EE 5368

Wireless Communication Systems

Exam #1
Fall 2009

Student name: Solution

SN #: ____________________

September 23, 2009

The University of Texas at Arlington

Problem 1. (20%)

(a) **Why IS-136 system has more voice channel/radio channel than AMPS system?**

IS-136 uses advanced encoding techniques hence it has more voice channel/radio channels than AMPS.

(b) **Point out three important mechanisms needed in order for CDMA systems works?**

For CDMA system to work it needs –

① Rake Receiver.

② Power Control.

③ Variable transmission rate.

(c) **The advantages for using GMSK modulation scheme?**

GMSK is modulation scheme used in GSM. It has the advantage that it can be amplified by a non linear amplifier and remain undistorted hence GMSK receivers are cheap.

(d) **What are intra-cell handover and inter-cell handover?**

Intracell handover – User is transferred from one channel to other within a cell.

Inter cell handover – User is transferred from one cell to another.

(e) **What is the difference between TDD, TDM, and TDMA?**

Time division duplexing – It is an application of time division multiplexing to separate uplink and downlink data. Uplink and downlink data occupy different time slots.

Time division multiplexing – In TDM each user transmits signal in different time slots in the channel.

Time division multiple access – It allows several users to share same frequency channel by dividing signal into different time slots

(f) What is the definition of wide-sense (weakly) stationary?

A process is WSS if its mean function is constant & it auto correlation, $R_X(t,s)$ must be function of $|s-t|$. Any strictly stationary process which has mean and a covarience is also WSS.

(g) Briefly explain Sampling theorem?

Sampling theorem states that in order to reconstrut analog signal back from digital, the sampling of original analog signal must be done at the rate twice the maximum frequency of the signal or f_s (sampling frequency) $\geq 2f_m$ (max. frequency)

(h) Brief explain Nyquist's first criterion for zero ISI?

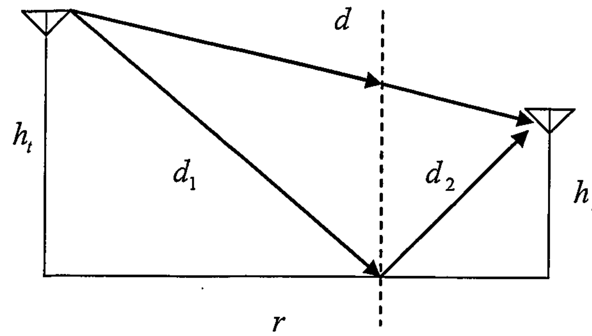
This states that zero ISI can be achived by choosing a pulse shape that has non-zero amplitude at its center (say $t=0$) and zero amplitudes at $t = \pm nT_b$ ($n = 1, 2, 3 \ldots$) where T_b = separation between scicessive ~~tar~~ transmitted pulses. Thus,

$$p(t) = \begin{cases} 1, & t=0 \\ 0, & t = \pm nT_b \end{cases}, \quad T_b = 1/R_b.$$

RBNA

Problem 2. (20%)

Two-ray model is show below.



where $\dfrac{P_r}{P_t} = G_t \cdot G_r \cdot (\dfrac{\lambda}{4\pi d})^2 \cdot |1+\rho \dfrac{d}{d_1+d_2} \cdot \exp(-j\Delta\phi)|^2$

Find:

(1) Find the plane-earth equation from $\dfrac{P_r}{P_t} \cong G_t\, G_r\, (\dfrac{\lambda}{4\pi\, d})^2\, |1+\rho \exp(-j\Delta\phi)|^2$ (Hint: Assume $\rho=-1$, Using Taylor Series Expansion, and assume $\dfrac{\Delta\phi}{2} < \dfrac{\pi}{6}$ to obtain the received power falls off with distance raised to the fourth power).

(2) Explain Fresnel zone ellipsoids?

(3) What is the break distance, d_B,?

(4) Consider an antenna transmitting a power of 10 W at 1 GHz. Calculate the received power at a distance of 500 m, if propagation is taking place in two-ray model? (Assume $G_t = G_r = 1\ dBd$, $h_t = 10m,\ h_r = 1.5m$)

(5) Consider an antenna transmitting a power of 10 W at 1 GHz. Calculate the received power at a distance of 3 Km, if propagation is taking place in two-ray model? (Assume $G_t = G_r = 1\ dBd$, $h_t = 10m,\ h_r = 1.5m$)

