Chap. 4 Dynamic channel allocation & power control

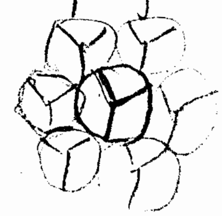
I. overview

1. In chap. 3, we discussed Fixed channel allocation with reuse constraints. $SIR \propto \frac{D_c}{R} = \sqrt{3C}$.

   Disadvantage : reduce the Number of channels that can be allocated to each cell $\xRightarrow{\text{imply}}$ reduce Traffic Capacity.

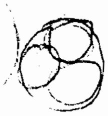
2. Various proposed strategies to improve system performance.

   (1). sectorization or directional antennas

   $\xRightarrow{\text{can}}$ reduce interferences

   (2). Reducing cell size (cell splitting)

   Improve system capacity, but increase Hand off.

   (refer to class Notes P. 1-13.)

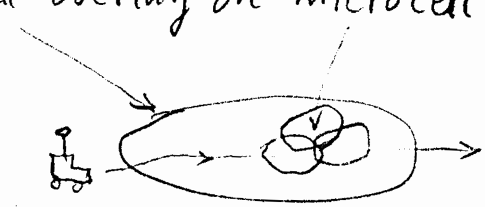
   cell size hierarchy

   ① Macrocell (cell size ≳ 1 Km)
   propagation model : Hata model.

   ② Microcell (cell size ≈ 600 m)
   propagation model : Ray Tracing model.

   ③ picocell (Indoor)
   measurement based linear regression model.

   (3). Macrocell overlay on Microcell systems

   improve Traffic-handling capacity.

3. Other methods used to improve system performance.

(1). Dynamic channel allocation (DCA)

⇒ reduce call blocking

(2) power control (PC)

⇒ reduce interference

(3). Hybrid DCA & PC

⇒ Great improvements in system performance

4. In this chapter, we discuss

(1). Simple DCA strategy

for low-tier wireless system, DECT (Digital European Cordless Telephone)

designed to handle pedestrian traffic.

(Cellular system is High-tier wireless sys.)

(2) Examples on power control algorithm.

DBA (Distributed Balancing Algorithm)

DPC (Distributed power control algorithm)

II. Dynamic channel allocation

1. Definitions of FCA & DCA

FCA: Groups of channels are assigned permanently to given calls following a prescribed reuse pattern.

DCA: channels may be allocated to users in a cell in accordance with varying traffic demands. There are variety of Techniques.

Note: A channel allocation scheme is said to be better than another in a given load region, if its blocking-probability load curve is below the other's curve.

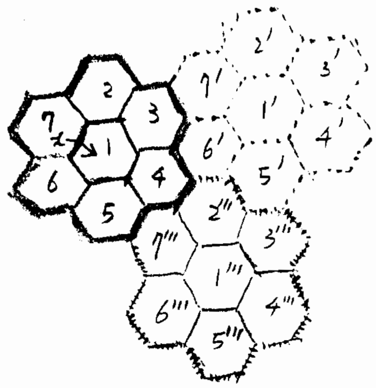
2. An overview of DCA & FCA performance

(1). At low & moderate traffic condition ⇒
All DCA schemes out-perform FCA scheme

(2). At heavy loads ⇒
FCA scheme is perform better because channel locking leads to performance degradation

(3) Hybrid schemes have been proposed which convert to FCA at high loads.

3. Example of channel locking concept.



(1). stringent borrow strategy

If channel x in cell 7 is borrowed by cell 1

than channel x is looked out from all cells surrounding cell 1

and cells 7' and 7'''.

It can not be borrowed by cells surrounding cell 7' & cell 7'''.

This borrow strategy is Too stringent!

(2). Directional locking

In the same situation → the cells on the far side of cell 7', away from cell 1, such as cells 1' and 2', could borrow channel x from cell 7' without violating the reuse condition.

(Also cells 1''' and 6''', could borrow channel x from cell 7''').

*. Using directional locking improve the DCA performance.

4. We focus on one DCA scheme

BDCL (Borrowing with Directional Channel Locking) is a scheme for which analysis is possible, making it easier to explain the operation of the algorithm.

The complete scheme is defined by three specific features.

