2.4 Audio channels multiplexed in time (TDM), PAM signals.

T1 System: digital signal level 1 (DS1), 1.544 Mbps (North America & Japan)
outside North America & Japan.

30 channel PCM system: 2.048 Mbps

Every 6th frame has

\[ 7 \times 24 = 168 \text{ info bits, plus} \]

2 + signaling bits + 1 framing bit (EACH)

All other frames have \[ 8 \times 24 = 192 \text{ info bits} + 1 \text{ framing bit.} \]

\[ f_n = 8 \text{ KHz, 8 bit PCM, 24 channel} \]

\[ 64 \times 24 = 1.536 \text{ Mbit/sec/Frame} \]

\[ \text{value} = 8000 \text{ Frames/sec} \text{ Total}=1.544 \text{ Mbps} \]
Example of TDM System (Digital Telephone System)

Each voice signal goes thru a BPF (200-3400 Hz) 
Bandpass
8 bits/sample, 8 KHz sampling rate.

\[ \text{64 KBPS/Voice channel, Frame Synch., } 0.125 \text{ ms} \]

\[ \frac{24 \text{ channels} \times 24 \times 64 = 1.536 \text{ MBPS}}{64} = 8 \text{ KBPS} \]

FIGURE 3.68 Digital multiplexing scheme for digital telephone. (a) T1 frame, (b) Digital multiplexing.

includes one signalling bit (For Synchronization)

Framing bits

\[ 24 \times 64 = 1536 \text{ KBPS} \]

\[ 1536 + 8 = 1544 \text{ KBPS or } 1.544 \text{ MBPS} \]

\[ \text{DS1: 24 Voice channels, 64 kbps each, T1 channel} \]

\[ \text{DS2: 4 T1 channels, T2 channel} \]

\[ \text{DS3: 6 T3 channels, T4 channel} \]

\[ \text{DS4: 274.176 MBPS} \]

\[ \text{6.312 MBPS} \]

\[ \text{44.736 MBPS} \]

(4032 Voice channels)

Signalling bit: To establish calls and for synchronization.

T1 Carrier: For short transmission distances.

T4 & T5: For long
Figure 3-40 North American digital TDM hierarchy.
\[ w(k) - \hat{w}_\varphi(k) = \text{Reconstruction Error} \]

Feedback predictor

\[ \hat{w}(k) = \text{original signal} \]
\[ \hat{\hat{w}}_\varphi(k) = \text{predicted value of } w(k) \]
\[ \hat{w}_\varphi(k) = \text{Reconstructed value of } w(k) \]

\[ \hat{d}(k) = w(k) - \hat{w}_\varphi(k) = \text{prediction error} \]
\[ \hat{d}_\varphi(k) = \text{quantized value of } d(k) \]
\[ \hat{w}_\varphi(k) = \hat{\hat{w}}_\varphi(k) + \hat{d}_\varphi(k) \]
\[ \hat{d}(k) - \hat{d}_\varphi(k) = \text{quantization error} = w(k) - \hat{\hat{w}}_\varphi(k) + \hat{\hat{d}}_\varphi(k) - \hat{d}_\varphi(k) \]
\[ \hat{m}_q(k) = a_1 m_q(k-1) + a_2 m_q(k-2) + a_3 m_q(k-3) + \ldots + a_N m_q(k-N) \]

Nth order linear predictor

\(a_1, a_2, a_3, \ldots, a_N\) : predictor coefficients (weights)

Feedback predictor (causal predictor)

Predictor Dimension:

\[ \begin{array}{cccccc}
2-D predictor & x & x & x & x & x \\
(Quantization error) & x & x & x & x & x \\
\rightarrow & y & x & x & x & x \\
(Reconstruction error) & x & x & x & x & x \\
(2-D digital image) & & & & & x
\end{array} \]
$$\frac{\text{SNR Improvement of DPCM over PCM}}{\text{Peak amplitude of } m(t)}$$

$$2p = \frac{\text{Peak amplitude of } l(t)}{2p}$$

Assume uniform quantization & same # of Q levels ($l$)

$$E \left( \frac{(\Delta V)_{PCM}}{12} \right)^2 = \frac{m_p^2}{3L^2}$$

$$(MSQ E)_{PCM} = \frac{1}{12} \left( \frac{\Delta V}{m_p^2} \right)^2 \frac{2m_p}{L}$$

$$(MSQ E) = \frac{1}{12} \left( \frac{\Delta V}{m_p^2} \right)^2 \frac{2\rho}{3L^2}$$

$$(MSQ E)_{DPCM} = \left( \frac{\Delta V}{m_p^2} \right)^2 \frac{d^2}{d^2} \rho < m_p$$

