Phasor Notation-Review

Horizontal projection on real axis

\[ a(t) = A \cos(\omega t + \theta_0) \]

Projection on vertical axis

\[ b(t) = A \sin(\omega t + \theta_0) \]

Complex variable =

\[ \tilde{c}(t) = a(t) + jb(t) \]

\[ = A \cos(\omega t + \theta_0) + jA \sin(\omega t + \theta_0) \]

\[ = A[\cos(\omega t + \theta_0) + j \sin(\omega t + \theta_0)] \]
Phasor Notation-Review …

Euler’s Identity

\[ e^{j\alpha} = \cos \alpha + j \sin \alpha \]

\[ \therefore c(t) = Ae^{j(\omega t + \theta)} = \underline{Ae^{j\theta}} e^{j\omega t} \]

\[ \overline{A} = Ae^{j\theta} = A \angle \theta = A[\cos \theta + j \sin \theta] \]
Phasor Notation-Review …

Small Example illustrating phasors

\[ V_s(t) \rightarrow i(t) \rightarrow L \rightarrow R \rightarrow C \]

**KVL**

\[ v_s(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int_{-\infty}^{t} i(x) dx \]

---

Phasor Notation-Review …

We assume \( i(t) \) and \( v(t) \) are sinusoidal

\[ \therefore \quad i(t) = I_m \cos(\omega t + \theta_i) \]

\[ v_s(t) = V_m \cos(\omega t + \theta_v) \]

We define a complex instantaneous current and voltage

\[ \tilde{i}(t) = I_m \cos(\omega t + \theta_i) + j I_m \sin(\omega t + \theta_i) \]

\[ \tilde{v}_s(t) = V_m \cos(\omega t + \theta_v) + j V_m \sin(\omega t + \theta_v) \]

Then

\[ \tilde{i}(t) = I_m e^{j(\omega t + \theta_i)} = I_m e^{j\theta_i} e^{j\omega t} \]

\[ \tilde{v}_s(t) = V_m e^{j(\omega t + \theta_v)} = V_m e^{j\theta_v} e^{j\omega t} \]
Phasor Notation-Review …

We substitute complex instantaneous currents and voltage in KVL, but take real parts.

\[
\text{Real} \left\{ v(t) = R \dot{i}(t) + L \frac{d\dot{i}(t)}{dt} + \frac{1}{C} \int_{x=-\infty}^{t} i(x) dx \right\}
\]

The validity of the KVL holds if we remove the “Real” operator, thus,

\[
V_m \angle \theta_v e^{j\omega t} = R I_m \angle \theta_i e^{j\omega t} + j \omega L I_m \angle \theta_i e^{j\omega t} + \frac{1}{j \omega C} I_m \angle \theta_i e^{j\omega t}
\]

Since \( e^{j\omega t} \) can never be zero, we factor it out

\[
V_m \angle \theta_v = \left( R + j \omega L + \frac{1}{j \omega C} \right) I_m \angle \theta_i
\]

\[
\therefore \overline{V} = \overline{Z} \overline{I}
\]
Phasor Notation-Review …

Note algebraic operations related to differentiation and integration

\[
\frac{d(I_m e^{j\theta} e^{j\omega t})}{dt} = I_m e^{j\theta} \frac{de^{j\omega t}}{dt} = j\omega (I_m e^{j\theta} e^{j\omega t})
\]

\[
\int_{-\infty}^{\infty} i(t)dx = \int_{-\infty}^{\infty} I_m e^{j\theta} e^{j\omega x} dx = I_m e^{j\theta} \frac{e^{j\omega x}}{j\omega} \bigg|_{-\infty}^{\infty} = \frac{1}{j\omega} (I_m e^{j\theta} e^{j\omega t})
\]

11.3 Instantaneous Power and Average Power

Definition

\[ p(t) = v(t)i(t) \quad \{\text{instantaneous} \} \]

units of watts

Periodic Functions

Function repeats every “T” seconds.

\[ v(t) = v(t + T) \]

\[ i(t) = i(t + T) \]

and \[ p(t) = v(t + T)i(t + T) \]
Average Power
Defined as the energy in one period divided by the period.

\[ P = \frac{1}{T} \int_{x=t}^{t+T} p(x)dx = \frac{1}{T} \text{[energy in one period]} \]

Special Case: Sinusoidal functions

\[ v(t) = V_m \cos(\omega t + \theta_v) \]
\[ i(t) = I_m \cos(\omega t + \theta_i) \]

where: \( 2\pi f = \omega \), radian frequency

\[ f \text{ : frequency in Hz (cps)} \]
\[ T = \frac{1}{f} \text{, Period in sec} \]
\[ \theta_v, \theta_i \text{ : phase angle shift} \]

\[ p(t) = V_m \cos(\omega t + \theta_v) \cdot I_m \cos(\omega t + \theta_i) \]

Note trig identity:

\[ \cos \alpha \cos \beta = \frac{1}{2} \left[ \cos(\alpha - \beta) + \cos(\alpha + \beta) \right] \]

Let \( \alpha \equiv \omega t + \theta_v \) and \( \beta \equiv \omega t + \theta_i \)

\[ \therefore p(t) = \frac{V_m I_m}{2} \left[ \text{Not a function of time} \quad + \quad \text{sinusoidal with frequency twice fundamental} \right] \]
Now, we determine the average power per period

\[
P = \frac{1}{T} \int_{x=t}^{t+T} \frac{V_m I_m}{2} \left[ \cos(\theta_v - \theta_i) + \cos(2\omega x + \theta_v + \theta_i) \right] dx
\]

\[
eq \frac{1}{T} \left[ \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) t^{t+T} \right] + \frac{V_m I_m}{2} \int_{x=t}^{t+T} \cos(2\omega x + \theta_v + \theta_i) dx
\]

\[
P = \frac{1}{2} V_m I_m + \text{zero}
\]

(Why is the second term zero??)