(1). specification of lock directions

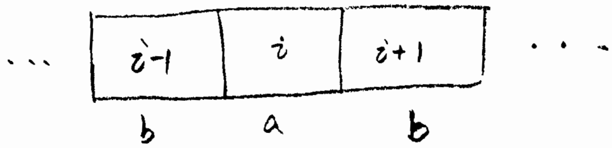
(2). channel ordering

(3). Immediate channel reallocation.

Channel ordering : ① channel assigned to a cell are numbered.
② The lowest-numbered channel has the highest priority to be assigned to the next local call.
③ The highest-numbered channel has the highest priority to be borrowed by neighboring cells.

Immediate channel reallocation :
① packing of local-call ordering.
② local channel terminates & there is ongoing call using a borrowed channel. The call move to the local call & release the borrowed channel.
③ packing of borrowed channel.
④ A channel is unlocked by termination of a call in an interfering cell, any call on a borrowed channel or a higher-order channel is switched to this channel.

5. One-dimensional BDCL scheme.



There are a total of 2m channels in the system.

(1). For FCA case:

(i) If all m channels in cell i are occupied than a new call arrives it will be blocked.

(ii) The blocking probability

channels available

$$P_B = \frac{A^N/N!}{\sum_{n=0}^{N} A^n/n!} \quad \text{(Erlang-B Formula)}$$

Traffic intensity

(2). For DCA case:

(i). i may borrow a channel from $i-1$ or $i+1$, if available

If i borrows a channel from $i-1$,

than the corresponding channel in $i+1$ is then locked.

(ii) The blocking of calls at cell i occurs, if all 2m channels covering this three-cell system are in use.

(iii) The blocking probability

Derived in the following section

6. Analysis of the one-dimension, 3-cell group in isolation, leads to a (tight) upper bound on the blocking probability

(1). total $2m$ channels assigned to the group.

$i-1$	i	$i+1$

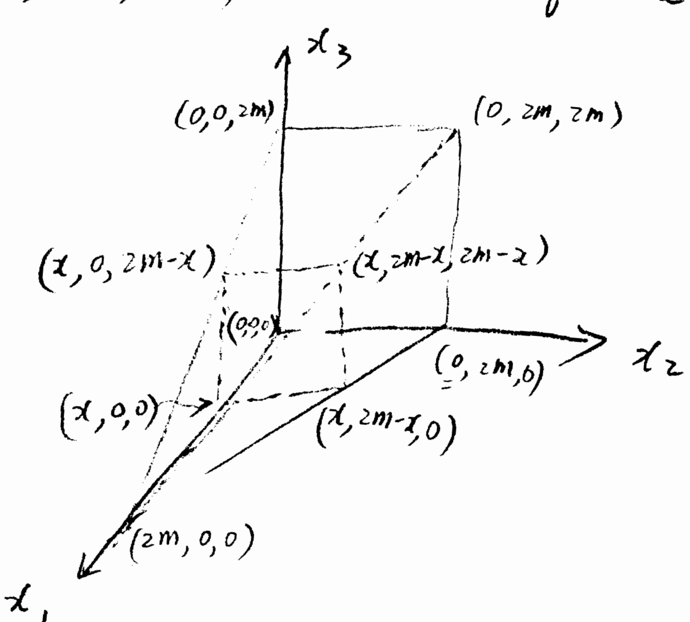
let x_1 channels occupied in cell i

x_2 channels occupied in cell $i+1$

x_3 channels occupied in cell $i-1$

than

(x_1, x_2, x_3) is the vector of states (Number of channels occupied)



At $x_1 = 2m-1 \xrightarrow{\text{there are}} (1+1)^2 = 2^2 = 4$ states

$x_1 = 2m-2 \longrightarrow (2+1)^2 = 3^2 = 9$

$\vdots$

$x_1 = 0 \longrightarrow (2m+1)^2$

The total possible states $\sum_{K=1}^{2m+1} K^2 = (2m+1)(2m+2)[2(2m+1)+1]/6$

EX: For $m=1$, with $2m=2$ channels avalible in the system, there is a total of 14 states.