Signal power $\sim (\text{peak value})^2$

Power of $m(t)$, $P_m \sim m_p^2$

Power of $l(t)$, $P_l \sim \rho^2$

SNR increase in dB (DPCM over PCM) in $10 \log_{10} \left( \frac{P_m}{P_l} \right)$ dB
Product Preview
ADPCM Transcoder
Conforms to G.721 and 11.301-1987

The MC145532 Analog Differential Pulse Code Modulation (ADPCM) Transcoder pro-
vides a low-cost, full-duplex, single-channel transducer to/from 8 kbit PCM channel
from data either 16 kbit, 24 kbit, 32 kbit, or 64 kbit streams.

- Complies with CCITT Recommendation G.721 (Geneva 1988)
- Complies with FSK Advanced Transcoder Standard (11.301-1987)
- Full-Duplex, Single-Channel Operation
- μ-Law or A-Law Coding in Pin Selectable
- Synchronous or Asynchronous Operation.
- Black Interface with Any Member of Motorola's PCM Codec/Filter Micro-Circuit
- Family of Other Industry Standard Codes
- Serial PCM and ADPCM Data Transfer Rates from 64 kbits to 6.12 kbits
- Power Down Capability for Low Current Consumption
- The Reset State, as Specified in the Standards, is Automatically Initiated
  When the RESET Pin is Released
- Single Time Slot Assignment Timing for Transcoder Applications
- Single 9-Volt Power Supply
- 16-Pin Package

BLOCK DIAGRAM

DEVICE DESCRIPTION

An Analog Differential PCM (ADPCM) Transcoder is used to reduce the data rates required to transmit a PCM encoded
voice signal without degrading the voice fidelity and intelligibility
of the PCM signal.

The transcoder is used on 64 kbit data streams which
represent either voice or voice band data signals that have been
captures at a similar band as the original voice band. The transcoder uses a
filter to attempt to predict the next PCM input value based on
previous PCM input values. The error between the predicted and
the true PCM input value is the information that is sent
to the other end of the line. Hence the word differential, since
the ADPCM data stream is the difference between the
true PCM input value and the predicted value. The term "adaptive"
refers to the filter that is performing the prediction. It is
adaptive in that it can transfer functions changes based on the
PCM input data. That is, it adapts to the statistics of the signals
presented to it.

ADPCM Speech coder

G.721 (1986) 32 kbit/s
ANSI (T1.301-1987) 32 kbit/s
(T1Y1/87-040) — 24 kbit/s
(Modified quantization from the 32 kHz technique
μ-law and A-law companding)

ENCODING/DECODING RATES

The MC145532 allows for the encoding and decoding of
data rates of 8, 12, 16, or 24 kbit/s by simple back. Each
data sample which is provided to the input is accompanied by
an indication of the rate at which it is to be encoded or decoded.
This allows for the data to be encoded or decoded at
the rate of the input stream. The rate at which the data is
encoded or decoded is determined by the pin rates.

The 8 kbit rate allows for data to be passed directly
through the pin. The 24 kbit rate is either the 8 kbit rate or
the 16 kbit rate. The 32 kbit rate is either the 8 kbit rate or
the 16 kbit rate and the 12 kbit rate is either the 8 kbit rate or
the 16 kbit rate. The 16 kbit rate is quantized by quantizing
the 32 kbit rate and/or the 24 kbit rate.
Fig. 1. VT 45 A functional block diagram (encoder) ABL TV DS-3 Codec
6.4 Delta Modulation (DM)

Prediction of $m(k)$ is $m_q(k-1)$

$$m_q(k) = m_q(k-1) + d_q(k) \quad (6.30)$$

$$m_q(k-1) = m_q(k-2) + d_q(k-1)$$

$$m_q(k) = m_q(k-2) + d_q(k) + d_q(k-1)$$

By iteration (assume $m_q(0)=0$)

$$m_q(k) = \sum_{m=0}^{k} d_q(m) \quad (6.31)$$

Receiver (demodulator) is an accumulator (adder)
Delta Modulation (contd.)