The instantaneous power \(p(t)\) entering a circuit
Exercises in Section 11.3

Exercise 11.3-1 (pg 500)

Determine the instantaneous power delivered to an element and sketch p(t) when the element is (a) a resistor R, (b) and inductor L. The voltage across the element is \( v(t) = V_m \cos(\omega t + \theta) \).

a) Resistor R

\[
\begin{align*}
i(t) &= \frac{v(t)}{R} = \frac{V_m}{R} \cos(\omega t + \theta) \\
\therefore \quad \theta_i &= \theta_v = \theta, \quad I_m = \frac{V_m}{R}
\end{align*}
\]

\[
p_R(t) = \frac{V_m I_m}{2} \left[ \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i) \right]
\]

\[
p_R(t) = \frac{V_m I_m}{2} + \frac{V_m I_m}{2} \cos(2\omega t + 2\theta)
\]

\[
p_R(t) = \frac{V_m^2}{2R} [1 + \cos(2\omega t + 2\theta)]
\]
b) Inductor L:

\[
I(t) = \frac{jV_L}{\omega L} = \frac{V_m}{\omega L} \left( \theta - 90^\circ \right) = I_m \left( \theta - 90^\circ \right)
\]

\[
\therefore i(t) = \frac{V_m}{\omega L} \cos(\omega t + \theta - 90^\circ)
\]

\[
\therefore \theta_i = \theta - 90^\circ, \theta_i = \theta, I_m = \frac{V_m}{\omega L}
\]

\[
p_L(t) = \frac{V_m I_m}{2} \left[ \cos(\theta_i - \theta) + \cos(2\omega t + \theta) \right]
\]

\[
= \frac{V_m^2}{2\omega L} \left[ \cos(90^\circ - \cos(2\omega t + 20 - 90^\circ)) \right]
\]

\[
p_L(t) = \frac{V_m^2}{2\omega L} \cos(2\omega t + 20 - 90^\circ)
\]

11.4 Effective value of a periodic waveform

\[
V_0 = \frac{V_o}{R}
\]

\[
I_o = \frac{V_o}{R}
\]

\[
p(t) = V_0 = \frac{V_o^2}{R}
\]

Energy over one period T

\[
W = \int_{-T/2}^{T/2} p(x)dx = \int_{-T/2}^{T/2} \frac{V_o^2}{R} dx
\]

\[
= \frac{V_o^2}{R} T
\]

For the source (period T)

\[
\dot{i}(t) = \frac{V(t)}{R}
\]

\[
p(t) = \frac{V(t)}{R}
\]

\[
W = \int_{-T/2}^{T/2} \frac{V(t)^2}{R} dx
\]

For one period

\[
W_0 = W
\]

We set the energies equal and seek an expression of \( V_0 \) to realize this equality (i.e., \( W_0 = W \))
\[ W_o = \frac{V_o^2}{R}T = W = \int_{x=0}^{T} \frac{v^2(x)}{R} dx \]

\[ \therefore V_o^2 = \frac{1}{T} \int_{x=0}^{T} v^2(x) dx \]

\[ V_{\text{eff}} = V_o = \sqrt{\frac{1}{T} \int_{x=0}^{T} v^2(x) dx} \]

For a sinusoidal voltage source
\[ v(t) = V_m \cos(\omega t + \theta) \]

\[ \therefore V_{\text{eff}} = \sqrt{V_m^2} = \sqrt{V_{\text{rms}}} \]

Note that \( \cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha) \)

\[ V_{\text{eff}} = \sqrt{\frac{1}{T} \int_{x=0}^{T} \frac{V_m^2}{2} [1 + \cos(2\omega x + 2\theta)] dx} \]

\[ = \sqrt{\frac{1}{T} \int_{x=0}^{T} \frac{V_m^2}{2} dx + \frac{1}{T} \int_{x=0}^{T} \frac{V_m^2}{2} \cos(2\omega x + 2\theta) dx} \]

\[ V_{\text{eff}} = V_{\text{rms}} = \] for sinusoidal functions
\[ P_{\text{avg}} = \frac{V_m I_m \cos(\theta_v - \theta_i)}{2} \]
\[ = \left( \frac{V_m}{\sqrt{2}} \right) \left( \frac{I_m}{\sqrt{2}} \right) \cos(\theta_v - \theta_i) \]
\[ P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) \]

or

\[ P_{\text{avg}} = V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i) \]

Exercises section 4
Exercise 11. 4-1 (pg 502)

Find the effective value of the following currents:
(a) $\cos 3t + \cos 3t$
(b) $\sin 3t + \cos(3t + 60^\circ)$
(c) $2\cos 3t + 3\cos 5t$

(a) $\cos 3t + \cos 3t = 2\cos 3t$

$$I_{\text{eff}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

(b) $\sin 3t + \cos(3t + 60^\circ) = \cos(3t - 90^\circ) + \cos(3t + 60^\circ)$

In phasor form

$$\lvert -90^\circ \rvert + \lvert 60^\circ \rvert = -j1 + 0.5 + j\frac{\sqrt{3}}{2}$$

$$= 0.5 - j0.134$$

$$= 0.518 - 15.003^\circ$$

$$\therefore I_{\text{eff}} = \frac{0.518}{\sqrt{2}} = 0.366$$

Exercise 11, 4-1 cont’d

(c) $i(t) = 2\cos 3t + 3\cos 5t$

$$i^2(t) = 4\cos^2 3t + 12\cos 3t \cos 5t + 9\cos^2 5t$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} i^2(x)dx}$$

$$= \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} \left[ 4\cos^2 3x + 12\cos 3x \cos 5x + 9\cos^2 5x \right] dx}$$

$$= \sqrt{\frac{1}{2} \left[ \int 4\cos^2 3xdx + \int 12\cos 3x \cos 5xdx + \int 9\cos^2 5xdx \right]}$$

$$= \sqrt{\frac{1}{2} \left[ \frac{4}{2} + \frac{9}{2} \right]} = \sqrt{\frac{13}{2}}$$

$$I_{\text{eff}} = 2.55 \text{ A}$$

$$\therefore I_{\text{eff}} = \sqrt{I_{\text{eff}}^2 + I_{\text{eff}}^2}$$

$\cos \alpha \cos \beta = \frac{1}{2} \left[ \cos(\alpha + \beta) + \cos(\alpha - \beta) \right]$
11.5 Complex Power

Derivation (Not presented this way in text)

Passive Convention:

Assume: \( v(t) = V_m \cos(\omega t + \theta_v) \)
\( i(t) = I_m \cos(\omega t + \theta_i) \)

Without loss of generality, we reference the voltage to the current by subtracting \( \theta_i \) from both

\[ v(t) = V_m \cos(\omega t + \theta_v - \theta_i) \]
\[ i(t) = I_m \cos(\omega t + \theta_i - \theta_i) = I_m \cos(\omega t) \]

\[ p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos(\omega t) \]

This is our average power term

This is a double freq. sinusoidal term

Consider the second term. It can be considered as the projection on the “Real” axis of a point rotating in a complex plane with radius \( \frac{V_m I_m}{2} \), at a frequency of \( 2\omega \)

Let’s define an instantaneous quantity that is complex as follows:

\[ s(t) = \frac{V_m I_m}{2} \left[ \cos(2\omega t + \theta_v - \theta_i) + j \sin(2\omega t + \theta_v - \theta_i) \right] \]

\[ = \frac{V_m I_m}{2} e^{j(2\omega t + \theta_v - \theta_i)} = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} e^{j2\omega t} \]