Consider the Blocking states of cell i

ch 1

ch 2

Blocking states

x_1 (i)	x_2 ($i+1$)	x_3 ($i-1$)
2 (1,2)	0	0
0	2 (1,2)	0
0	0	2 (1,2)
1 (1)	0	1 (2)
1 (1)	1 (2)	0
0	1 (2)	2 (1,2)
0	2 (1,2)	1 (1)
1 (1)	1 (2)	1 (1)
0	2 (1,2)	2 (1,2)

$\Big\}\ 9$

Non-Blocking states

x_1	x_2	x_3
0	0	0
1	0	0
0	1	0
0	0	1
0	1 (2)	1 (1)

$\Big\}\ 5$

(2). A total of $2m$ channels, the blocking probability of cell i for the one-dimensional BDCL algorithm

$$B_i = \sum_{x_1+x_3 \le 2m} P[x_1, 2m-x_1, x_3] + \sum_{x_1+x_2 < 2m} P[x_1, x_2, 2m-x_1]$$

(3) The desired steady-state probability $P[x_1, x_2, x_3]$

(i) with the poisson call arrival - exponential holding time assumption and

(ii) the state-transition diagram and possible states calculation

a special case of transitions in a Markov chain (refer to Yeung & Yum 1995 paper)

$$P[x_1, x_2, x_3] = G^{-1} \prod_{j=1}^{3} \frac{A_j^{x_j}}{x_j!} \; ; \quad A_j \triangleq \frac{\lambda_j}{\mu_j}$$

The average No. of call attempts/unit time

Erlang intensity

coupling term

product form

The average call length

(even though the state occupancies x_1, x_2, x_3 of the three cells $i, i-1$, and $i+1$ are clearly coupled, the probability of state occupancy acts as if they are independent.)

G^{-1} is the coupling term.

since the total probability $\sum_{\text{all states}} P[x_1, x_2, x_3] = 1$.

Hence the quantity $G = \sum_{\text{all states}} \left(\prod_{j=1}^{3} \frac{A_j^{x_j}}{x_j!} \right)$

Based on the above equations, we obtain

$P[x_1, x_2, x_3]$ & B_i

Given B_i, the probability of blocking B of the entire N-cell system

$$B = \sum_{i=1}^{N} \lambda_i B_i \Big/ \sum_{i=1}^{N} \lambda_i$$

For the uniform traffic case, $\lambda_i = \lambda$ & $\mu_i = \mu$, $\Rightarrow B = B_i$

EX: one-dimension, three-cell group system

$m = 1$ ($2m = 2$ channels are available in the system).

Assumption of homogeneous (uniform) traffic conditions.

Blocking States x_1	x_2	x_3	$P[x_1, x_2, x_3]$
2	0	0	$G^{-1}A^2/2$
0	2	0	
0	0	2	
1	0	1	$G^{-1}A^2$
1	1	0	
0	1	2	$G^{-1}A^3/2$
0	2	1	
1	1	1	$G^{-1}A^3$
0	2	2	$G^{-1}A^4/4$

Non-Blocking states x_1	x_2	x_3	$P[x_1, x_2, x_3]$
0	0	0	G^{-1}
1	0	0	$G^{-1}A$
0	1	0	
0	0	1	
0	1	1	$G^{-1}A^2$

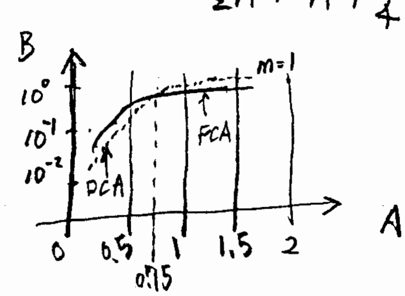
$$\sum_{\text{all states}} P[x_1, x_2, x_3] = \sum_{\substack{\text{blocking}\\\text{states}}} P[x_1, x_2, x_3] + \sum_{\substack{\text{non-blocking}\\\text{states}}} P[x_1, x_2, x_3] = 1$$

$$G^{-1}\left[\left(3\times\frac{A^2}{2} + 2A^2 + 2\times\frac{A^3}{2} + A^3 + \frac{A^4}{4}\right) + \left(1 + 3A + A^2\right)\right] = 1$$

$$G = \left[\left(\frac{7}{2}A^2 + 2A^3 + \frac{A^4}{4}\right) + \left(1 + 3A + A^2\right)\right] = 1 + 3A + \frac{9}{2}A^2 + 2A^3 + \frac{A^4}{4}$$

$$\text{DCA} \Rightarrow B = \sum_{\substack{\text{blocking}\\\text{states}}} P[x_1, x_2, x_3] = G^{-1}\left(\frac{7}{2}A^2 + 2A^3 + \frac{A^4}{4}\right) = \frac{\frac{7}{2}A^2 + 2A^3 + \frac{A^4}{4}}{1 + 3A + \frac{9}{2}A^2 + 2A^3 + \frac{A^4}{4}} \quad \#$$

$$\text{FCA} \Rightarrow P_B = \frac{A^N/N!}{\sum_{n=0}^{N} A^n/n!} \Rightarrow B = \frac{A}{1+A} \quad \#$$



