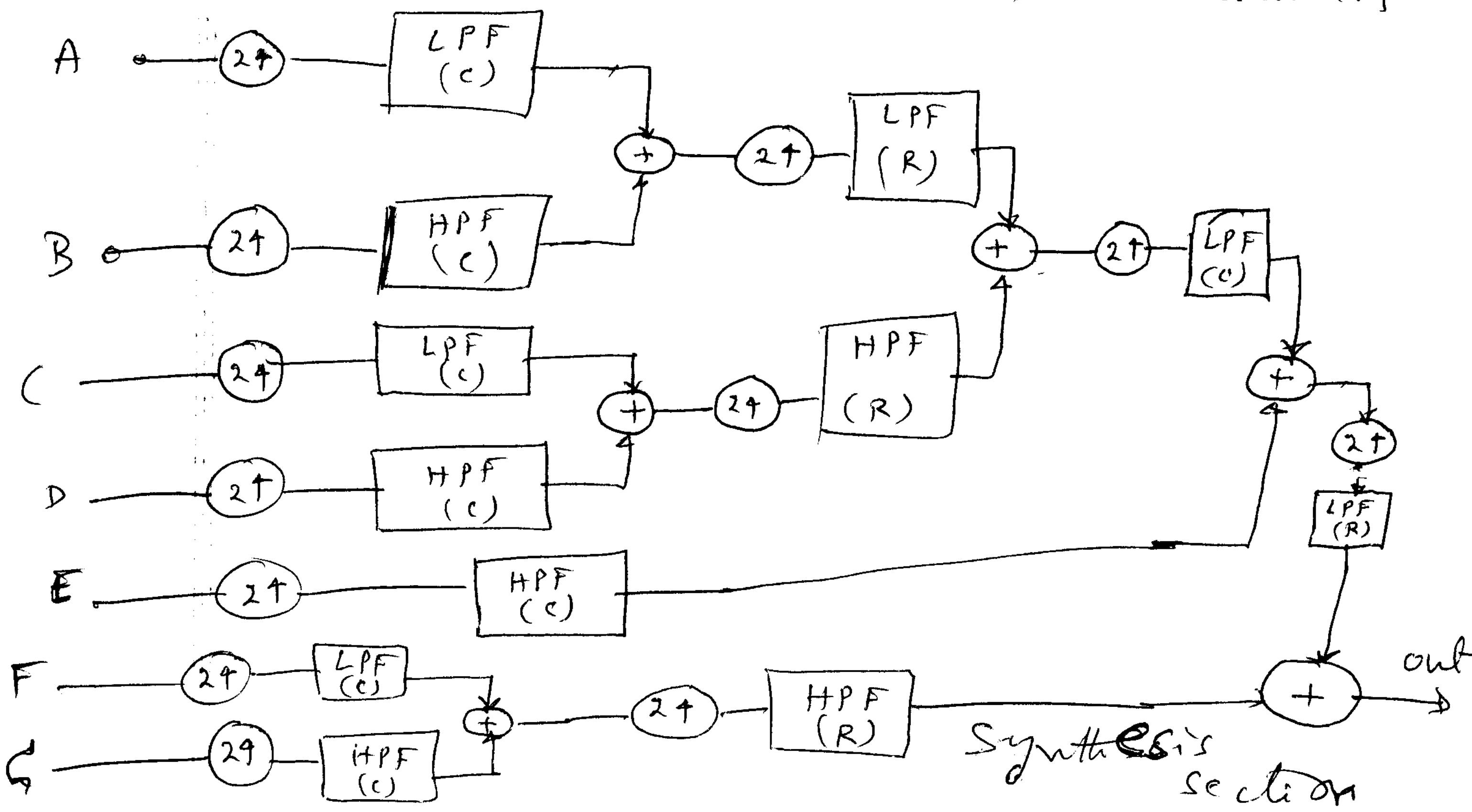
$$\text{FOISD III stage} = \frac{40.96}{218.45} = 0.1875$$

$$\text{FOISD (I+II+III) stages} = 0.3437$$

Prob 3



Decimation or sub-sampling filters
In
LPF (Rows)
2:1↓
HPF (Rows)
↓2:1
LPF (c)
HPF (c)
2↓
E
F
G
2↓ Decimation 2:1
A
B
C
D
LPF (R)
HPF (R)
2↑ Inter-polation 2:1
R along rows
C along columns
(Analysis section)