Now, we define \[ \tilde{S} = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} \]

\[ = \frac{V_m I_m}{2} [\theta_v - \theta_i] \]
or \( \bar{S} = \left( \frac{V_{\text{rms}}}{\sqrt{2}} \right) \left( \frac{|I_{\text{rms}}|}{\sqrt{2}} e^{j\theta} \right) = \left( V_{\text{rms}} e^{j\theta} \right) \left( I_{\text{rms}} e^{j\theta} \right) \)

\[ \bar{S} = VI \quad \text{Where } * \text{ indicates the conjugation of the complex variable } \bar{I} \]

\[ \text{Note: } \bar{S} = \frac{P}{V} + j \frac{Q}{V} \]

Avg. Power, Watts  Reactive power, VAR

\( \bar{S} \equiv \text{Complex Power (VA)} \)

Thus, when we know the phasor voltage across an element and the phasor current through the element, we can calculate the complex power. Consider again the element in phasor form.

\[ \bar{V} = \bar{Z} \bar{I} \]

Ohm's Law:

\[ \bar{V} = \bar{Z} \bar{I} \]

or \( \bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{V|\theta_v|}{I|\theta_i|} \)

\[ = \frac{V}{I} |\theta_v - \theta_i| = R + jX \]
Also, $\bar{T} = \frac{\bar{V}}{\bar{Z}}$

Now, $\bar{S} = \bar{V}^* = \sqrt{V^* Z} = \frac{\bar{V} \bar{Z}^*}{\bar{Z} \bar{Z}} = \frac{|\bar{V}|^2}{|\bar{Z}|}$

$= |\bar{V}|^2 = |\bar{V}| \times |\bar{Z}| \theta - \theta$.

### Conservation of power

The net sum of power produced is equal to the net sum of power absorbed.
My Convention

Source

\[ \bar{S} = \bar{V}_s \bar{I} = P_s + jQ_s \]

Load

Note how current is related to voltage polarity

\[ \bar{S} = \bar{V}_L \bar{I} = P_L + jQ_L \]

Then

\[ P_{S\cdot Q_S} + \text{Supply} \]

Absorbed

\[ P_{L\cdot Q_L} + \text{Absorbed} \]

Supply

The above convention is explained slightly different than what’s in the text, but the results are the same conservation of power

Exercises in section 11.5
**Exercise 11.5-1 (pg. 509)**

Determine the average power delivered to each element of the circuit shown in fig. E 11.5-1. Verify that the average power is conserved.

\[
\tilde{I} = \frac{12\angle 0^\circ}{10 + j8} = 0.937\angle 38.66^\circ, \quad Z=10+j8
\]

\[
P_{\text{loc}} = |\tilde{I}|^2 R = \left(\frac{0.937}{\sqrt{2}}\right)^2 (10) = 4.39 \text{ watts (absorbed)}
\]

**Exercise 11.5-1 cont’d**

Consider inductor

\[
V_L = Z_L I
\]

\[
S_L = V_L I^* = Z_L I^* = Z_L |I|^2 = (8\angle 90^\circ) \left(\frac{0.937}{\sqrt{2}}\right)^2 = 3.512\angle 90^\circ
\]

\[
= 3.512 (\cos 90^\circ + j \sin 90^\circ) = 0 + j3.512 = P_L + jQ_L
\]

\[
\therefore P_L = 0
\]

Consider Source

\[
P_s = V_I \cos(\theta_s - \theta) = \left(\frac{12}{\sqrt{2}}\right) \left(\frac{0.937}{\sqrt{2}}\right) \cos 38.66^\circ
\]

\[
= 4.39 \text{ watts (supplied)}
\]

Note that I used the Source/Load convention concept.

Conservation of reactive power also holds.
Exercise 11.5-2 (pg. 509)

Determine the complex power delivered to each element of the circuit shown in fig. E 11.5-2. Verify that complex power is conserved.

\[ Z = 10 + j(8-5) = 10 + j3 \]
\[ I = \frac{12\sqrt{2}}{10 + j3} = 0.813\angle -16.699^\circ \text{ (rms)} \]
\[ S_s = \overline{V_s} I = \left(\frac{12}{\sqrt{2}}\right) \left[(0.813 \angle -16.699^\circ)^*\right] = 6.896\angle 16.699^\circ \]
\[ S_s = 6.606 + j1.982 \text{ VA} \]

Both terms are positive, both are supplied.

\[ P_{\text{real}} = I^2 \times 10 = (0.813)^2 (10) = 6.606 \text{ watts} \]
\[ Q_{C} = I^2 \times (-5) = -3.303 \text{ VAR} \]
\[ Q_{L} = I^2 \times (8) = 5.284 \text{ VAR} \]

Net = 1.981
11.6 Power Factor

\[ S = VI \left[ \cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i) \right] \]

\[ \theta = \theta_v - \theta_i, \text{ referred to as the power factor angle} \]

\[ \cos \theta = \text{Power factor} \]

\[ \sin \theta = \text{Reactive factor} \]

\[ \theta_v - \theta_i \begin{cases} > 0, \text{current lags voltage, lagging p.f} \\ < 0, \text{current leads voltage, leading p.f} \\ = 0, \text{current in phase with voltage} \end{cases} \]

Power factor is most significant in relation to loads served by electric utilities, when a load has a low power factor, nearly all the energy from the source oscillates back and forth with the load at twice the fundamental frequency. The conductors carrying the current must be of sufficient capacity to accommodate this current flow without over heating. By increasing the power factor, the resulting current can be reduced.

Review Examples

11.6-1: parallel loads (pg 513)
11.6-2: power factor correction (pg 514)
11.6-3: complex power (pg 515)
Exercise 11.6-1 (pg 517)

A circuit has a large motor connected to the a.c power lines. 
(\(\omega = 2\pi 60 = 377 \text{ rad/s}\))

The model of the motor is a resistor of 100\(\Omega\) is series with an inductor of 5H. Find the power factor of the motor.

\[
\begin{align*}
100\Omega \\
\sqrt{100^2 + (1885)^2} = 1887.7 \\
\theta = \angle (100 + j1885) = 86.96^\circ \\
\end{align*}
\]

\[\text{pf} = \frac{\text{Real part}}{\text{Magnitude}} = \frac{100}{1887.7} = \frac{1}{18.877} = \frac{1}{18.877}\]
Exercise 11.6-2 (pg 517)