III. power control

1. In order to increase capacity:
   (1) channel reuse
      (a) sectorization
      (b) cell splitting
      (c) Microcell concept (e.g., refer to William Lee's paper)
   (2) Trunking system
   (3) DCA (Dynamic Channel Allocation)
   (4) P.C. (power control strategy)

2. Compare two distributed power control strategies
   (1) The distributed balancing algorithm (DBA)
   (2) The distributed power control algorithm (DPC)

These two strategies are :-

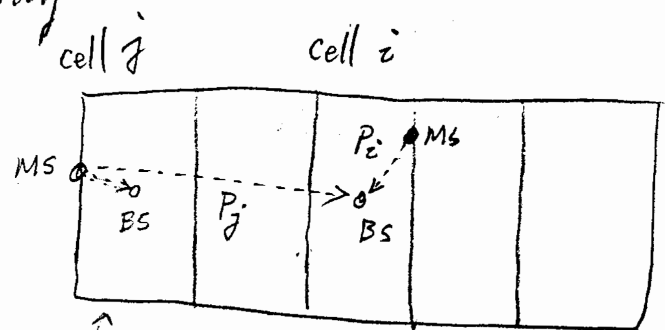
Transmitter powers are individually adjusted so that the SIR at all receivers is as large as possible.
Finally, proof that the resultant SIR turns out to be the same at all receivers.

For simplicity, we only focus on uplink power control algorithms. However, they are equally applicable to downlink power control.

Focusing on average power only

$$SIR_i \triangleq \frac{P_i / d_{ii}^n}{\sum_{j=1, j\neq i}^{M} P_j / d_{ij}^n}$$

↑ the SIR in Cell i



(one dimensional, 2-cell reuse uplink case.)

$$SIR_i = \frac{P_i G_{ii}}{\sum_{j=1}^{M} P_j G_{ij} - P_i G_{ii}} = \frac{P_i}{\sum_{j=1}^{M} P_j (G_{ij}/G_{ii}) - P_i}$$

where $G_{ij} \equiv \frac{1}{d_{ij}^n}$ (Gain factor)

the first algorithm

(a) For DBA strategy case

The objective is to adjust the transmitted power value, P_i, $1 \le i \le M$, in a distributed fashion, each of the M mobiles working in conjunction with its base station, to attain the maximum achievable SIR. It means

"The maximum over all the mobile powers of the minimum SIR."

find the min γ_i $1 \le i \le M$

$$\gamma^* = \max_{\text{all powers}} \left\{ \min \gamma_i \right\}, \quad 1 \le i \le M$$

maximum achievable SIR (γ^*) for all M mobiles using the same channel.

DBA algorithm makes the following power adjustment

TBA method definition $\Rightarrow$ $$P_i^{(n)} = c^{(n-1)} P_i^{(n-1)} \left[1 + \frac{1}{\gamma_i^{(n-1)}} \right]$$

where

$$\gamma_i^{(n-1)} = \frac{P_i^{(n-1)}}{\sum_{j=1}^{M} P_j^{(n-1)} r_{ij} - P_i^{(n-1)}}$$

where $$r_{ij} \equiv \frac{G_{ij}}{G_{ii}} \quad \& \quad c^{n-1} > 0$$

(The same value is to be selected by all the mobiles.)

Therefore

$$P_i^{(n)} = c^{(n-1)} \sum_{j=1}^{M} P_j^{(n-1)} r_{ij}$$

the vector form is

$$\underset{\text{vector}}{\bar{P}^{(n)}} = c^{(n-1)} \underset{\text{matrix}}{\bar{\bar{R}}} \underset{\text{vector}}{\bar{P}^{(n-1)}}$$

$\bar{\bar{R}}$: non-negative matrix with unit diagonal entries.

