

2-9 Consider the average fade duration equation (2-39). Take the case of a vehicle moving at a speed of 100 km/hr. The system frequency of operation is 1 GHz. Say the ratio $\rho = 1$. Show the average fade duration is 8 msec, as noted in the text. Now let the received signal amplitude be 0.3 of the rms value. Show the average fade duration is now 1 msec.

$$\text{Case (i)} \quad \rho = 1, \quad f_m = \frac{v}{\lambda} = \frac{100 \times 10^3 / (60 \times 60)}{3 \times 10^8 / 10^9} = 92.59 \text{ Hz} \quad \tau_f = \frac{e-1}{92.59 \times \sqrt{2\pi}} = 7.4 \text{ msec.}$$

$$\text{Case (ii)} \quad \rho = 0.3, \quad f_m = 92.59 \text{ Hz} \quad \tau_f = \frac{e^{0.3^2} - 1}{0.3 \times 92.59 \times \sqrt{2\pi}} = 1.35 \text{ msec.}$$

2-11 Indicate the condition for flat fading for each of the following data rates:

8 kbps, 40 kbps, 100 kbps, 6 Mbps.

Indicate which, if any, radio environments would result in flat fading for each of these data rates.

Data Rate	Symbol Interval	Flat Fading Condition	Radio Environment for Flat Fading
8 kbps	125 μsec	$\tau_{\text{av}} < 25 \mu\text{sec}$	Most environments
40 kbps	25 μsec	$\tau_{\text{av}} < 5 \mu\text{sec}$	Some urban environments and most suburban and rural environments
100 kbps	10 μsec	$\tau_{\text{av}} < 2 \mu\text{sec}$	Most suburban and rural environments
6 Mbps	0.167 μsec	$\tau_{\text{av}} < 0.033 \mu\text{sec}$	Some indoor picocellular environments

2-12 a. Consider the transversal filter equalizer of Fig. 2-20. A training sequence of K bin digits is used to determine the $2N + 1$ tap gains, as described in the text. Show that under a minimum mean-squared performance objective, the optimum choice of gains is given by (2-53).

Under a minimum mean-squared performance objective, we want to choose the gains h_n , $-N \leq n \leq N$, such that $\sum_{j=1}^K (s_j - \hat{s}_j)^2$ is minimum.

To obtain the optimum choice of tap gains, we find the solution of

$$\frac{\partial}{\partial h_l} \sum_{j=1}^K (s_j - \hat{s}_j)^2 = \sum_{j=1}^K 2(s_j - \hat{s}_j) \frac{\partial \hat{s}_j}{\partial h_l} = 0$$

Since $\hat{s}_j = \sum_{n=-N}^N h_n r_{j-n}$ and $\frac{\partial \hat{s}_j}{\partial h_l} = r_{j-l}$, the above equation becomes

$$\begin{aligned} & \sum_{j=1}^K 2(s_j - \hat{s}_j) r_{j-l} = 0 \\ \Rightarrow & \sum_{j=1}^K s_j r_{j-l} = \sum_{j=1}^K \hat{s}_j r_{j-l} \\ \Rightarrow & \sum_{j=1}^K s_j r_{j-l} = \sum_{j=1}^K \sum_{n=-N}^N h_n r_{j-n} r_{j-l} \quad -N \leq l \leq N \end{aligned}$$

b. Show the vector form of (2-53) is given by (2-56), with the solution given by (2-57).

Define $R_{l,n} \equiv \sum_{j=1}^K r_{j-n} r_{j-l}$ and $g_l \equiv \sum_{j=1}^K s_j r_{j-l}$, $-N \leq l, n \leq N$. Equation (2-53) then takes on the simpler looking form

$$g_l = \sum_{n=-N}^N h_n R_{l,n} \quad -N \leq l \leq N$$

In matrix notation,

$$\begin{pmatrix} g_{-N} \\ g_{-N+1} \\ \vdots \\ g_N \end{pmatrix} = \begin{pmatrix} R_{-N,-N} & R_{-N,-N+1} & \cdots & R_{-N,N} \\ R_{-N+1,-N} & R_{-N+1,-N+1} & \cdots & R_{-N+1,N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N,-N} & R_{N,-N+1} & \cdots & R_{N,N} \end{pmatrix} \begin{pmatrix} h_{-N} \\ h_{-N+1} \\ \vdots \\ h_N \end{pmatrix} \quad \text{or} \quad \mathbf{g} = \mathbf{R}\mathbf{h}$$

Multiply \mathbf{R}^{-1} to both sides, we have

$$\mathbf{R}^{-1}\mathbf{g} = (\mathbf{R}^{-1}\mathbf{R})\mathbf{h} \quad \text{or} \quad \mathbf{h} = \mathbf{R}^{-1}\mathbf{g}$$

2-15 Show the optimum maximal-ratio combining gain for the k th diversity branch is given by (2-64). Explain the statement that the SIR is then the sum of the SIRs, summed over the N diversity branches.