1) $\dfrac{Pr}{Pt} = Gt \cdot Gr \left(\dfrac{\lambda}{4d\pi}\right)^2 \left|1 + \rho \dfrac{d}{d_1+d_2} \cdot \exp(-j\Delta\phi)\right|^2$

let $\rho=-1$ and $\dfrac{d}{d_1+d_2}=1$

$Pr/Pt = Gt\, Gr \left(\dfrac{\lambda}{4\pi d}\right)^2 \left|1-\exp(-j\Delta\phi)\right|^2$

$\Delta\phi = \dfrac{2\pi}{\lambda}\Delta d$

$\Delta d = d_1 + d_2 - d.$

After some algebra.

$$d_1+d_2 = \sqrt{r^2+(h_t+h_r)^2}$$

$$d^2 = r^2 + (h_t-h_r)^2$$

$$d_1+d_2 = d\sqrt{1+\frac{4h_th_r}{d^2}}$$

If $d^2 \gg 4h_th_r$, $\frac{d_1+d_2}{d} \simeq 1$

Therefore amplitude $\rightarrow 1$ $\left(\frac{d_1+d_2}{d}=1\right)$

$$\Delta d = d_1+d_2-d$$
$$= \left(d\sqrt{1+\frac{4h_th_r}{d^2}} - d\right)$$

$(1+x)^{1/2} \simeq 1+\frac{x}{2}$ by Taylor series expansion

$$\therefore \Delta d \simeq \frac{2h_th_r}{d} \Rightarrow$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta d = \frac{2\pi \times 2h_th_r}{d\lambda}$$

$$\Delta\phi = \frac{4\pi h_th_r}{\lambda d}$$

Assuming the phase difference term is small $\Delta\phi \ll 0.6$ radian

$$\Delta\phi \simeq \frac{4\pi h_th_r}{\lambda d} \leq 0.6 \Rightarrow d \geq 21\frac{h_th_r}{\lambda}$$

Also, $|1-e^{-j\Delta\phi}|^2 = |1-\cos\Delta\phi+j\sin\Delta\phi|^2$

$$= (1-\cos\Delta\phi)^2 + (\sin\Delta\phi)^2$$
$$= 1-2\cos\Delta\phi + \cos^2\Delta\phi + \sin^2\Delta\phi$$

$$= 2(1-\cos\Delta\phi)$$

$$= 4\sin^2(\Delta\phi/2)$$

$$|1-e^{-j\Delta\phi}|^2 \approx \left(\frac{\Delta\phi}{2}\right)^2 \cancel{4}$$

$$\approx (\Delta\phi)^2.$$

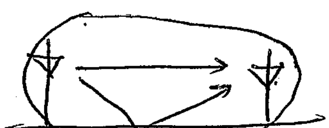
$$\therefore P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 (\Delta\phi)^2.$$

$$= P_T G_T G_R \frac{(h_t h_r)^2}{d^4}$$

$$P_R = K(d)^{-4}$$

2). Fresnel zone ellipsoids.

The indirect path via each circle is $\frac{n\lambda}{2}$ greater than the direct path.



$$\Delta\phi = \frac{2\pi}{\lambda}\Delta d$$

if $\Delta\phi = n\pi$ $\sin\left(\frac{\Delta\phi}{2}\right) = \pm 1$

then $\Delta d = \frac{n\lambda}{2}$.

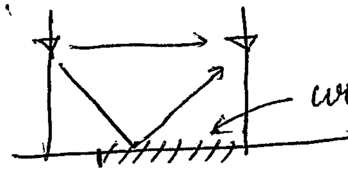
Again two ray approach

$$\frac{P_R}{P_T} \simeq G_T G_R \left(\lambda/4\pi d\right)^2 |1-e^{-j\Delta\phi}|^2$$

$$= G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 4\sin^2\left(\frac{\Delta\phi}{2}\right) \begin{cases} \frac{\Delta\phi}{2} = \frac{n\pi}{2} \Rightarrow \sin\left(\frac{\Delta\phi}{2}\right) = \pm 1 \\ \frac{\Delta\phi}{2} = n\pi \Rightarrow \sin\left(\frac{\Delta\phi}{2}\right) = 0. \end{cases}$$

3) The break distance d_B is the first fresnel ellipse just touch the grount.

1^{st} fresnel zone $\Rightarrow$ $\Delta d = \frac{\lambda}{2}$



Also $\Delta d = \frac{2 h_t h_r}{d}$

$\therefore d_B = \frac{4 h_t h_r}{\lambda}$.

4) $P_t = 10 W$.

$f = 1 GHz$. $\therefore \lambda = c/f = 0.3 m$.