(by using Perron-Frobenius theorem the largest eigen values $z^* > 1$ & $|z_j| < z^*$.)

Now, proof $\lim_{n\to\infty} \gamma_i^{(n)} = \gamma^* = \dfrac{1}{z^*-1}$

let each mobile independently choose an initial transmitting power value. $\bar{P}^{(0)}$

Iteration $\{\bar{P}^{(n)} = C^{(n-1)} \bar{\bar{R}} \bar{P}^{(n-1)}\}$ n times.

$$\bar{P}^{(n)} = \bar{\bar{R}}^n \left[\prod_{j=0}^{n-1} C^{(j)}\right] \bar{P}^{(0)}$$

since $\bar{\bar{R}}$ is a $(M\times M)$ matrix which has M eignvalues z_j, $1 \le j \le M$. Each has its corresponding eignvector $\bar{\phi}_j$.

Therefore, $\bar{P}^{(0)} = \sum_j a_j \bar{\phi}_j$ where $a_j = (\bar{P}^{(0)}, \bar{\phi}_j)/(\bar{\phi}_j, \bar{\phi}_j)$.

$$(\bar{x}, \bar{y}) = \sum_j x_j y_j$$

let

$$\bar{\bar{R}}^n \bar{P}^{(0)} = \sum_j a_j \bar{\bar{R}}^n \bar{\phi}_j$$

$$\bar{\bar{R}} \bar{\phi}_j = z_j \bar{\phi}_j \quad \text{(Definition of eignvalue \& eignvector)}$$

$\Downarrow$ imply

$$\bar{\bar{R}}^n \bar{\phi}_j = z_j^n \bar{\phi}_j$$

$$\bar{\bar{R}}^n \bar{P}^{(0)} = \sum_j a_j z_j^n \bar{\phi}_j$$

Since z_1 is $z^* > 1$, the largest of the M eigenvalues. for a large n iteration $(z^*)^n$ must dominate all the terms in the sum over j. $\Rightarrow \sum_j a_j z_j^n \bar{\phi}_j \simeq a_1 (z^*)^n \bar{P}^*$

where $\bar{P}_* = \bar{\phi}_1$

Hence $\lim_{n\to\infty} \bar{\bar{R}}^n \bar{p}^{(0)} \simeq a_1 (z^*)^n \bar{p}^*$

Therefore $\bar{p}^{(n)} = \bar{\bar{R}}^n \left[\prod_{j=0}^{n-1} c^{(j)}\right] \bar{p}^{(0)}$ becomes

$$\lim_{n\to\infty} \bar{p}^{(n)} = \bar{p}^* \cdot a_1 \cdot \lim_{n\to\infty} (z^*)^n \left[\prod_{j=0}^{n-1} c^{(j)}\right]$$

choose to constant $c^{(j)}$ on the j^{th} iteration so that the limit of $\bar{p}^{(n)}$ is bounded.

one possible way is $c^{(j)} = \dfrac{1}{\max\{\bar{p}^{(j)}\}}$

since

$$P_i^{(n)} = c^{(n-1)} \underbrace{P_i^{(n-1)}} \left[1 + \frac{1}{\gamma_i^{(n-1)}}\right]$$

For large n case

$$P_i^{(n)} = c^{(n-1)} P_i^* a_1 (z^*)^n \prod_{j=0}^{n-2} c^{(j)}$$

Then

$$P_i^{(n)} = c^{(n-1)} \left[c^{(n-2)} P_i^* a_1 (z^*)^{n-1} \prod_{j=0}^{n-3} c^{(j)} \right] \cdot \left[1 + \frac{1}{\gamma_i^{(n-1)}}\right]$$

$$= c^{(n-1)} \left[1 + \frac{1}{\gamma_i^{(n-1)}}\right] \cdot P_i^* a_1 (z^*)^{n-1} \prod_{j=0}^{n-2} c^{(j)}$$

