I. Performance of Digital Modulation in slow, Flat-Fading channels

ref: "Wireless Communications: principles & practice" 2nd Edition, Theodore S. Rappaport. PP. 340-344.

(1) simple analytical solution for computing BER (bit error rates) in slow flat-fading channel exist.

(2) performance evaluation in Frequency selective channels and computation of outage probabilities are often made through computer simulations.

◎ To evaluate the probability of error of any digital modulation scheme in a slow, flat-fading channel, one must average the probability of error of the particular modulation in AWGN channels over the possible ranges of signal strength due to fading.

The probability of error in slow, flat-fading channe

$$P_{e\_fading} = \int_0^\infty P_{e\_AWGN}(\gamma)\, p_{fading}(\gamma)\, d\gamma$$

$P_{e\_AWGN}(\gamma)$ is the probability of error for an arbitrary modulation at a specific value of signal-to-noise ratio γ. $\gamma = \alpha^2 E_b/N_0$

$p(\gamma)$ is the probability density function of γ due to the fading channel.

E_b & N_o are constants that represent the average energy per bit and noise power density in a non-fading AWGN channel. and the random variable α^2 is used to represent instantaneous power values of fading channel.

For Rayleigh fading channels, the fading power α^2 is

$$p(\gamma) = \frac{1}{\Gamma_0} \exp\left(-\frac{\gamma}{\Gamma_0}\right) \quad \gamma \geq 0 \qquad \text{where } \Gamma_0 = \frac{E_b}{N_o}\,\overline{\alpha^2}$$

For $\overline{\alpha^2} = 1$. $\quad \underline{\Gamma_0 = \frac{E_b}{N_o}}$.

For Example :—

Given: the probability of error of non coherent orthogonal FSK in an AWGN channel

$$P_{e\text{-}AWGN}(\gamma) = \frac{1}{2} \exp\left(-\frac{1}{2}\gamma\right)$$

Find: the probability of error of non coherent orthogonal FSN in Rayleigh fading channel.

$$P_{e\text{-}Raigh\,fading} = \int_0^\infty P_{e\text{-}AWGN}(\gamma) \cdot p(\gamma)\, d\gamma$$

Supplement chap 2-3

$$P_{e\text{-Rayleigh fading}} = \int_0^\infty \frac{1}{2}\exp\left(-\frac{1}{2}\gamma\right)\cdot \underbrace{\frac{1}{\Gamma_0}\exp\left(-\frac{\gamma}{\Gamma_0}\right)} d\gamma$$

$$= \frac{1}{2+\Gamma_0}.$$

with 2-branch Max-Ratio combining, γ distribution is $\frac{\gamma}{\Gamma_0^2} e^{-\frac{\gamma}{\Gamma_0}}$

$$P_{e\text{-selection}} = \int_0^\infty \frac{1}{2}\exp\left(-\frac{1}{2}\gamma\right)\cdot \underbrace{\frac{\gamma}{\Gamma_0^2} e^{-\frac{\gamma}{\Gamma_0}}} d\gamma$$

$$= \frac{2}{(2+\Gamma_0)^2}$$