![Diagram of Delta Modulator and Demodulator](image)

Train of narrow pulses

(1 bit code)

\[ d(t) \]

Amplifier-integrator

Low-pass filter

\[ r(t) \]

(c) Delta modulator

(d) Delta demodulator

(e) Error \( d(t) \)

Figure 6.20 Delta modulation.

(DM) Transfer function

\[
\frac{R}{c} \left( \frac{1}{jwC} \right) \approx \frac{1}{jwRC}
\]

when \( R \gg \frac{1}{jwC} \), i.e.,

\[
\approx \frac{1}{jw} \frac{1}{RC}, \quad \left( \frac{1}{jw} \right. \text{ implies integration}
\]

6-19
Thresholding

Step size too large when \( m(t) \) is changing slowly. Gravel-like noise, pulses alternate in sign.

Step size is too small when \( m(t) \) is changing fast. (Slope overload), pulses have the same polarity.

Max slope that \( m_q(t) \) can follow is \( \sigma / T_0 \).\( \quad T_0 = \text{sampling interval, } f_s = (1 / T_0) = \text{sampling rate} \)

No slope overload if

\[
\left| \frac{d}{dt} m(t) \right|_{\text{max}} \leq \sigma f_s
\]
When \( m(t) \) changes too fast there is slope overload. (See Fig. 3.60.) Maximum slope that can be followed by \( m_\alpha(t) \) is \( \frac{S_0}{T_0} \). **Slope Overload**

DM output: Sequence of narrow pulses

\[ m_\alpha(t) = \text{staircase approximation of } m(t). \]

**Figure 3.60** Illustration of slope overload. (a) Illustration of \( m(t) \) and \( m_\alpha(t) \) with step change in \( m(t) \). (b) Error between \( m(t) \) and \( m_\alpha(t) \).

\[ d(t) = m(t) - m_\alpha(t) \]

**Demodulation of DM**

Integrate \( x_c(t) \) to get \( m_\alpha(t) \). Apply LPF to \( m_\alpha(t) \) to suppress the discrete jumps in \( m_\alpha(t) \).
Example of slope overload

Consider tone modulation

\[ m(t) = A \cos \omega t \]

\[
\left| \frac{d}{dt} m(t) \right|_{\text{max}} = \omega A \leq \frac{\delta f_a}{\omega} \quad (6.32)
\]

or

\[ A_{\text{max}} \leq \frac{\delta f_a}{\omega} \quad (6.33) \]

for no slope overload.

Amplitude of \( m(t) \) \( A \sim \frac{1}{\omega} \).

Voice signal, \( B\omega = 4 \text{ KHz} \).

Using \( \omega = 2\pi \times 4000 \text{ gives} \)

\[ A = \frac{1}{2\pi \times 4000}, \text{ very small} \]

See Jager.

\[ [A_{\text{max}}]_{\text{voice}} = \frac{\delta f_a}{2\pi \times 800} = \frac{\delta f_a}{\omega r} \quad (6.34) \]

\[ f_r = 800 \text{ Hz} \]
Voice spectrum decays with frequency. Voice spectrum decreases as $1/w$ up to 2000 Hz and as $1/w^2$ beyond 2000 Hz. Two integrators in cascade:

\[ R_1 \frac{1}{C_1} \quad \text{and} \quad \quad R_2 \frac{1}{C_2} \]

Single integrator up to 2000 Hz (100 - 2000 Hz)
Double integrator beyond 2000 Hz

Choose:
\[ R_1 C_1 = \frac{1}{200 \pi}, \quad \text{freq}_{\phi_1} = 100 \text{ Hz} \]
\[ \omega_1 \gg \frac{1}{R_1 C_1} \]
\[ R_2 C_2 = \frac{1}{4000 \pi}, \quad \text{freq}_{\phi_2} = 2000 \text{ Hz} \]
\[ \omega_2 \gg \frac{1}{R_2 C_2} \]

See page 6-19.
Adaptive Delta Modulation (ADM) step size or weight $\delta_0$ is made a variable.