A circuit has a load impedance $Z = 50 + j80 \Omega$ as shown below. Determine the power factor of the uncorrected circuit. Determine the impedance $Z_c$ required to obtain a corrected power factor of 1.0

$$
\bar{Z}_L = 50 + j80 = 94.34 [57.995^\circ]
$$

$$
pf = \cos(57.995^\circ) = ______________________
$$

Exercise 11.6-2 cont’d

Circuit for corrected pf

$$
\bar{Z}_{eq} = \frac{(-jX_c)(50 + j80)}{50 + j80 - jX_c} = \frac{X_c (80 - j50)}{50 + j(80 - X_c)}
$$

$$
= \frac{X_c (80 - j50) [50 - j(80 - X_c)]}{50^2 + (80 - X_c)^2}
$$

$$
= \frac{X_c \left\{ [(80)(50) - 50(80 - X_c)] - j \left[ 80(80 - X_c) + 50^2 \right] \right\}}{50^2 + (80 - X_c)^2}
$$
Exercise 11.6-2 cont’d

For \( pf = 1 \), the imaginary component of \( Z_{\text{eq}} \) must be zero.

\[
\therefore \quad 80(80 - X_C) + 50^2 = 0
\]

\[
80 - X_C = -\frac{50^2}{80}
\]

\[
X_C = \quad \text{__________________________}
\]

\[
X_C = \quad \text{______________} \Omega
\]

\[
\therefore \quad Z_C = \quad \text{______________} \Omega
\]

Exercise 11.6-2 cont’d

Another way to compute \( Z_C \)

\[
Z_L = \frac{1}{Y_L}\quad \text{or} \quad Y_L = \frac{1}{Z_L} = \frac{1}{R_L + jX_L} = \frac{R_L - jX_L}{(R_L + jX_L)(R_L - jX_L)} = G_L - jB_L
\]

\[
Y_C = \frac{1}{Z_C} = \frac{1}{-jX_C} = j \frac{1}{X_C} = jB_C
\]
Exercise 11.6-2 cont’d

For pf=1, \( \therefore -jB_L + jB_C = 0 \)

or \( B_L = B_C \)

\[
\frac{X_L}{R_L^2 + X_L^2} = \frac{1}{X_C} \Rightarrow X_C = \frac{R_L^2 + X_L^2}{X_L} = \frac{R_L^2}{X_L}
\]

\( \therefore X_C = \frac{50^2}{80} + 80 = 111.25 \Omega \)

\( Z_C = -j111.25 \Omega \)

---

Exercise 11.6-3 (pg 518)

Determine the power factor for the total plant of example 11.6-1 (pg 513) when the resistive heating load is decreased to 30 KW. The motor load and the supply voltage remain as determined in example 11.6-1

Load ① \( \overline{S}_1 = 30 \) KW

Load ② \( \theta_2 = \cos^{-1}(0.86) = 30.68^\circ \)

\[
P_2 = 100\cos \theta_2 = (100)(0.86) = 86 \text{ KW}
\]

\[
Q_2 = 100\sin \theta_2 = (100)(0.51) = 51 \text{ KVAR}
\]

Total:

\[
\overline{S}_T = \overline{S}_1 + \overline{S}_2 = 30 + 86 + j51
\]

\[
= 116 + j51 = 126.7(23.73^\circ)
\]

New \( pf = \cos \theta_T = \cos(23.73^\circ) = 0.915 \)
Exercise 11.6-4 (pg 518)

A 4 KW, 110V load, as shown in fig. 11.6-5 (pg 514), has a power factor of 0.82 lagging. Find the value of the parallel capacitor that will correct the pf to 0.95 lagging when $\omega = 377$ rad/s

\[
\begin{align*}
\text{P}=4 \text{ KW} \\
\text{Pf}=0.82 \text{ lagging} \\
\text{110 V} \\
\hline
\end{align*}
\]

\[\theta = \cos^{-1}(0.82) = 34.915^\circ\]

\[|S| = \frac{P}{\cos(\theta)} = \frac{4}{\cos(34.915)} = 4.878 \text{ VA}\]

\[Q = P \tan(\theta) = 4 \times 0.698 = 2.792 \text{ KVAR}\]

\[S_L = \sqrt{V^2 - I_L^2} = \sqrt{(110)^2 - (\frac{2.792}{2.034})^2} = 4000 - j2792\]

\[Z_L = \left|\frac{|V|}{S_L}\right| = \frac{(110)^2}{4000 - j2792} = 2.48 + j34.915^\circ\]

\[\theta_{\text{corrected}} = \cos^{-1}(|\text{pf}_{\text{corrected}}|) = \cos^{-1}(0.95) = 18.195^\circ\]

Exercise 11.6-4 cont’d

Load: $|S_L| = \frac{P_L}{\cos(\theta_L)} = \frac{4}{\cos(34.915)} = 4.878 \text{ VA}$

\[\theta_L = \cos^{-1}(0.82) = 34.915^\circ\]

\[|S_L| = \frac{P_L}{\cos(\theta_L)} = \frac{Q_L}{\sin(\theta_L)}, \text{ or } Q_L = P_L \frac{\sin(\theta_L)}{\cos(\theta_L)}\]

\[Q_L = P_L \tan(\theta_L) = 4 \times 0.698 = 2.792 \text{ KVAR}\]

\[S_L = \sqrt{V^2 - I_L^2} = \sqrt{(110)^2 - (\frac{2.792}{2.034})^2} = 4000 - j2792\]

\[Z_L = \left|\frac{|V|}{S_L}\right| = \frac{(110)^2}{4000 - j2792} = 2.48 + j34.915^\circ\]

The corrected power factor angle

\[\theta_{\text{corrected}} = \cos^{-1}(|\text{pf}_{\text{corrected}}|) = \cos^{-1}(0.95) = 18.195^\circ\]
Exercise 11.6-4 cont’d

We will use the second method illustrated in EXERCISE 11.6-2

\[ \vec{V}_c = G_L - jB_L + jB_c = |\vec{V}_c|, \theta_c = \frac{1}{|Z_c|}, \theta_c \]

\[ \therefore \theta_c = -\tan^{-1}\left[ \frac{-B_L + B_c}{G_L} \right] \text{ or } -\tan\theta_c = \frac{B_L + B_c}{G_L} \]

\[ -\tan\theta_c = \frac{-B_L + B_c}{G_L} = \frac{X_L}{R_L^2 + X_L^2} + \frac{1}{X_c} = \frac{-X_L X_c + (R_L^2 + X_L^2)}{R_L X_c} \]

Exercise 11.6-4 cont’d

\[ -R_L X_c \tan \theta_c = -X_L X_c + (R_L^2 + X_L^2) \]

\[ X_c = \frac{R_L^2 + X_L^2}{X_L - R_L \tan \theta_c} = \frac{2.034^2 + 1.42^2}{2.034 - 1.42 - (2.034) \tan(18.195^0)} \]

\[ X_c = \frac{6.1536}{1.42 - 0.6685} = 8.188 \Omega \]

\[ C = \frac{1}{\omega X_c} = \frac{1}{(377)(8.188)} = \mu F \]
11.7 The power superposition principle

