Integration of pulses

At this point we will simply make some statements about integration of pulses.

Previous results relate to a single pulse. In practice, we will receive a number of pulses from a target during a scan.

\[
\text{number of pulses} = \frac{\theta_b \cdot \text{PRF}}{\dot{\theta}_s}
\]

\[
\dot{\theta}_s \leftarrow \text{Antenna Beamwidth (Deg)}
\]

\[
\dot{\theta}_s \leftarrow \text{Angular scan rate (Deg/sec)}
\]

For example if \( \dot{\theta}_s = 5 \text{ rpm} \)

\[
= \left( \frac{360^\circ}{\text{rev}} \right) \left( \frac{5 \text{rev}}{\text{min}} \right) \left( \frac{1 \text{min}}{60 \text{sec}} \right)
\]

\[
= (6)(5) = 30^\circ / \text{sec}
\]

and \( \theta_b = 1.5^\circ \), then

\[
\text{number of pulses} = \frac{\theta_b \cdot \text{PRF}}{\dot{\theta}_s} = \frac{1.5 \text{}(300)}{30} = 15 \text{ pulses}
\]

Hence, we can sum pulses to improve S/N ratio. Summing the echo pulses is called integration.
Two techniques:
1. Pre-detection
2. Post-detection

Pre-detection is coherent but more difficult. In this case, voltages add in phase such that the signal is \( n \cdot V_{\text{single}} \) or signal power \( n^2 P_{\text{single}} \). Noise tends to add as \( n \cdot N_{\text{single}} \).

Post-detection is incoherent but some improvement in \( S/N \) can be obtained.

The \((S_0/N_0)_{\text{min}}\) shown previously was a per-pulse value. Consider what effect integration has.
EE 5331

We can express as \((S/N)_{\text{min}} = \frac{(S/N)}{nE_i(n)}\)

Number of pulses integrated

For a predetection integrator,
\(E_i(n) \equiv 1\), while for a postdetection integrator,
\(n^{-\frac{1}{2}} \leq E_i(n) \leq 1\).

To illustrate, notice from Fig. 9-8 of the text that \((S/N)_{\text{min}} = 15.8 \text{ dB}\) is required to obtain \(P_{\text{FA}} = 10^{-12}\) and \(P_0 = 0.9\). From the handout, if 1000 pulses are postdetection integrated (square law detector), \(nE_i(n) \equiv 130\)

so \((S/N)_{\text{min}} = 15.8 - 10 \log(130) = -5.3 \text{ dB}\)

Note that we can rewrite the equation for \(R_{\text{max}}\) as

\[R_{\text{max}} = \left[ \frac{P_t + G^2 r^2 \sigma ^2 \cdot nE_i(n)}{(4\pi)^3 kTB F_n L (S/N)} \right]^{\frac{1}{4}}\]

Before going on to MTI and pulse Doppler, we will look at one more version of the radar equation.

In the above equation \(P_t\) is "peak power". \(P_{\text{avg}} = P_t \tau F_p = P_t \frac{\tau}{T}\)

Since \(\frac{\tau}{T}\) = "duty cycle"
Hence, \( R_{\text{max}} = \left( \frac{P_{\text{av}} - G^2 \sigma^2 n E_i(n)}{(4\pi)^3 k T_{\text{m}}(B, L) L(\text{SNR})} \right)^{\frac{1}{4}} \)

or in terms of energy per pulse since \( E_r = \text{energy per pulse} = \frac{P_{\text{av}}}{f_p} \)

so \( R_{\text{max}} = \left[ \frac{E_r G^2 \sigma^2 n E_i(n)}{(4\pi)^3 k T_{\text{m}}(B, L) L(\text{SNR})} \right]^{\frac{1}{4}} \)

**Fundamental difference?**

<table>
<thead>
<tr>
<th>MTI - moving target indicator</th>
<th>Ambiguous Doppler, Unambiguous range, Low PRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse Doppler</td>
<td>Unambiguous Doppler, Ambiguous range, High PRF</td>
</tr>
</tbody>
</table>
Pulse compression

What is the purpose of pulse compression?
Pulse compression allows a radar system to transmit a pulse of relatively long duration and low peak power to attain the range resolution and detection performance of a short-pulse, high-peak power system.

\[ \text{Range resolution achievable} \]
\[ T_r \propto \frac{c}{2B} \]

\[ c - \text{speed of light} \]
\[ B - \text{bandwidth of the transmitted waveform} \]

For a pulse radar with pulse length \( T \), \( T_r = \frac{cT}{2} \)

If we modulate the pulse either frequency or phase, we can obtain a pulse of width \( T \) but which has a bandwidth \( B \gg \frac{1}{T} \). If we let \( \tau = \frac{1}{B} \), \( T_r = \frac{c\tau}{2} \). This implies that by modulating a pulse of width \( T \) we can obtain an effective pulse width of \( \tau \). The ratio \( T/\tau \) is referred to as the pulse compression ratio.
Chirp pulse system

Oscillator → Modulator → Dispenser (SAW) → Amp → Duplexer

Further processing:

Pulse filter (SAW) → Amp
The system shown is an example of an FM chirp-pulse system. The purpose of the dispersive delay (SAW) device is to spread out the frequency information as:

For example, consider a sinc pulse modulation:

If we pass this through the dispersive delay device, the time domain signal becomes of the form or dispersion could be in other direction.
The pulse compression filter is a matched filter that does the opposite of the dispersion device, i.e.,
goes from

\[ \text{in this case the matched filter (correlator) behaves as} \]

Example:
Consider a pulse compression radar operating at Ku-band utilizing a 200ns pulse. \( \Delta f = \frac{500}{2} = 30 \text{MHz} \)
Assume now that the 1200ns pulse is chirp modulated over a bandwidth of 500 MHz. This improves \( \Delta \tau \) to
\[ \Delta \tau = \frac{500}{24} = 2.1 \text{ ns}. \]
Two types of radar applications (that are most common)

**Surveillance/Tracking**

- Moving Target Indicator: high
- Emphasizes high range and low PRF
- Return power range of moving target

![Diagram of clutter and time range](image)
MTI and Pulsed Doppler

MTI - Moving Target Indicator
PRF is too slow to unambiguously reconstruct the Doppler frequency because emphasis is on unambiguous range measurements.