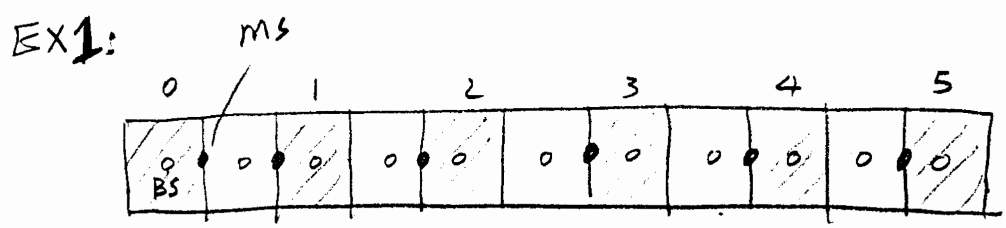
compare them

$$z^* = 1 + \frac{1}{\gamma_i^{(n-1)}}$$

$$\lim_{n\to\infty} \gamma_i^{(n-1)} \equiv \gamma^* = \frac{1}{z^* - 1}$$

#

EX1:



$\bar{\bar{R}}$ is defined as $r_{ij} = \left(\frac{d_{ii}}{d_{ij}}\right)^n$. n: path loss exponent

IF $n=4$

$$\begin{array}{c}
r_{ij} \\
\bar{\bar{R}} = \begin{bmatrix}
\left(\frac{0.5}{0.5}\right)^4 & \left(\frac{0.5}{1.5}\right)^4 & \left(\frac{0.5}{3.5}\right)^4 & \left(\frac{0.5}{5.5}\right)^4 & \left(\frac{0.5}{7.5}\right)^4 & \left(\frac{0.5}{9.5}\right)^4 \\
\left(\frac{0.5}{1.5}\right)^4 & \left(\frac{0.5}{0.5}\right)^4 & \left(\frac{0.5}{1.5}\right)^4 & \left(\frac{0.5}{3.5}\right)^4 & \left(\frac{0.5}{5.5}\right)^4 & \left(\frac{0.5}{7.5}\right)^4 \\
\left(\frac{0.5}{3.5}\right)^4 & \left(\frac{0.5}{2.5}\right)^4 & \left(\frac{0.5}{0.5}\right)^4 & \left(\frac{0.5}{1.5}\right)^4 & \left(\frac{0.5}{3.5}\right)^4 & \left(\frac{0.5}{5.5}\right)^4 \\
\left(\frac{0.5}{5.5}\right)^4 & \left(\frac{0.5}{4.5}\right)^4 & \left(\frac{0.5}{2.5}\right)^4 & \left(\frac{0.5}{0.5}\right) & \left(\frac{0.5}{1.5}\right)^4 & \left(\frac{0.5}{3.5}\right)^4 \\
\left(\frac{0.5}{7.5}\right)^4 & \left(\frac{0.5}{6.5}\right)^4 & \left(\frac{0.5}{4.5}\right)^4 & \left(\frac{0.5}{2.5}\right)^4 & \left(\frac{0.5}{0.5}\right)^4 & \left(\frac{0.5}{1.5}\right)^4 \\
\left(\frac{0.5}{9.5}\right)^4 & \left(\frac{0.5}{8.5}\right)^4 & \left(\frac{0.5}{6.5}\right)^4 & \left(\frac{0.5}{4.5}\right)^4 & \left(\frac{0.5}{2.5}\right)^4 & \left(\frac{0.5}{0.5}\right)^4
\end{bmatrix}
\end{array}$$

$$= \begin{bmatrix}
1 & \frac{1}{3^4} & \frac{1}{7^4} & \frac{1}{11^4} & \frac{1}{15^4} & \frac{1}{19^4} \\
\frac{1}{3^4} & 1 & \frac{1}{3^4} & \frac{1}{7^4} & \frac{1}{11^4} & \frac{1}{15^4} \\
\frac{1}{7^4} & \frac{1}{5^4} & 1 & \frac{1}{3^4} & \frac{1}{7^4} & \frac{1}{11^4} \\
\frac{1}{11^4} & \frac{1}{9^4} & \frac{1}{5^4} & 1 & \frac{1}{3^4} & \frac{1}{7^4} \\
\frac{1}{15^4} & \frac{1}{13^4} & \frac{1}{9^4} & \frac{1}{5^4} & 1 & \frac{1}{3^4} \\
\frac{1}{19^4} & \frac{1}{17^4} & \frac{1}{13^4} & \frac{1}{9^4} & \frac{1}{5^4} & 1
\end{bmatrix} \quad \#$$