Superposition Principle
The response to all sources acting together is equal to the sum of the responses to each source acting alone.

\[ i(t) = i_1(t) + i_2(t). \]

Instantaneous Power
\[ p(t) = i^2 R = (i_1 + i_2)^2 R = (i_1^2 + i_2^2 + 2i_1i_2)R \]

Average Power
\[
\begin{align*}
P &= \frac{1}{T} \int_{x=0}^{T} p(x)dx = \frac{R}{T} \int_{0}^{T} (i_1^2 + i_2^2 + 2i_1i_2)dx \\
&= \frac{R}{T} \int_{0}^{T} i_1^2 dx + \frac{R}{T} \int_{0}^{T} i_2^2 dx + \frac{2R}{T} \int_{0}^{T} i_1i_2 dx \\
&= P_1 + P_2 + \frac{2R}{T} \int_{0}^{T} i_1i_2 dx
\end{align*}
\]

Note that \( P_1 \) is due to source \( v_1(t) \) and \( P_2 \) is due to \( v_2(t) \).

What about the third term?
For sinusoidal functions, let's assume
\[
\begin{align*}
i_1(t) &= I_{m1} \cos(\omega t + \theta_1) \\
i_2(t) &= I_{m2} \cos(\omega t + \theta_2)
\end{align*}
\]
Then
\[
\frac{2R}{T} \int_0^T I_{m1} I_{m2} \cos(m\omega x + \theta_1) \cos(n\omega x + \theta_2) \, dx
\]
\[
\frac{R}{T} \int_0^T I_{m1} I_{m2} \left[ \cos \left( (m - n)\omega x + (\theta_1 - \theta_2) \right) + \cos \left( (m + n)\omega x + (\theta_1 + \theta_2) \right) \right] \, dx
\]
\[
\therefore P_{12} = \begin{cases} 
0, & \text{for } m \neq n \\
RI_{m1} I_{m2} \cos(\theta_1 - \theta_2), & \text{for } m = n
\end{cases}
\]

**Observations**

When the frequencies are different for each source, the total power (average) delivered to an element is equal to the sum of the average power delivered to an element by each source acting alone. This situation is referred to as “Superposition of Average Power.”

When all sources have the same frequency the principle of “Power Superposition” does not hold. In this case, the total current is found first, then the total average power is calculated using total current.

---

**Exercises in section 11.7**
Exercise 11.7-1 (pg 521)

Determine the average power absorbed by the resistor in Fig. 11.7-2a for these two cases

(a) \( v_1(t) = 12 \cos 3t \) and \( v_2(t) = 2 \cos 3t \)

(b) \( v_1(t) = 12 \cos 4t \) and \( v_2(t) = 4 \cos 3t \)

Solution:

(a) Note that frequencies of both sources are same

\[ I_1 = \frac{12}{6 + j6} = \] ____________

\[ i_1(t) = \sqrt{2} \cos(3t - 45^\circ) \]

(b) \[ I_2 = \frac{-4}{6 + j6} = \] ____________
Exercise 11.7-1 (cont’d)

The phasor total is
\[ I = I_1 + I_2 = \sqrt{2} \angle -45^\circ + 0.4714 \angle 135^\circ = 0.9428 \angle -45^\circ \]

\[ P_{6\Omega} = \frac{R |I|^2}{2} = \frac{12}{6 + j8} = 1.2 |53.13^\circ | \]

b) Note the frequencies of sources are different

This is same as part (a) for \( i_B(t) \)

\[ 4 \angle 0^\circ \quad \therefore \quad I_2 = 0.4714 \angle 135^\circ \]

\[ P_2 = \frac{R |I_2|^2}{2} = \frac{12}{6 + j6} = \frac{12}{12} = 1 \quad \text{W} \]

\[ \therefore \quad P = P_1 + P_2 = \text{W} + \text{W} = \text{W} \]
11.8 Maximum power transfer theorem

Review section 5.6 (pg. 174) as to why maximum power transfer is important. It is important from a signal transmission point of view (electronics and communication industries)

The issues

a) Efficiency- Electric Power Industry
b) Effectiveness- Maximum power transfer to maintain maximum signal strength at the load

Thevenin’s circuit with load

\[
\bar{Z}_t = R_t + jX_t \\
\bar{Z}_L = R_L + jX_L
\]

Power delivered to resistance of load, \( R_L \)

\[
P_L = \frac{I_m^2 R_L}{2}
\]

where

\[
T = \frac{\bar{V}_t}{(R_t + R_L) + j(X_t + X_L)} \quad & \quad \bar{V}_t = \frac{|\bar{V}_t|}{\sqrt{(R_t + R_L)^2 + (X_t + X_L)^2}}
\]

\[
I_m = \frac{1}{\sqrt{(R_t + R_L)^2 + (X_t + X_L)^2}} \bar{V}_t
\]

\[
\therefore P_L = \frac{1}{2} R_L
\]

For the power delivered to be a maximum, \( R_L \) and \( X_L \) must be allowed to take on whatever value is necessary to allow maximum power transfer. \( \bar{V}_t \) and \( \bar{Z}_t \) are fixed.
\[ \Delta P_L = \frac{\delta P_L}{\delta R_L} \Delta R_L + \frac{\delta P_L}{\delta X_L} \Delta X_L = 0 \]

\[ \therefore \frac{\delta P_L}{\delta R_L} = 0 \quad \text{and} \quad \frac{\delta P_L}{\delta X_L} = 0 \quad \text{Two equations two unknowns} \]

First, let's determine \( \frac{\delta P_L}{\delta X_L} = 0 \)

We observe that the denominator cannot be zero, nor is \( |V| \) or \( R_L \), otherwise \( P_L \) would be zero.

\[ \therefore X_t + X_L = 0 \quad \text{or} \quad X_L = -X_t \]

Now, let us determine \( \frac{\delta P_L}{\delta R_L} \) knowing that \( X_t + X_L = 0 \)

\[ \frac{\delta P_L}{\delta R_L} = \frac{\delta}{\delta R_L} \left[ \frac{|V_t|^2}{2} \times R_L \right] = \frac{|V_t|^2}{2} \left[ \frac{1}{(R_t + R_L)^2} + \frac{-2R_L}{(R_t + R_L)^3} \right] \]

\[ = \frac{|V_t|^2}{2} \left[ \frac{R_t + R_L - 2R_L}{(R_t + R_L)^3} \right] = \left[ \frac{R_t - R_L}{(R_t + R_L)^3} \right] = 0 \]

\[ \therefore R_t - R_L = 0 \quad \text{or} \quad R_L = R_t \]

\[ Z_L = Z^* \]
Exercises in section 11.8

Exercise 11.8-1 (Pg 523)

