

3), [25 points]

Symbol	Probability
S_0	0.4
S_1	0.2
S_2	0.2
S_3	0.1
S_4	0.1

- Find Huffman Code and get its average code length.
- Find Minimum variance Huffman Code and get its average code length.
- Find the Redundancy.
- Encode the sequence: $S_1S_2S_0S_1S_4S_1S_3$

Using Huffman and Minimum Variance Huffman Codes.

Now, suppose the SECOND bit is received in error at the receiver.

Decode using both Huffman and Minimum Variance Huffman codes.

How many characters are received in error before the first correctly decoded character in each case?

4), [15 points]

For an alphabet $A = \{a_1, a_2, a_3\}$ with probabilities

$$P(a_1) = 0.7, P(a_2) = 0.2, P(a_3) = 0.1$$

Design a 3-bit Tunstall code

5), [20 points]

Given the probability model in the following table, find the real valued tag for the sequence: $S_1S_1S_3S_2S_1$

letter	probability
S_1	0.2
S_2	0.3
S_3	0.5

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Index	Codebook entry
1	a
2	b
3	h
4	i
5	s
6	t
7	th
8	hi
9	is
10	sb
11	bh
12	ha
13	at
14	tb
15	bi
16	isb
17	bhi
18	isbh
19	hat
20	tbi
21	it
22	tbis
23	sbh
24	his
25	sbha
26	at

6 3 4 5 2 3 1 6 2 9 11 16 12 14 4 20 10 8 23 13
 t h i s b h a t b i s bh isb ha tb i tbi sb hi sbh at

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Codebook entry

1	a
2	b
3	r
4	y
5	ϕ
6	aϕ
7	ϕb
8	ba
9	ar
10	rϕ
11	ϕa
12	arr
13	ra
14	ay
15	yϕ
16	ϕby
17	yϕb
18	bar
19	rr
20	ray
21	ya
22	arϕ
23	

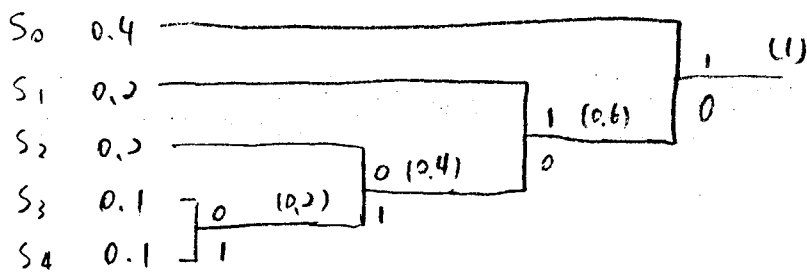
ϕba

abbar barray ϕby ϕbarray arϕbay

1, 5, 2, 1, 3, 5, 9, 3, 1, 4, 7, 15, 8, 3, 13, 4, 9, 7, 14

3.

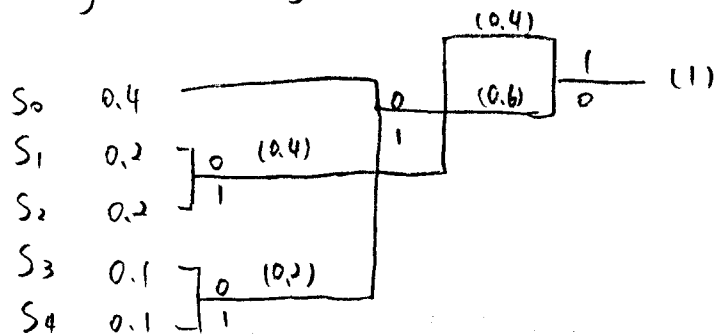
(a)



S_0	1
S_1	01
S_2	000
S_3	0010
S_4	0011

Average Code length: $0.4 \times 1 + 0.2 \times 2 + 0.2 \times 3 + 0.1 \times 4 + 0.1 \times 4 = 2.2$ bits/symbol

(b)



S_0	00
S_1	10
S_2	11
S_3	010
S_4	011

Average Code length: $0.4 \times 2 + 0.2 \times 2 + 0.2 \times 2 + 0.1 \times 3 + 0.1 \times 3 = 2.2$ bits/symbol.