**FIGURE 3.65** Adaptive delta modulator

- $m(t)$
- $d(t)$
- $\delta(t)$
- $x(t)$
- $x_0(t)$

make amplifier gain always positive.

when $m(t)$ changes rapidly (increasing or decreasing rapidly) $x_c(t)$ pulses have the same polarity. \( \therefore \) LPF output will be very large. Hence large $\delta_0$.

**FIGURE 3.62** Adaptive DM receiver

- $x_c(t)$
- $m_0(t)$

make amplifier gain always positive.

when $m(t)$ is slowly changing or nearly constant, $x_c(t)$ pulses alternate in sign. \( \therefore \) LPF output will be small. Hence small $\delta_0$. 
Output SNR

Error $d(t)$ in DH lies in the range $(\sigma, \sigma)$. Ignore slope overload. Granular noise power $\bar{e}^2$

$$\bar{e}^2 = \frac{1}{2\sigma} \int_{-\sigma}^{\sigma} x^2 dx = \frac{1}{3} \int_{0}^{\infty} x^2 dx$$

Assume $P_{SD}$ of output noise limited to $B = BW$ of Voice signal $m(t)$.

$$N_0 = \left(\frac{\sigma^2}{3}\right) \frac{B}{f_0} = \frac{\sigma^2 B}{3 f_0} \quad (6.35)$$

Output SNR $S_0 = \frac{m^2(t)}{\bar{e}^2 B}$

$$\frac{S_0}{N_0} = \frac{3 f_0}{\sigma^2 B} m^2(t) \quad (6.37)$$
\[ m_p = \left| m(t) \right|_{\text{max}} \]

No slop overload for Voice Signals

\[ m_p = \frac{S_f}{\omega_r} \quad (6.38) \]

\( \omega_r = 2\pi \times 800 \text{ rad/sec} \)

\[ \frac{1}{c^2} = \frac{f_n^2}{\omega_r^2 m_p^2} \quad (6.37) \]

becomes

\[ \frac{S_0}{N_0} = \left( \frac{3}{\omega_r^2 B m_p^2} \right)^\infty m^2(t) \quad (6.39) \]

Min. Transmission BW

For Voice Signals

\[ B = 4000 \text{ Hz} = \left( f_n / 2 \right) = B_T \quad (6.40) \]

\[ \frac{S_0}{N_0} = \frac{150}{\pi^2} \left( \frac{B_T}{B} \right)^3 \left( \frac{m^2(t)}{m^2} \right) \]

For Double integration

\[ \frac{S_0}{N_0} = 5.34 \left( \frac{B_T}{B} \right)^5 \left( \frac{m^2(t)}{m^2} \right) \quad (6.41) \]
**Comparison of DM with PCM 6-25**

![Graph showing comparison of DM and PCM](image)

Figure 6.22 Comparison of DM and PCM.

\[
\text{DM: } \text{SNR} \sim \left( \frac{B_T}{B} \right)^3 \text{ single integration}
\]

\[
\sim \left( \frac{B_T}{B} \right)^5 \text{ double integration}
\]

\[
\text{PCM: } \text{SNR} \sim \text{exponentially with } \frac{B_T}{B}
\]

*(See Eq. 6.22)*

DM is superior to PCM at low SNR but is inferior at high SNR.
**DPCM Applications**

Speech coders, image coders, video coders

**Speech:**

G.721

G.723.1 Dual rate speech coder for multimedia communication at 5.3 and 6.3 Kbps.

G.726 ADPCM, speech coding at 40, 32, 24 and 16 Kbps

G.722 7KHz audio coding within 64 Kbp (Two band ADPCM)

**Images:** Independent Lossless Coding (JPEG)

**Video:**

H.261, H.263, MPEG-1, MPEG-2, GA-HDTV,
MPEG-4 Interframe (picture) motion compensated prediction.

G.728 Low-delay code-excited linear prediction (LDCELP) - 3KHz (narrowband) audio at 16 Kbps

APT-X100 Audio coding system (4 equal subbands - 4 band QMF filterbank-all-pole backward adaptive prediction)

**ITU-T & ISO/IEC standards**
DPCM CODING

**Differential Pulse Code Modulation**

\[ e_p(n) = \text{prediction error} = x(n) - x_p(n) \]

\[ Q(n) = \text{Quantization error} = e_p(n) - e_{pq}(n) \]

\[ \tilde{x}(n) = \text{Reconstructed signal} = e_{pq}(n) + x_p(n) \]

\[ x(n) = \text{Input signal} \]

\[ x_p(n) = \text{Predicted value of } x(n) \]

**Feedback Prediction**

**Adaptive Quantizer**

**Reconstruction error**

\[ = x(n) - \tilde{x}(n) \]

\[ = e_p(n) - e_{pq}(n) = q(n) \]

\[ = x_p(n) + e_p(n) - \left[ e_{pq}(n) + x_p(n) \right] \]

= quantization error

Adaptive DPCM involves adaptive predictor and/or adaptive quantizer.