From (2-61)
$$SIR \equiv |s|^2 / I_0 = \left| \sum_{k=1}^N g_k a_k \right|^2 / \sum_{k=1}^N |g_k|^2 |n'_k|^2$$

According to Schwartz's inequality for complex numbers,

$$\left| \sum_{k=1}^N c_k^* d_k \right| \leq \left(\sum_{k=1}^N |c_k|^2 \right)^{1/2} \cdot \left(\sum_{k=1}^N |d_k|^2 \right)^{1/2}$$

Choose $c_k = a_k^* / |n'_k|$ and $d_k = g_k |n'_k|$, we have

$$\begin{aligned} \left| \sum_{k=1}^N a_k g_k \right| &\leq \left(\sum_{k=1}^N |a_k|^2 / |n'_k|^2 \right)^{1/2} \cdot \left(\sum_{k=1}^N |g_k|^2 |n'_k|^2 \right)^{1/2} \\ \Rightarrow \quad SIR &\equiv \left| \sum_{k=1}^N a_k g_k \right|^2 / \sum_{k=1}^N |g_k|^2 |n'_k|^2 \leq \sum_{k=1}^N |a_k|^2 / |n'_k|^2 \end{aligned}$$

SIR is maximized when equality is obtained with $d_k = Kc_k$, i.e. $g_k |n'_k| = K a_k^* / |n'_k|$ or $g_k = K a_k^* / |n'_k|^2$, which is the expression in (2-64).

Since the SIR in this case is $\sum_{k=1}^N |a_k|^2 / |n'_k|^2$, and the SIR on the k th diversity branch is just given by $|a_k|^2 / |n'_k|^2$, we can state that the resultant SIR is the sum of the SIRs over the N diversity branches.

- 2-13 a. Work out a simple example of the transversal filter equalizer: Say the equalizer has three taps to be found using the minimum mean-squared performance objective. Choose a set of $K = 10$ arbitrarily-chosen binary digits as the training sequence and then let some of these digits be received in "error", i.e. some are converted to the opposite polarity. Find the "best" set of taps in this case. Try to choose the training sequence so that there are equal numbers of +1 and -1 digits. Compare the tap coefficients with those found using the approximation of (2-58).

Choose the training sequence $\{s_j\} = [1 \ 1 \ -1 \ 1 \ -1 \ -1 \ -1 \ 1 \ 1 \ -1]$

And the received sequence $\{r_j\} = [1 \ 1 \ 1 \ 1 \ -1 \ -1 \ -1 \ -1 \ 1 \ -1]$, with the third and eighth bits received in errors.

$$q_l = \sum_{j=1}^K s_j r_{j-l}$$

No of taps = 3.

$$K = 10, \quad -N \leq l \leq N \quad \therefore -1 \leq l \leq 1$$

$$q_{-1} = \sum_{j=1}^{10} s_j r_{j+1} = 3$$

$$q_0 = \sum_{j=1}^{10} s_j r_j = 6$$

$$q_1 = \sum_{j=1}^{10} s_j r_{j-1} = -1$$

$$\therefore \mathbf{q} = \begin{bmatrix} 3 \\ 6 \\ -1 \end{bmatrix}$$

$$R = \begin{bmatrix} R_{-1,-1} & R_{-1,0} & R_{-1,1} \\ R_{0,-1} & R_{0,0} & R_{0,1} \\ R_{1,-1} & R_{1,0} & R_{1,1} \end{bmatrix}$$

$$R_{l,n} = \sum_{j=1}^K r_{j-n} r_{j-l} \quad -N \leq l, n \leq N \quad -1 \leq l, n \leq 1$$

$$R_{-1,-1} = \sum_{j=1}^K r_{j+1} r_{j+1} = 9$$

$$\begin{aligned}
 R_{-1,0} &= \sum_{j=1}^{10} A_j A_{j+1} = 3 \\
 R_{-1,1} &= \sum_{j=1}^{10} A_{j-1} A_{j+1} = 2 \\
 R_{0,-1} &= \sum_{j=1}^{10} A_{j+1} A_j = 3 \\
 R_{0,0} &= \sum_{j=1}^{10} A_j A_j = 10 \\
 R_{0,1} &= \sum_{j=1}^{10} A_{j-1} A_j = 3 \\
 R_{1,-1} &= \sum_{j=1}^{10} A_{j+1} A_{j-1} = 2 \\
 R_{1,0} &= \sum_{j=1}^{10} A_j A_{j-1} = 3 \\
 R_{1,1} &= \sum_{j=1}^{10} A_{j-1} A_{j-1} = 9 \\
 \therefore R_{L,n} &= \begin{bmatrix} 9 & 3 & 2 \\ 3 & 10 & 3 \\ 2 & 3 & 9 \end{bmatrix} \\
 g &= R h \\
 \therefore h &= R^{-1} g \\
 &= \begin{bmatrix} 9 & 3 & 2 \\ 3 & 10 & 3 \\ 2 & 3 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ -1 \end{bmatrix} \\
 \therefore \begin{bmatrix} h_{-1} \\ h_0 \\ h_1 \end{bmatrix} &= \begin{bmatrix} 0.198 \\ 0.652 \\ -0.372 \end{bmatrix}
 \end{aligned}$$

By using the approximation of (2-58), the tap coefficients

$$h_{approx} = \frac{1}{10} \cdot g = \frac{1}{10} \cdot \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.6 \\ -0.1 \end{pmatrix}$$

The approximated coefficients obtained by (2-58) are very close to the exact coefficients obtained by (2-57). The mean-squared difference given by the approximated tap coefficients is just 0.0763 higher than the optimum value.

In general, the higher the order of diversity, the better the performance. However, as the ratio of local-mean power to threshold increases, the improvement becomes less significant. For a ratio larger than three, there is no much difference between four- and eight-fold diversities. Besides, if the order of diversity is high enough, it will be hard to improve the performance further by more diversity branches.