Ex (continue):

The corresponding 6 eigenvalues of $\bar{\bar{R}}$

$\lambda = [1.0137, 1.006, 1, 0.999, 0.994, 0.987]$

$z^* = 1.0137$

↑ the maximum eigen value

$$\lim_{n\to\infty} \gamma_i^{(n-1)} \equiv \gamma^* = \frac{1}{z^*-1} = 73.3\ (18.7\ \text{dB})$$

Since the eigen values are quite closely spaced together,

$$z_1 \approx z_i,\ i \neq 1.$$

The proof of convergence (for iteration n large enough)

$$z_1^n \gg z_i^n,\ i \neq 1.$$

In this case $z_1 \approx z_i$, n has to be very large for $z_1^n \gg z_i^n$.

Therefore, the convergence is expected to be very slow.

$$\frac{z_2}{z_1} = \frac{1.006}{1.0137} = 0.992$$

$$\frac{z_2^n}{z_1^n} = 0.52 \quad \text{for } n=60.$$

If z_1 & z_2 are both less than 1, the convergence is much more rapid.

$$\text{If } \lambda_i = z_i - 1, \quad \frac{\lambda_2}{\lambda_1} = \frac{z_2-1}{z_1-1} = \frac{0.006}{0.0137}$$

$$\frac{\lambda_2^n}{\lambda_1^n} = \left(\frac{0.006}{0.0137}\right)^n = 0.0015, \text{ for } n=8 \quad (\text{fast convergence}).$$

(b). The second algorithm called

DPC Algorithm (Distributed power control)

The new power iteration equation

DPC method definition $P_i^{(n)} = c^{(n-1)} P_i^{(n-1)} / \gamma_i^{(n-1)}$

The matrix-vector form

$$\bar{P}^{(n)} = c^{(n-1)} \bar{\bar{A}} \bar{P}^{(n-1)}$$

$\bar{\bar{A}}_{m\times m}$ matrix elements $a_{ij} \equiv \frac{G_{ij}}{G_{ii}}$, $i \neq j$; $a_{ii} = 0$

$$\bar{\bar{A}} = \bar{\bar{R}} - \bar{\bar{I}} .$$

The eigenvalues $\lambda_j = z_j - 1$

the eigenvalues of the matrix $\bar{\bar{R}}$.

the eigenvalues of the matrix $\bar{\bar{A}}$.

The DPC algorithm converges to the same SIR (γ^*) as the DBA Algorithm

$$\lambda^* = \frac{1}{z^*-1}$$

$$\gamma^* = \frac{1}{z^*-1} = \frac{1}{\lambda^*}$$

The convergence rate is $\frac{\lambda_2}{\lambda_1}$, a much small number the $\frac{z_2}{z_1}$

Consider again the 11-cell example from Ex1. the six mobiles are assigned the same channel, assuming a reuse of two. The relative powers at each mobile, relative to the one with the highest power.

On convergence of the SIR

mobile	0	1	2	3	4	5
relative power	0.86	1	0.214	0.052	0.016	0.004

The problems associated with implementing the previous two algoriths are the gain coefficients.

$c^{(j)} = \dfrac{1}{\max(\bar{P}^{(j)})}$, where by $\max(\bar{P}^{(j)})$ we mean the maximum among the M transmitted powers on the j^{th} iteration.

we need to synchronously pass their respective power values on the j^{th} iteration to each other.

(C). The third algorith

(A modified DPC algorithm converges with Asynchronous implementation that mobiles do Not pass their respective power values to each other)

Foschini-Miljanic iteration equation

$$P_i^{(n)} = \gamma P_i^{(n-1)} / \gamma_i^{(n-1)}$$

γ is a "target" SIR, replacing $c^{(n-1)}$.

This target SIR is defined as $\gamma \le \gamma^*$.

all mobile iteration are then locally carried out, with no inter-mobile communication required, as is the case in using $c^{(n-1)}$

$$P_{i,dB}^{(n)} = P_{i,dB}^{(n-1)} + (\underbrace{\gamma_{dB}}_{\text{SIR targeted}} - \underbrace{\gamma_{i,dB}^{(n-1)}}_{\text{SIR measured}})$$

The algorithms are appropriate for TDMA/FDMA and the uplink power control algorithm in IS-95 CDMA systems.