**Transmitter**

**VWL Encoder**

**Transmitter**

**Receiver**

\[ \left( \text{Same as at the} \right. \ ]
DPCM

\[ x_p(n) = \text{function of previously reconstructed pels} \]
\[ = f[x(n-1), x(n-2), \ldots, x(n-k)] \]

Order of the predictor = number of previously reconstructed pels used in the prediction. k-th order predictor. Dimension of the predictor is dependent on the locations of the previously reconstructed pels, i.e., pels in the same line means 1-D predictor. Pels in the horizontal and vertical directions implies 2-D predictor. Pels in the horizontal, vertical and temporal domains implies 3-D predictor.

Linear predictor.

\[ x_p(n) = \sum_{l=1}^{k} a_l x(n-l) \]

(\text{linear combination of } k \text{ previously reconstructed pels})

\[ a_l = \text{predictor weights (coefficients)} \]
\[ \sum_{l=1}^{k} a_l = 1. \]

Predictor weights generally reflect the correlation of \( x(n) \) with the neighborhood pels.

DPCM evaluation

1) Sensitive to image statistics
2) Sensitive to channel errors
3) Performs poorly at high compression ratios (i.e., low bit rates)

Image artifacts:

Granularity: Grainy structure in uniform regions
Slope overload: Near high contrast edges, prediction error greater than quantizer saturation levels. Effect: blurred edges. Visible losses in edge/contrast rendition.

edge-busyness: Caused at less sharp edges where the reconstructed pels on adjacent scan lines have different quantization levels. In interframe/interfield coding, this causes blurred moving edge.

To overcome DPCM artifacts:

1) introduce adaptive predictors

2) Incorporate human visual sensitivity in the quantizer design

3) Increase the range and the number of levels of the quantizer

Adaptive DPCM techniques

\[
\begin{array}{cccccc}
\text{n-1} & x^n & x^c & x^p & \text{previous line} \\
\text{n} & x^d & x_r & \text{present line} \\
\end{array}
\]

\[\uparrow \text{under prediction}\]

1) Compute \( |x - A_k|, |x - B_k|, |x - C_k|, \& |x - D_k| \)

Choose min \( |x - A_k|, |x - B_k|, |x - C_k|, \& |x - D_k| \) for prediction of x. To avoid overhead choose prediction of present pel based on the previous pel prediction.

2) For a block of pels say \((4 \times 4), (4 \times 8), (8 \times 8), (8 \times 16)\) or \((16 \times 16)\) apply various predictors and choose the predictor which yields minimum of the absolute sum of prediction errors of the block. This needs an overhead to indicate the predictor chosen.


Both lines are from the same field.

\[
R = E \left[ \frac{x(n-k) x(n-1) x(n-2) \ldots x(n-k) x(n-k)}{x(n) x(n-1) x(n-2) \ldots x(n-k) x(n-k)} \right]
\]

\[
\sigma_f^2 = \sigma_x^2 - \frac{1}{k} \sum_{i=1}^{k} a_i E \left[ x(n) x(n-I) \right]
\]

Once the predictor is designed, based on the statistical distribution of the prediction error, the quantizer is designed. This process assumes that both the predictor and quantizer are independent of each other. In the actual DPCM process the prediction is based on previously reconstructed pixels (i.e., causal predictor) as these are available at the receiver. Similar to quantizer design, predictor design can be based on minimizing any meaningful distortion.