$h_t = 10 m$, $h_r = 1.5 m$; $\underline{G_r = G_t = 1 dBd = 3.15 dBi}$ *

$= 2.065$.

find P_r at $d = 500 m$

i) find d_B.

$$d_B = \frac{4 h_t h_r}{\lambda} = \frac{4 \times 10 \times (1.5)}{0.3} = 200 m.$$

Since $d > d_B$, we can use 2 ray model.

(ii) $$\Delta\phi = \frac{4\pi h_t h_r}{\lambda d} = \frac{4\pi \times 10 \times (1.5)}{0.3 \times 500} = 1.25$$

$\Delta\phi > 0.6$.

$\therefore$ we cannot use simple formula.

$$\therefore P_r = P_t \cdot G_t \cdot G_r \left(\frac{\lambda}{4\pi d}\right)^2 4 \sin^2 \frac{\Delta\phi}{2}$$

$$= 10^4 \times 2.065 \times 2.065 \times \left(\frac{0.3}{4\pi \times 500}\right)^2 4 \sin^2\left(\frac{1.2}{2}\right)$$

$$P_r = 1.33 \times 10^{-4} \text{ m.}$$

$$\boxed{P_r = -38.759 \text{ dBm}}$$

5) $d = 3\text{ Km}$.

$P_T = 10W$

$f = 1\text{ GHZ} \quad \therefore \lambda = 0.3m$

$G_t = G_r = 1\text{dBd} = 3.15\text{dBi} = 2.065$ *

$h_t = 10m, \; h_r = 1.5m$

$P_r = ?$ at $d = 3\text{Km}$.

i) $\Delta\phi = \dfrac{4\pi h_t h_r}{\lambda d}$

$$= \frac{4 \times \pi \times 10 \times 1.5}{0.3 \times 3K}$$

$$= 0.2$$

Since $\Delta\phi < 0.6$ we can use

$$P_r = P_t \, G_t \cdot G_r \left(\frac{h_t \cdot h_r}{d^2}\right)^2$$

$$= 10^4 \times (2.065)^2 \times \left(\frac{10 \times 1.5}{(3 \times 10^3)^2}\right)^2$$

$$\boxed{P_r = 1.185 \times 10^{-7} \text{ mW} = -69.26 \text{ dBm}}$$

Problem 3. (20%)

Signal transmission through a system:

(1) A nonlinear system $y(t) = x(t) + 0.0002 \cdot x^2(t)$

Find $y(t)$ and $Y(f)$, where input signal is $x(t) = 1000 \cdot \operatorname{sin} c\,(1000\pi t)$

(2) A linear distortion system caused by multipath with different delays. (Assume TX , Interactive objects, and RX are all fixed. We only consider two-path model for simplicity).
Find

(a) The magnitude and the phase characteristics of $H(f) = \exp(-j2\pi f t_d) + \alpha \cdot \exp[-j2\pi f (t_d + \Delta t)]$?

(b) How do we name this kind of short-term fading and what kind of distortion caused by multipath effects (hint: time-invariant distortion or time-varying distortion)?

(3) A linear distortion system caused by Doppler shift with the same path delays. (Assume TX and Interactive objects are fixed. However, RX is moves towards to the TX. We only consider two-path model for simplicity).

(a) Describe and plot a well-known effect of "beating" of two oscillations with slightly different frequency. Channel response is $h(t) = \cos[2\pi t(f_c - \frac{v}{\lambda}\cos 0^o)] + \cos[2\pi t(f_c - \frac{v}{\lambda}\cos 45^o)]$ where v is the mobile velocity and λ is the wavelength?

(b) How do we name this kind of short-term fading and what kind of distortion caused by Doppler effects?

$$y(t) = x(t) + 0.0002\, x^2(t)$$

$$x(t) = 1000 \operatorname{sinc}(1000\pi t)$$

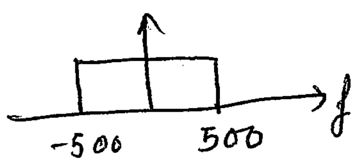
$$\therefore y(t) = 1000 \operatorname{sinc}(1000\pi t) + 0.0002 \times (1000)^2 \operatorname{sinc}^2(1000\pi t)$$

$$y(t) = 1000 \operatorname{sinc}(1000\pi t) + 200 \operatorname{sinc}^2(1000\pi t)$$

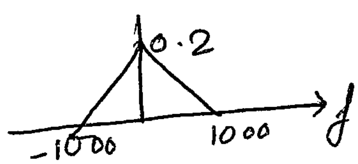
taking fourier transform

$$Y(f) = \underbrace{\pi\, \mathrm{rec}(f/500)}_{Y_1(f)} + \underbrace{0.2\, \Delta(f/1000)}_{Y_2(f)}$$