For the circuit of Fig 11.8-1, find $\overline{Z}_L$ to obtain the maximum power transferred when the Thevenin equivalent circuit has $\overline{V}_t = 100[0^\circ]$ and $\overline{Z}_t = 10 + j14\Omega$. Also determine the maximum power transferred to the load.

\[
\overline{Z}_L = \overline{Z}_t^* = 10 - j14\Omega
\]

\[
P_L = \frac{|\overline{V}_t|^2}{2} \frac{R}{(R_t + R_L)^2} = \frac{|\overline{V}_t|^2}{2} \frac{R_L}{(2R_L)^2} = \frac{|\overline{V}_t|^2}{8R_L}
\]

\[
P_L = \boxed{\frac{|\overline{V}_t|^2}{8R_L}} \quad \text{W}
\]
Exercise 11.8-2 (Pg 523)

A television receiver uses a cable to connect the antenna to the TV, as shown in Fig. E 11.8-2, with \( v_s = 4 \cos \omega t \) mV. The TV station is received at 52 MHz. Determine the average power delivered to each TV set if

(a) the load impedance is \( Z = 300 \Omega \)

(b) two identical TV sets are connected in parallel with \( Z = 300 \Omega \) for each set

(c) two identical sets are connected in parallel and \( Z \) is to be selected so that maximum is delivered at each set.

Exercise 11.8-2 (cont’d)

(a) \( Z = 300 \Omega \)

\[ I = \frac{4 \times 10^{-3}}{200 + 300} = 8 \times 10^{-6} \text{ A} \]

\[ P = \frac{I^2 Z}{2} = \frac{(8 \times 10^{-6})^2 (300)}{2} = \left( \frac{64 \times 10^{-12}}{2} \right) (300) \]

\[ P = 9.6 \times 10^{-3} \text{ W} \]
Exercise 11.8-2 (cont’d)

(b) \[ Z_p = \frac{300}{2} = 150 \Omega \]
\[ T = \frac{4 \times 10^{-3}}{200 + 150} = \frac{4 \times 10^{-3}}{350} = 11.43 \times 10^{-9} \]
\[ P_t = \frac{(11.43 \times 10^{-9})^2}{2} = 9.8 \times 10^{-9} \]
\[ \therefore P_{TV} = P_t / 2 = 10 \times 10^{-9} W \]

(c) For max, pwr transfer, \( Z_p = 200 \Omega \)
\[ \therefore Z_{TV} = 2Z_p = 400 \Omega \]
\[ P_t = \frac{(4 \times 10^{-3})^2}{(8)(200)} = 16 \times 10^{-6} = 10 \times 10^{-9} W \]
\[ P_{TV} = P_t / 2 = 15 \Omega \]

11.9 Coupled Inductors

Consider the arrangement of two coupled coils

Coil “1” is energized by source, \( V_1 \), resulting in current entering terminal “a”. The terminals of coil “2” are left open.
For coil 1:
\[ v_i = L_i \frac{di_i}{dt} \text{ where } L_i = \text{Self inductance of coil 1.} \]

Also, \[ v_i = \frac{d\lambda_i}{dt} \text{ where } \lambda_i = \text{All magnetic flux linking coil 1.} \]
\[ \lambda_i = N_i \phi_i \]

Also, \[ \phi_i = C_1 N_i i_i \text{ where } N_i i_i = \text{Ampere-turns} \]

\[ C_1 \text{ is a "constant" related to the magnetic properties and geometry of the "core"} \]
\[ C_1 = \frac{d\phi_i}{dN_i i_i} = \frac{\phi_i}{N_i i_i} \text{ (linear and passes thru origin)} \]

Also, 
\[ \phi_i = \phi_{i1} + \phi_{i2} \]

Where \( \phi_{i1} \) is leakage flux and \( \phi_{i2} \) is flux linking coil 2

Back to \( v_i \)
\[ v_i = L_i \frac{di_i}{dt} = \frac{d\lambda_i}{dt} = N_i \frac{d\phi_i}{dt} = \]
\[ \therefore L_i = \text{self inductance of coil 1.} \]

Consider voltage \( v_2 \)
\[ v_2 = \frac{d\lambda_{21}}{dt} = N_2 \frac{d\phi_{21}}{dt} = N_2 \frac{dC_21 N_i i_i}{dt} = \]

we define \( M_{21} = \text{mutual inductance} \)
Note that $C_1 = C_{11} + C_{21}$, $\phi_1 = \phi_{11} + \phi_{21}$

$\lambda_1 = N_1 \phi_1 = N_1 (\phi_{11} + \phi_{21}) = N_1 \phi_{11} + N_1 \phi_{21}$

$= N_1 (C_{11}N_{11}) + N_1 (C_{21}N_{11})$

$= C_{11}N_1^2 i_{11} + C_{21}N_1^2 i_{11}$

$= (L_{11} + L_{21}) i_{11}$

$\therefore L_1 = L_{11} + L_{21}$ \{Self Inductance has two Components\}

Now, let’s open coil one terminals and energize coil two with $V_2$

\[ v_2 = L_2 \frac{di_2}{dt} = \frac{d\lambda_2}{dt} = \frac{dN_2 \phi_2}{dt} = C_2N_2^2 \frac{di_2}{dt}, \quad L_2 = C_2N_2^2 \] \{Self inductance of coil 2\}

Let’s look at $v_1$

\[ v_1 = \frac{d\lambda_{12}}{dt} = \frac{dN_{12} \phi_{12}}{dt} = N_1 \frac{dC_{12}N_2 i_2}{dt} \]

$= C_{12}N_1N_2 \frac{di_2}{dt}$ where $M_{12} = C_{12}N_1N_2$

The core is such that $M_{12} = M_{23} = M(\text{linear & bilateral})$

What about energizing both coils

\[ v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \]

\[ v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]

Whether $M \frac{di_1}{dt}$ and $M \frac{di_2}{dt}$ are negative or positive depends on how the coils are wound
Dot Convention

Each current produces a flux that flows in the same direction, i.e., additive.