**Predictor design**

Minimize mean square prediction error i.e.,

\[
E[\varepsilon^2_f(n)] = E \left[ \left( x(n) - x_r(n) \right)^2 \right]
\]

Assume

\[ x(n) \approx \hat{x}(n) \quad \text{(negligible quantization error)}. \]

\[ x_r(n) = \sum_{i=1}^{k} a_i x(n-I), \quad k \text{th order causal/linear predictor} \]

Variables are \[ a_l \quad l = 1, 2, \ldots, k \]
Hence
\[
E[e_i^2(n)] = E \left[ \left( x(n) - \sum_{i=1}^{K} a_i x(n-i) \right)^2 \right] = E \left[ \left( x(n) - x_p(n) \right)^2 \right]
\]

Assume
\[
E[x(n)] = 0, \text{ variance } = \sigma_x^2
\]

Set
\[
\frac{\partial}{\partial a_i} E[e_i^2(n)] = 0, \text{ for } i = 1, 2, \ldots, K
\]

Show
\[
R = \begin{bmatrix} R_{x} & E[x(n)x(n-1)] & \cdots & E[x(n)x(n-K)] \\ E[x(n)x(n-1)] & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ E[x(n)x(n-K)] & \cdots & E[x(n-K)x(n-K)] \end{bmatrix}
\]

where
\[
a = [a_1, a_2, \ldots, a_K]^T
\]

\[
R = \left( E[x(n)x(n-1)], E[x(n)x(n-2)], \ldots, E[x(n)x(n-K)] \right)
\]

Show
\[
\sigma_{pe}^2 = \sigma_x^2 - \frac{\sigma_T^2}{C} = \text{Prediction error variance}
\]

\[
E \left[ \left( x(n) - \sum_{i=1}^{K} a_i x(n-i) \right) x(n-i) \right] = 0
\]

Line
\[
\sum_{i=1}^{K} a_i x(n-i) = x(n-1)
\]

P. Tischer. "Optimal predictors for image compression." (Abstract only) IEEE Transactions on Image Processing, Vol. 3, p. 225, March 1994. (Under review). [Criteria other than minimizing the MSPE are presented and a technique for designing the predictor is developed.]
\[ E \left[ x_n x_{n-1} \right] = E \left[ x_{n-1} \sum_{l=1}^{K} a_l x_{n-l} \right] \]
\[ \lambda = 1, 2, \ldots, K \]

\[ E \left[ x_n x_{n-1} \right] = a_1 E \left[ x_{n-1} x_{n-1} \right] + a_2 E \left[ x_{n-1} x_{n-2} \right] + \cdots + a_K E \left[ x_{n-1} x_{n-K} \right] \]

\[ E \left[ x_n x_{n-2} \right] = a_1 E \left[ x_{n-2} x_{n-1} \right] + a_2 E \left[ x_{n-2} x_{n-2} \right] + \cdots + a_K E \left[ x_{n-2} x_{n-K} \right] \]

\[ \vdots \]

\[ E \left[ x_n x_{n-K} \right] = a_1 E \left[ x_{n-K} x_{n-1} \right] + a_2 E \left[ x_{n-K} x_{n-2} \right] + \cdots + a_K E \left[ x_{n-K} x_{n-K} \right] \]

\[ Y = Ra \]

\[ (K \times 1) = (K \times K)(K \times 1) \]

\[
\begin{bmatrix}
E \left[ x_n x_{n-1} \right] \\
E \left[ x_n x_{n-2} \right] \\
\vdots \\
E \left[ x_n x_{n-K} \right]
\end{bmatrix}
= \begin{bmatrix}
E \left[ x_{n-1} x_{n-1} \right], E \left[ x_{n-1} x_{n-2} \right], \ldots, E \left[ x_{n-1} x_{n-K} \right] \\
E \left[ x_{n-2} x_{n-2} \right], E \left[ x_{n-2} x_{n-3} \right], \ldots, E \left[ x_{n-2} x_{n-K} \right] \\
\vdots \\
E \left[ x_{n-K} x_{n-K} \right], E \left[ x_{n-K} x_{n-K-1} \right], \ldots, E \left[ x_{n-K} x_{n-K} \right]
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_K
\end{bmatrix}
\]

\[ a = R^{-1} Y \]

\[ (K \times 1) = (K \times K)(K \times 1) \]
\[
\sigma_{\text{pe}}^2 = \sigma_x^2 - \frac{a^T X}{(1 \times K)(K \times 1)}
\]

\[
\sigma_{\text{pe}}^2 = E \left[ e_p^2(n) \right] = E \left[ \left( x(n) - \sum_{l=1}^{K} a_l x(n-l) \right)^2 \right]
\]