A
B
C
D
E
F
G
2↑
LPF (c)
HPF (c)
LPF (R)
HPF (R)
+
out
Synthesis section

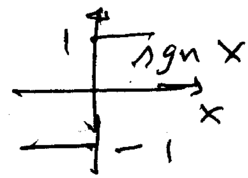
Compandor

Expander

$$c^{-1}(x) = \frac{x_{max}}{\mu}\left[(1+\mu)^{\frac{|x|}{x_{max}}} - 1\right] \operatorname{sgn} x$$

Consider $x > 0$ & let $c^{-1}(x) = y$

$$y = \frac{x_{max}}{\mu}\left[(1+\mu)^{x/x_{max}} - 1\right]$$



$$\left[\frac{\mu y}{x_{max}} + 1\right] = (1+\mu)^{x/x_{max}}$$

$$\ln\left(1 + \mu \frac{y}{x_{max}}\right) = \left[\frac{x}{x_{max}}\right] \ln(1+\mu)$$

$$x = \frac{x_{max}}{\ln(1+\mu)} \ln\left(1 + \mu \frac{y}{x_{max}}\right)$$

∴ compressor

$$c(x) = \left(\frac{x_{max}}{\ln(1+\mu)}\right) \ln\left(1 + \mu \frac{x}{x_{max}}\right)$$

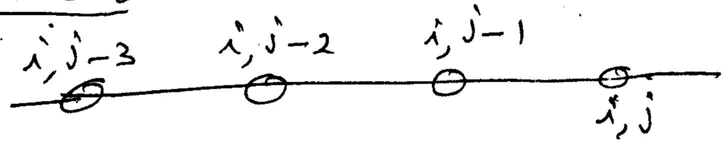
for $x > 0$

Similar derivation for $x < 0$

which yields Eq. (8-53)/p. 204 (Sayood)

Linear Predictor

DPCM



$$\hat{x}_{i,j} = a_1 x_{i,j-1} + a_2 x_{i,j-2} + a_3 x_{i,j-3}$$

Minimize $MSE = E\left[\left(x_{i,j} - \hat{x}_{i,j}\right)^2\right]$.

$$E\left[\left(x_{i,j} - \sum_{l=1}^{3} a_l x_{i,j-l}\right)^2\right]$$

Variables are a_1, a_2, a_3.

set $\left[\dfrac{\partial (MSE)}{\partial a_l}\right] = 0, \quad l = 1, 2, 3$

to get

$$E\left[2\left(x_{i,j} - \sum_{l=1}^{3} a_l x_{i,j-l}\right) - x_{i,j-K}\right] = 0, \qquad K = 1, 2, 3$$

$$E\left[x_{i,j} \; x_{i,j-K}\right] = E\left[x_{i,j-K} \sum_{l=1}^{3} a_l x_{i,j-l}\right]$$

$(K = 1, 2, 3)$

$$\begin{bmatrix} R_{xx}(1) \\ R_{xx}(2) \\ R_{xx}(3) \end{bmatrix} = \begin{bmatrix} R_{xx}(0) & R_{xx}(1) & R_{xx}(2) \\ R_{xx}(1) & R_{xx}(0) & R_{xx}(1) \\ R_{xx}(2) & R_{xx}(1) & R_{xx}(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\underline{r} = \underline{R} \; \underline{a}$$

or

$$\underset{(3\times 1)}{\underline{a}} = \underset{(3\times 3)}{\underline{R}^{-1}} \; \underset{(3\times 1)}{\underline{r}}$$