Pulsed Doppler - PRF is sufficiently fast to unambiguously reconstruct the Doppler frequency but too fast to obtain unambiguous range measurements.

We already discussed range ambiguities for pulse radar but what about Doppler ambiguities?

Consider a typical pulse radar waveform:

\[ T = 10^{-6} \text{ s} \]

\[ t_0 = 10^{-3} \text{ s} \]

How can we extract Doppler information?
We can measure Doppler on a pulse-by-pulse basis if \( f_d > \frac{1}{T} = 10^6 \text{Hz} \) but note that this only works for very fast moving targets since

\[
\frac{2v}{c} = 10^6 \Rightarrow v = 15,000 \text{m/s at } \lambda = 0.03 \text{cm}
\]

More typically, Doppler frequency is reconstructed from a sequence of pulses; e.g.,

\[
\sqrt{f_d}
\]

Notice in this case that the maximum Doppler frequency is given by \( f_d \leq \frac{1}{2T} \). In other words we must obtain at least 2 samples per period of the Doppler frequency. Hence, the need for high PRFs for pulsed Doppler radar of aircraft and other high speed targets.
Improved performance for both range and doppler determination can be achieved via:
- staggering the PRF
- switching between modes of operation

Ideally a surveillance or tracking radar will process the returns in terms of range bins and doppler bins:

<table>
<thead>
<tr>
<th>Range 1</th>
<th>Doppler 1</th>
<th>Doppler 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Remote sensing radars (May still, in some cases, be used for surveillance/tracking but most often only with imaging-type radars)
For regions that are statistically uniform with many nondominant scatterers, the idea of scattering coefficients can be applied.

\[ \sigma = (\sigma^0)A \] in that case.
When comparing results from different experimenters (who have measured clutter) it should be noted whether one-way or two-way antenna 3-dB beamwidths were used because the results will be somewhat different.

Aside:

Two different measurement scenarios

(1) Pulse limited

The pulse limited assumption is valid for \( \frac{c\tau}{2} \tan \theta < 2R \tan (\theta/2) \)

If strip is narrow enough in the \( \frac{c\tau}{2} \sec \theta \) dimension, then the area illuminated is approximately

\[ A = 2R \left( \frac{c\tau}{2} \right) \tan (\theta/2) \sec \theta, \]

i.e., a rectangle.
For $\phi_a < 10^\circ$, \(2R\tan(\phi_a/2) = 2R(\phi_a/2) = R\phi_a\)
so \(A = R(\phi_a/2)\phi_a\csc\Theta\)

(2) Beamwidth limited

If $\phi_a/2 \tan\Theta > 2R\tan(\phi_a/2)$ then the beamwidth assumption is valid.

In this case

\[
\frac{\theta_a}{2} \tan\Theta
\]

The area of an ellipse is

\[
\frac{\pi ab}{4}
\]

Hence,

\[
A = \pi R^2 \tan(\phi_a/2) \tan(\phi_a/2) \csc\Theta
\]

For small beamwidths,

\[
A = \frac{\pi R^2}{4} \phi_a \phi_a \csc\Theta
\]
What is the significance of these two measurement scenarios?

For simple scatterometer measurements, the illuminated area may be either beamwidth limited or pulse limited, depending on a number of factors. Near nadir the illuminated area is normally beamwidth limited.

Resolution cells as defined by $\frac{cT}{2}$ for pulse radar or $\frac{c}{2B}$ for frequency modulated radar.

Well off-nadir (pulse-limited case):

Side-looking airborne radar (SLAR) takes advantage of pulse limiting to generate maps of natural terrain.
Antenna generally has fan beam such that $\phi_{az}$ is small (e.g., a few degrees) and $\phi_{el}$ is large (e.g., $>45^\circ$).

Conceivably this type of radar could be used for surveillance. But in reality it is of limited usefulness for surveillance because of its poor resolution. For example, given an altitude of 3,000 m and a radar system with a pulsewidth of 200 ns, the resolution cell at $\Theta = 45^\circ$ has dimensions of

$$\frac{c_2}{2 \sec 45^\circ} = 21.2 \text{ m}$$

by

$$R(0.052) = \frac{b}{0.707} (0.052) = 221 \text{ m}$$

How can this resolution be enhanced? Generally a synthetic aperture technique is used.
Synthetic Aperture Radar (SAR)

By appropriately processing data collected along flight path length \( D \) we can create an antenna with effective beamwidth \( \phi_{\text{eff}} \). (More later)

Satellite-based SARs for remote sensing of the Earth with resolution of \( 3\times3 \) m are currently in operation. But even this resolution is insufficient for many military purposes, e.g., for locating antiaircraft weapons. The military reportedly has in operation space-borne SARs that have a resolution of \( 30\times30 \) cm