A helpful model

\[ v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \]
\[ v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]

Phasor Domain

\[ \bar{V}_1 = j\omega L_1 \bar{I}_1 + j\omega M \bar{I}_2 \]
\[ \bar{V}_2 = j\omega M \bar{I}_1 + j\omega L_2 \bar{I}_2 \]
**Coupling Coefficient**

Consider the product of $L_1L_2$

$$L_1L_2 = (C_1N_1^2)(C_2N_2^2) = C_1C_2(N_1N_2)^2$$

$$= \frac{C_M^2}{C_M^2} \left[ C_1C_2(N_1N_2)^2 \right] = \left( \frac{C_1C_2}{C_M^2} \right)(N_1N_2C_M)^2 \quad \text{where} \quad C_M \equiv C_{12} = C_{21}$$

We define $k^2 = \frac{C_M^2}{C_1C_2}$

$$\therefore L_1L_2 = \frac{M^2}{k^2} \quad \text{or} \quad k = \frac{M}{\sqrt{L_1L_2}}$$

Note range of $k$: $0 \leq k \leq 1$

- If $C_u = 0$, $k = 0$.
- If $C_u = C_i$, $k = 1$

---

**Use of Transformer (coupled coils)**

![Transformer Diagram](chart)

**Time Domain**

**Phasor Domain**
Reflection of Load

\[ \bar{V}_s = (R_1 + j\omega L_1) \bar{I}_1 - j\omega M \bar{I}_2 \]

\[ 0 = -j\omega M \bar{I}_1 + (Z_L + j\omega L_2) \bar{I}_2 \]

\[ \bar{I}_1 = \frac{\begin{bmatrix} \bar{V}_s \\ -j\omega M \\ 0 \end{bmatrix}}{(R_1 + j\omega L_1) \begin{bmatrix} -j\omega M \\ -j\omega M \\ (Z_L + j\omega L_2) \end{bmatrix}} \]

\[ \bar{I}_2 = \frac{\begin{bmatrix} (R_1 + j\omega L_1) \bar{V}_s \\ -j\omega M \\ -j\omega M \end{bmatrix}}{(R_1 + j\omega L_1) \begin{bmatrix} -j\omega M \\ -j\omega M \\ (Z_L + j\omega L_2) \end{bmatrix}} \]
Solving for $I_2$ in second equation:

$$I_2 = \left( \frac{j\omega M}{Z_4 + j\omega L_2} \right) I_1$$

Substitute into equation 1:

$$V_s = (R_1 + j\omega L_1)I_1 + \frac{Z_{11}I_1}{Z_{11} + Z_R}$$

Impedance of secondary reflected over to primary side.

Exercises in section 11.9
Exercise 11.9-1 (pg. 530)

Determine the voltage $V_0$ for the circuit of Fig. E 11.9-1. Hint: Write a single mesh equation. The current in each coil is the same and equal to the mesh current.

Exercise 11.9-1 (cont’d)

KVL: $j24I + j16I + j40I + j16I = 24\Omega$

$\therefore I = \frac{24}{j96}$

$V_0 = j40I + j16I = (j56)\left(\frac{24}{j96}\right)$

$= 14\Omega$

$\therefore V_0(t) =$ volts
Exercise 11.9-2 (pg 531)
Determine the voltage $v_0$ for the circuit of Fig E 11.9-2. Hint: The exercise is the same as exercise 11.9-1, except for the position of the dot on the vertical coil.

Exercise 11.9-2 (cont’d)

KVL: $j24I - j16I + j40I - j16I = 24|0^0$

$\therefore I = \frac{24}{j32}$

$V_0 = j40I - j16I = (j24) \left( \frac{24}{j32} \right)$

$= 18|0^0$

$\therefore v_0(t) =$ volts
Exercise 11.9-3 (pg 531)

Determine the current \( i_0 \) for the circuit of Fig E 11.9-3. Hint: The voltage across the vertical coil is zero because of the short circuit. The current through the horizontal coil induces a voltage in the vertical coil. Consequently, a current is produced in the vertical coil.

\[
\begin{align*}
&\text{KVL around perimeter of circuit:} \\
&240° - j24I + j16I_c = 0
\end{align*}
\]

or

1) \( j24I - j16I_c = 24 \)

KVL that includes vertical coil and short circuit:

2) \( j16I - j40I_c = 0, \quad \text{or} \quad I_c = \frac{16}{40} l = 0.4 \ l \)

From Eqn (1):

\[
\begin{align*}
&j24I - j16 \left( \frac{16}{40} \right) l = 24 \\
&I = \frac{24}{j17.6} = 1.3636\angle-90° \\
&\text{and} \quad I_c = 0.41 = 0.5454\angle-90°
\end{align*}
\]

\[
\begin{align*}
&I_0 = I + I_c = 1.3636\angle-90° + 0.5454\angle-90° \\
&= 1.909\angle-90° \\
&\therefore i_0(t) = \boxed{1.909\angle-90°} \quad \text{A}
\end{align*}
\]
Exercise 11.9-4 (pg 531)

Determine the current $i_0$ for the circuit of Fig E 11.9-4. Hint: This exercise is same as Exercise 11.9-3, except for the position of the dot on the vertical coil.

Exercise 11.9-4 (cont’d)

KVL around perimeter of circuit:

$$24\angle 0^\circ - j24I + j16I_c = 0$$

or

1) $j24I - j16I_c = 24$

KVL that includes vertical coil and short circuit:

2) $j16I - j40I_c = 0$, or $I_c = \frac{16}{40} = 0.41 I$

Solving for $I$: $I = 1.3636 - 90^\circ$

and $I_c = 0.41 = 0.5454 - 90^\circ$

$I_0 = I - I_c = 1.3636 - 90^\circ - 0.5454 - 90^\circ$

$= 0.8182 - 90^\circ$

$\therefore i_o(t) = \frac{0.8182 - 90^\circ}{A}$
Typical Two-Winding Transformer

Normal Operation

Power Transformer
Assortment of Small Transformers

Diagram showing various small transformers with numbered parts.
11.10 Ideal Transformer

What are the conditions that constitute an ideal transformer?

- $k \to 1.0$ (perfect coupling, no leakage flux)
- $C_1 = C_{11} + C_{21}$ but $C_{11} = 0$
  \[ C_2 = C_{22} + C_{12} \text{ but } C_{22} = 0 \]
  and $C_{21} \to \infty$

Thus, $N_1$ is developed with virtually no $N_1 i_1$

- Resistance is zero in each coil
Consider coupled coils again (open circuit side 2)

\[ \nabla_1 = j\omega L_1 I_1 \]
\[ \nabla_2 = j\omega M I_1 \]
\[ \frac{\nabla_2}{\nabla_1} = \frac{M}{L_1} \]

But \( C_{11} = 0 \) (No leakage flux, all linkage)
\[ \frac{\nabla_2}{\nabla_1} = \frac{C_M N_2}{C_M N_1} = \frac{N_2}{N_1} \]
or
\[ \frac{\nabla_2}{N_2} = \frac{\nabla_1}{N_1} \]
volts per turn are equal in each coil