\[
= E \left[ x_n^2 \right] - 2 \sum_{l=1}^{K} a_l E [x_n x_{n-l}]
\]

\[
+ \sum_{l=1}^{K} \sum_{m=1}^{K} a_l a_m E [x_{n-l} x_{n-m}]
\]

\[
\sigma_{\text{pe}}^2 = E \left[ x_n^2 \right] - 2 \sum_{l=1}^{K} a_l E [x_n x_{n-l}]
\]

\[
+ \sum_{l=1}^{K} \sum_{m=1}^{K} a_l a_m E [x_{n-l} x_{n-m}]
\]

\[
\sigma_{\text{pe}}^2 = E \left[ e_p^2(n) e_p^*(n) \right] = E \left( e_p(n) [x(n) - x_p(n)] \right)
\]

\[
= E \left[ e_p(n) x(n) \right] - E \left[ e_p(n) x_p(n) \right]
\]

\[
E \left[ e_p(n) x_p(n) \right] = \left( E \left[ e_p(n) \right] \right) \left( E \left[ x_p(n) \right] \right) = 0
\]

Since \( E \left[ e_p(n) \right] = 0 \) (Assume \( e_p(n) \) and \( x_p(n) \) are uncorrelated.)

\[
\sigma_{\text{pe}}^2 = E \left[ x(n) - x_p(n) \right] x(n)
\]

\[
= E \left[ x(n) \right] - E \left( \sum_{l=1}^{K} a_l x(n-l) \right) x(n)
\]

\[
= \sigma_x^2 - \sum_{l=1}^{K} a_l E \left[ x(n-l) \right] x(n)
\]

\[
= \sigma_x^2 - \sum_{l=1}^{K} a_l \tau_l
\]

\[
= \sigma_x^2 - \frac{a^T X}{(1 \times K)(K \times 1)}
\]

Q.E.D.
**Predictor weights**

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>-1/2</td>
<td>1/2</td>
<td>-</td>
</tr>
<tr>
<td>3/4</td>
<td>-3/8</td>
<td>5/8</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>-5/16</td>
<td>1/2</td>
<td>-</td>
</tr>
<tr>
<td>7/8</td>
<td>-1/2</td>
<td>1/2</td>
<td>1/8</td>
</tr>
<tr>
<td>1/2</td>
<td>1/8</td>
<td>1/4</td>
<td>1/8</td>
</tr>
<tr>
<td>3/4</td>
<td>-1/4</td>
<td>3/8</td>
<td>1/8</td>
</tr>
</tbody>
</table>

**MMSE predictor**

**Line (n-1)**

\[
p = \frac{3}{4} \hat{a} - \frac{1}{4} \hat{b} + \frac{3}{8} \hat{c} + \frac{1}{8} \hat{d}
\]

2-Dimensional, 4th order, causal & linear predictor

Adaptive predictor

\[\sigma^2 < Th_1\]
\[Th_1 \leq \sigma^2 \leq Th_2\]
\[Th_2 \leq \sigma^2 \leq Th_3\]
\[\sigma^2 > Th_3\]

*4 Activity classer*

(overhead to indicate quantizer switching in 256/512/256 pels)
Leak factor: 15/16, 31/32, 7/8

NEC DS-3 (44.736 MBPS) NTSC TV Codec. Proposed to T1.Y1.1.

T. Ishiguro et al. "Composite interframe coding of NTSC color TV signals." NTC, pp. 6.4-1 thru 6.4-4, Dallas, TX, 1976. (4 adaptive predictors and 5 uniform/2 nonuniform quantizers)

KDD DS-3 (44.736 MBPS) NTSC TV Codec. Proposed to T1.Y1.1.

Three adaptive predictors (i) intrafield, (ii) interfield, (iii) and MC interframe. Predictor selection is based on 12 Y samples and 6 samples of each color difference signal.

Eight quantizers. Quantizer selection is made every eight lines (stripe) controlled by buffer status.
Histogram of the prediction error.

Prediction errors tend to follow Laplacian distribution.
**Digital Leak:** To reduce the error propagation under noisy channel.

**Clip:** To bring the reconstructed signal within the allowable range.

\[
\frac{15}{16}, \frac{7}{8}, \frac{31}{32} \quad \text{(Leak factor close to one but } < 1)\
\]

**System used for studying transmission errors**