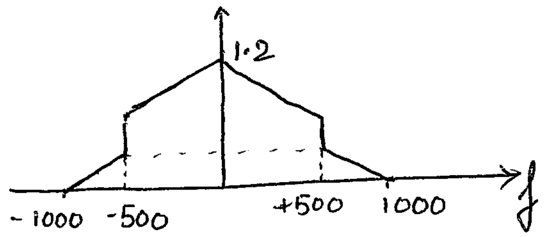
$Y_1(f) =$



$Y_2(f)$



$\therefore Y(f) =$



2) a) $H(f) = e^{-j2\pi f t_d} + \alpha e^{-j2\pi f(t_d + \Delta t)}$

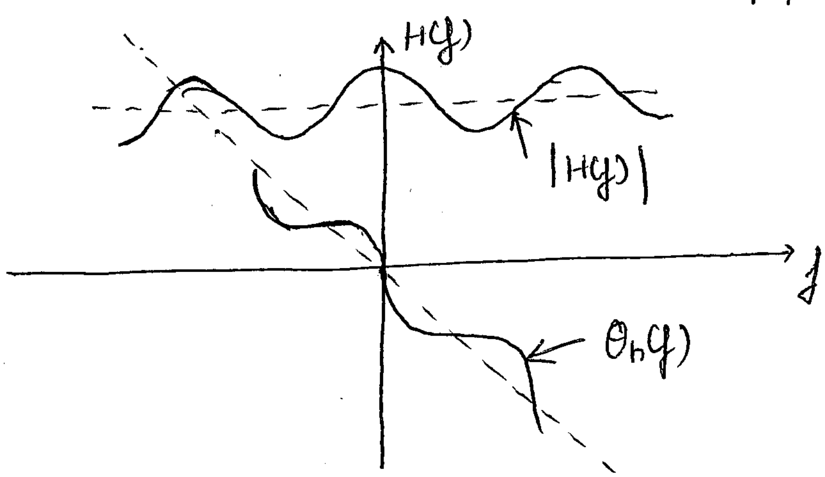
$$= e^{-j2\pi f t_d}\left(1 + \alpha e^{-j2\pi f \Delta t}\right)$$

$$= e^{-j2\pi f t_d}\left(1 + \alpha \cos 2\pi f \Delta t - j\alpha \sin 2\pi f \Delta t\right)$$

$$= \sqrt{1+\alpha^2+2\alpha\cos 2\pi f \Delta t}\; \exp\left[-j\left(2\pi f t_d + \tan^{-1}\left(\frac{\alpha \sin 2\pi f \Delta t}{1+\alpha\cos 2\pi f \Delta t}\right)\right)\right]$$

$$\therefore |H(f)| = \sqrt{1+\alpha^2+2\alpha\cos 2\pi f \Delta t}$$

phase $\theta_h(f) = 2\pi f t_d + \tan^{-1} \dfrac{\alpha . \sin 2\pi f \Delta t}{1+\alpha \cos 2\pi f \Delta t}$.



2) b) This type of short term fading is called frequency selective fading.

Multipath effects causes time-invariant distortion.

3). a) Assume only receiver (Rx) move. (Transmitter (Tx) and Interactive objects are fixed)

If the Rx moves away from the Tx with speed v, the distance d between Tx and Rx increases with that speed.

$$r(t) = a_0 \cos\left(\omega_c t - k_0 [d_0 + vt]\right)$$
$$= a_0 \cos\left(2\pi t\left(f_c - v/\lambda\right) - k_0 d_0\right)$$

Where k_0 is wavenumber, d_0 is the distance at time $t=0$

Doppler shift is $\nu_{max} = -\frac{v}{\lambda} = -f_c \cdot \frac{v}{c}$.

For two path model, as the Rx moves, it receives two waves that are each Doppler shifted, by different amounts. By fourier transformation to time domain, a well known effect of "beating" of two oscillations with slightly different frequencies. The frequency of this beating envelope is equal to the frequency difference between two carriers