Now, consider transformer with terminals of coil 2 short circuited

\[ \nabla_1 = j\omega L_1 I_1 - j\omega M I_2 \]
\[ 0 = -j\omega M I_1 + j\omega L_2 I_2 \]
From second equation
\[ M I_1 = L_2 I_2 \]
\[ C_M N_1 N_2 I_1 = C_2 N_2^2 I_2 = (C_{22} + C_M) N_2^2 I_2 \]
But \( C_{22} = 0 \) (No leakage flux, it's all linking)
\[ C_M N_1 N_2 \]
\[ \frac{C_M N_1 N_2}{C_M N_2^2} = \frac{I_2}{I_1} \]
\[ \frac{N_1}{N_2} = \frac{I_2}{I_1} \]
or
\[ \frac{N_1 I_1}{N_2 I_2} = \frac{N_1}{N_2} \]
Ampere turns are equal in each coil
The circuit symbol for an ideal transformer

(a) time domain

(b) Phasor domain

Turns Ratio

For above transformer

\[ n = \frac{N_2}{N_1} = \frac{\nabla_2}{\nabla_1} \text{ or } \nabla_2 = n\nabla_1 \]

\[ N_1\bar{T}_1 + N_2\bar{T}_2 = 0 \text{ or } \bar{T}_1 = \frac{N_2}{N_1}\bar{T}_2 = -n\bar{T}_2 \]

We need to examine these relationships in light of the dot convention.
Effect of relocating one dot

\[
\begin{align*}
\begin{align*}
\vec{V}_1 &= j\omega L_1 \bar{T}_1 \\
\vec{V}_2 &= -j\omega M \bar{T}_1 \\
\end{align*}
\end{align*}
\]

\[
\frac{\vec{V}_2}{\vec{V}_1} = -\frac{j\omega M \bar{T}_1}{j\omega L_1 \bar{T}_1} = -\frac{M}{L_1}
\]

\[
= -\frac{N_2}{N_1} \quad \text{for } k = 1
\]

\[
\therefore \vec{V}_2 = \quad =
\]

Short circuit Analysis

\[
\begin{align*}
\vec{V}_1 &= j\omega L_1 \bar{T}_1 - j\omega M \bar{T}_2 \\
0 &= -j\omega M \bar{T}_1 + j\omega L_2 \bar{T}_2 \\
\end{align*}
\]

From second equation:

\[
M \bar{T}_1 = L_2 \bar{T}_2 \quad \text{or} \quad \frac{\bar{T}_2}{\bar{T}_1} = \frac{M}{L_2} = \quad =
\]

\[
\frac{\bar{T}_2}{\bar{T}_1} = \frac{N_1}{N_2} \quad \text{or} \quad \bar{T}_2 N_2 = \bar{T}_1 N_1
\]

\[
\frac{N_1}{N_2} \bar{T}_1 = \bar{T}_2 \quad \text{or} \quad \bar{T}_1 =
\]
Summary of Possibilities

\[ V_2 = nV_1 \]
\[ I_1 = -nI_2 \]

Exercises in section 11.10
Exercise 11.10-1 (Pg 536)

Determine the impedance of $Z_{ab}$ for the circuit of Fig. E11.10-1. All the transformers are ideal.

$$Z_{ab} = Z + \frac{1}{4} \left( Z + 9 \left( Z + \frac{Z}{4} \right) \right) = \left( 1 + \frac{1}{4} + \frac{9}{4} + \frac{9}{16} \right) Z$$

$$= \frac{65}{16} Z = 4.063 Z$$

Extra Exercises-A

Find $V_1$ and $I_1$ for the circuit below when $n=5$

$$V_1, V_2 \rightarrow 1: \frac{V_2}{V_1} \rightarrow 1: n \text{ where } n = \frac{V_2}{V_1} = 5 \text{ [n is positive, why?] }$$

$$N_1 I_1 + N_2 I_2 = 0 \rightarrow I_1 = -\frac{N_2}{N_1} I_2 = -nI_2$$

$I_1 = -nI_2$

$V_2 = nV_1$
Extra Exercises-A (cont’d)

\[ Z_{eq} = \frac{\vec{V}_1}{\vec{I}_1} = \frac{\vec{V}_2}{n\vec{I}_2} = \left(\frac{1}{n^2}\right) \left( -\frac{\vec{V}_2}{\vec{I}_2} \right) \]

But \(100 - j75 = -\frac{\vec{V}_2}{\vec{I}_2}\)

\[ \therefore Z_{eq} = \left(\frac{1}{5^2}\right)(100 - j75) = 4 - j3 \]

\[ \vec{I}_1 = \frac{10|0^0}{(1 + j3) + (4 - j3)} = \frac{10|0^0}{5} = 2|0^0 \]

\[ \vec{V}_1 = Z_{eq} \vec{I}_1 = (4 - j3)(2) = 8 - j6 = 10|-36.87^0 \]

Extra Exercises-B

Determine \(v_2\) and \(i_2\) for the circuit shown below when \(n = 2\).

Note that \(i_2\) does not enter the dotted terminal.

Note that \(n = \frac{v_2}{v_1} = 2\) (positive value, why?)
Extra Exercises-B (cont’d)

\[ -j5 \]
\[ \frac{5}{1 : n} \]
\[ 2 \]

\[ \vec{V}_2 = 2\vec{V}_1 \]
\[ \vec{I}_1 = 2\vec{I}_2 \]

Approach will be to find \( \vec{I}_1 \) and \( \vec{V}_1 \), then \( \vec{I}_2 \) and \( \vec{V}_2 \). Finally, express \( \vec{I}_2 \) and \( \vec{V}_2 \) in time domain.

\[ Z_{eq} = \frac{\vec{V}_1}{\vec{I}_1} = \frac{\vec{V}_2/2}{\vec{I}_2} = \left( \frac{1}{4} \right) \left( \frac{\vec{V}_2}{\vec{I}_2} \right) = \frac{2 + j0.2}{4} \]
\[ = 0.5 + j0.05 \]
\[ \vec{I}_1 = \frac{-j5}{5 - j5 + 0.5 + j0.05} = \frac{5}{5.5 - j4.95} = 0.676\text{[41.987°]} \]
\[ \vec{V}_1 = Z_{eq}\vec{I}_1 = (0.5 + j0.05)(0.676[41.987°]) \]
\[ = 0.340[47.698°] \]
\[ \vec{I}_2 = \frac{1}{2}\vec{I}_1 = 0.338[41.987°] \]
\[ \vec{V}_2 = 2\vec{V}_1 = 0.68[47.698°] \]
\[ i_v(t) = 0.338\cos(10t + 41.987°) \]
\[ v_v(t) = 0.68\cos(10t + 47.698°) \]