(c) $H = -0.4 \log_2 0.4 - 2 \times 0.2 \log_2 0.2 - 2 \times 0.1 \log_2 0.1 = 2.122$ bits/symbol

Redundancy = $2.2 - 2.122 = 0.078$ bits/symbol

(d) $S_1 S_2 S_0 S_1 S_4 S_1 S_3$

Huffman: $\begin{array}{cccccccc} & 0 & & & & & & & \\ & \times & 000 & 1 & 01 & 0011 & 01 & 0010 & \\ & & 00 & 000 & 1 & 01 & 0011 & 01 & 0010 & \\ \times & S_2 & \times & S_3 & \times & S_0 & \times & S_4 & \times & S_1 & \times & S_3 & \end{array}$

First four characters
 $A_1 A_2 A_0 A_1$ received
wrong.

2, 3, 4 are wrong

Minimum Variance:

$\begin{array}{cccccccc} & 1 & & & & & & & \\ & \times & 11 & 00 & 10 & 011 & 10 & 010 & \\ & & 11 & 11 & 00 & 10 & 011 & 10 & 010 & \\ \times & S_2 & \times & S_2 & \times & S_0 & \times & S_1 & \times & S_4 & \times & S_1 & \times & S_3 & \end{array}$

one is wrong

4	letter	Probability
First Iteration	a_1	0.7
	a_2	0.2
	a_3	0.1

Second Iteration	Letter	Probability
	a_2	0.2
	a_3	0.1
	$a_1 a_1$	0.49
	$a_1 a_2$	0.14
	$a_1 a_3$	0.07

Third Iteration	Letter	Probability	code
	a_2	000	0.2
	a_3	001	0.1
	$a_1 a_2$	010	0.14
	$a_1 a_3$	011	0.07
	$a_1 a_1 a_1$	100	0.343
	$a_1 a_1 a_2$	101	0.098
	$a_1 a_1 a_3$	110	0.049

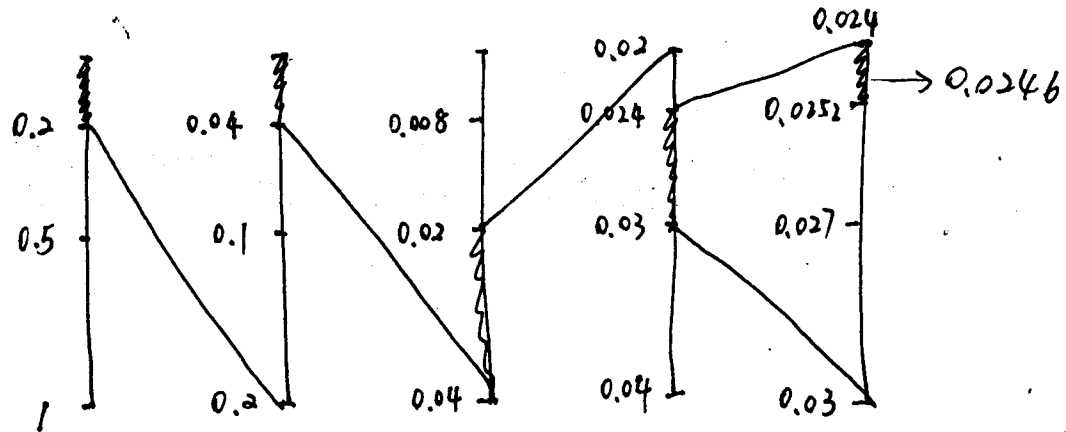
5. $S_1 S_1 S_3 S_2 S_1$

$$F_x(0) = 0$$

$$F_x(1) = 0.2$$

$$F_x(2) = 0.5$$

$$F_x(3) = 1$$



$$l^{(0)} = 0$$

$$x_1 = 1$$

$$u^{(0)} = 1$$

$$l^{(1)} = 0 + 1 \cdot F_x(0) = 0$$

$$x_2 = 1$$

$$u^{(1)} = 0 + 1 \cdot F_x(1) = 0.2$$

$$l^{(2)} = 0 + 0.2 F_x(0) = 0$$

$$x_3 = 3$$

$$u^{(2)} = 0 + 0.2 F_x(1) = 0.04$$

$$l^{(3)} = 0 + 0.04 F_x(2) = 0.02$$

$$x_4 = 2$$

$$u^{(3)} = 0 + 0.04 F_x(3) = 0.04$$

$$l^{(4)} = 0.02 + 0.02 F_x(1) = 0.024$$

$$x_5 = 3$$

$$u^{(4)} = 0.02 + 0.02 F_x(2) = 0.03$$

$$l^{(5)} = 0.024 + 0.006 F_x(0) = 0.024$$

$$u^{(5)} = 0.024 + 0.006 F_x(1) = 0.0252$$

$$x_6 = 1$$

$$\text{tag} = \frac{l^{(5)} + u^{(5)}}{2} = 0.0246$$