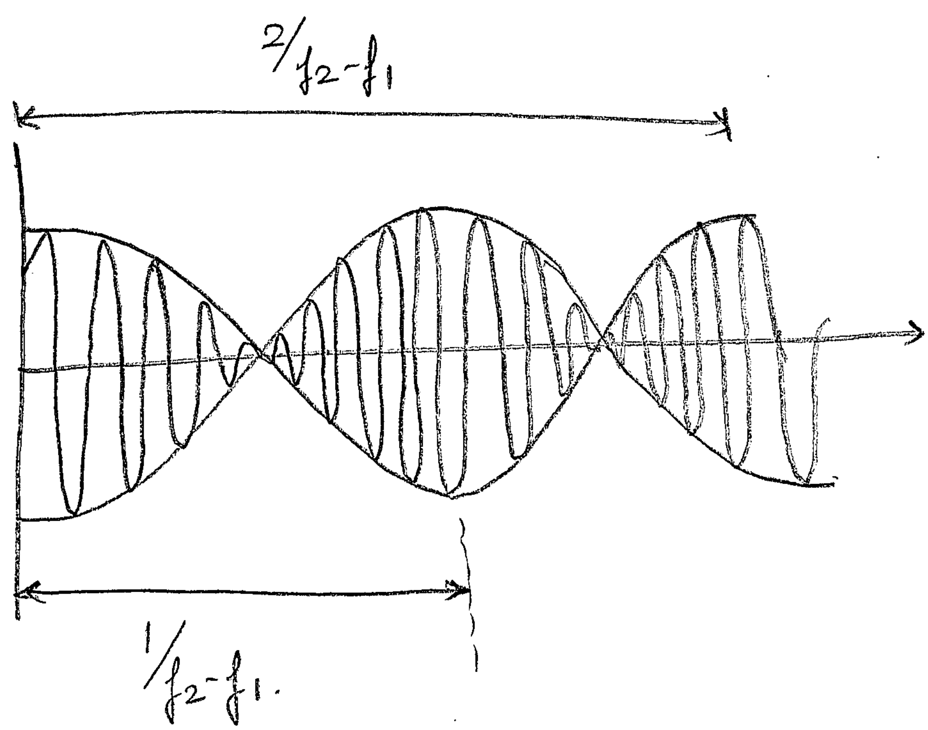
In our case

$$h(t) = \cos\left[2\pi t\left(f_c - \frac{v}{\lambda}\cos 0^\circ\right)\right] + \cos\left[2\pi t\left(f_c - \frac{v}{\lambda}\cos 45^\circ\right)\right]$$

$$= 2\left\{\cos\left[\frac{2\pi t\left(f_c - v/\lambda\right) + 2\pi t\left(f_c - \frac{v}{\sqrt{2}\lambda}\right)}{2}\right] \times \right.$$

$$\left.\cos\left[\frac{2\pi t\left(f_c - v/\lambda\right) - 2\pi t\left(f_c - \frac{v}{\sqrt{2}\lambda}\right)}{2}\right]\right\}.$$

$$= \cancel{2\left[\cos\left(2\pi t f_c\right.\right.}$$



Problem 4. (20%)

Four received power measurements were taken at distances of 100m, 200m, 900m, and 2 km from a transmitter. These measured values at these distances are –0dBm, -20dBm, -30dBm, and –40dBm, respectively. It is assumed that the path loss for these measurements follows the model below

$$PL(d)[dB] = \overline{PL}(d) + X_\sigma = \overline{PL}(d_0) + 10\, n \log(\frac{d}{d_0}) + X_\sigma$$

Where $d_0 = 100$ m and the standard deviation of log normal shadowing is X_σ.

(a) Find the minimum least square error (MLSE) estimate for the path loss exponent, n.

(b) Calculate the standard deviation about the mean value, X_σ.

(c) Estimate the area-mean received power at d=1 km using the resulting model.

$$\text{MLSE} \rightarrow J(n) = \sum_{i=1}^{4} (P_r - \hat{P}_r)^2$$

P_r = received power

and $P_r(d)[dB] = P_r(d_0) - 10n\log_{10}(d/d_0)$

$\therefore \hat{P}_r(100) = 0 - 10n\log(100/100) = 0$

$\hat{P}_r(200) = 0 - 10n\log(200/100) = -3.01n$

$\hat{P}_r(900) = 0 - 10n\log(900/100) = -9.54n$

$\hat{P}_r(2000) = 0 - 10n\log(2000/100) = -13.01n$

$$\therefore J(n) = (0-0)^2 + (-20+3.01n)^2 + (-30+9.54n)^2 + (-40+13.01n)^2$$

for minima, $\frac{dJ(n)}{dn} = 0 \quad \therefore 538.6n - 1733.6 = 0$

$\Rightarrow n = 3.21$

(b) $\sigma^2 = \frac{J(n)}{4} \Rightarrow \sigma = 5.25$

(c) Power at 1 Km.

$$P_r(1000\,m) = P_r(d_0) - 10n\log(d/d_0)$$

$$= 0 - 10 \times 3.21 \times \log_{10}(1000/10)$$

$$= 0 - 32.1$$

$$= -32.1\,dBm.$$