EE 5306
Electromagnetic Theory

Ph.D. Diagnostic Exam
Spring 2007
1. (30 points) The Maxwell's equations in differential form are given by:

\[ \nabla \cdot D(\mathbf{r}, t) = \rho_{s}(\mathbf{r}, t) \]
\[ \nabla \cdot B(\mathbf{r}, t) = 0 \]

\[ -\nabla \times E(\mathbf{r}, t) = \frac{\partial}{\partial t} B(\mathbf{r}, t) \]

\[ \nabla \times H(\mathbf{r}, t) = J(\mathbf{r}, t) + \frac{\partial}{\partial t} D(\mathbf{r}, t) \]

(a) Derive (show the derivation steps) the integral form of time-varying Maxwell's equations.

(b) Derive the Poynting Theorem in a source-free medium with parameters \( \varepsilon_1, \mu_1, \) and \( \sigma_1. \)

(c) Derive the vector Helmholtz equation (i.e. the time-harmonic complex vector wave equation) in a source-free medium with parameters \( \varepsilon_1, \mu_1, \text{ and } \sigma_1. \)
2. (30 points) The electric field of a uniform plane wave propagating in a dielectric medium is given by

\[ E(z, t) = -\hat{x} 7 \cos \left(2 \pi \times 10^9 t - \frac{20 \pi z}{3} \right) - \hat{y} 7 \sin \left(2 \pi \times 10^9 t - \frac{20 \pi z}{3} + \frac{\pi}{2} \right) \text{ [V/m]} \]

(a) Determine the frequency (in Hz), wavelength (in m), phase velocity and polarization of the wave.

(b) Determine the associated time-varying magnetic field \( B(z, t) \).

(c) Determine the direction and average power density carried by the wave.
3. (30 points) A plane wave incident on a plane interface at \( z = 0 \) has the form (time-harmonic)
\[
\mathbf{\hat{E}}_i = \hat{y}H_0 \exp(-jk_xx + jk_zz).
\]
The reflected and transmitted waves are given by
\[
\mathbf{\hat{E}}_r = \hat{y}R_vH_0 \exp(-jk_xx - jk_zz), \quad \text{and}
\]
\[
\mathbf{\hat{E}}_t = \hat{y}T_vH_0 \exp(-jktxx + jktzz), \quad \text{respectively, where}
\]
\[
k_x = k\sin\theta, \quad k_z = -k\cos\theta, \quad k_{tx} = k_t\sin\theta_t, \quad \text{and} \quad k_{tz} = k_t\cos\theta_t.
\]
\( k \) and \( k_t \) are the wavenumber of the upper and lower media respectively.

Derive the reflection coefficient, \( R_v \).