Transmit digital data over a channel.
Binary data '1' and '0' bits.
Assign a unique pulse to each bit.
Binary: only two symbols
M-ary: M symbols (see Sec. 7.7)

7.1 Digital Communication System

1. Source data, text, speech, audio, fax, video, images etc.
2. Multiplexer: several sources are combined (interleaved) in the multiplexer before transmission.
3. Line coder
4. Regenerative Repeater
\[ T_b = \text{Timing interval}, \quad R_e = \left( \frac{1}{T_b} \right) = \text{pulses/sec} \]

Figure 7.1 Some line codes. (a) On-off (R2). (b) Polar (R2). (c) Bipolar (R2). (d) On-off (NR2). (e) Polar (NR2).

- \text{On-off (R2)}
- \text{Polar (R2)}
- \text{Bipolar (R2)}
- \text{On-off (NR2)}
- \text{Polar (NR2)}

R2: Return to Zero
NR2: Non Return to Zero

\text{On-off (R2)} [\text{bit 1} \text{ pulse}] [\text{bit 0} \text{ no pulse}]

\text{Polar (R2)} [\text{bit 1} \text{ pulse } p(t)] [\text{bit 0} \text{ - } p(t)]

\text{Bipolar (AMI) (R2)} [\text{bit 1} \text{ alternate in sign}] [\text{bit 0} \text{ no pulse}]

\text{On-off (NR2)} [\text{Non Return to Zero}]

\text{Polar (NR2)}
<table>
<thead>
<tr>
<th>System</th>
<th>Rate (Mbits/s)</th>
<th>Digital Signal No.</th>
<th>Voice Channels</th>
<th>Medium</th>
<th>Line Code</th>
<th>Repeater Spacing (miles)</th>
<th>Maximum System Length (miles)</th>
<th>System Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1.544</td>
<td>DS-1</td>
<td>24</td>
<td>Wire pair</td>
<td>Bipolar RZ</td>
<td>1</td>
<td>50</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>T1C</td>
<td>3.152</td>
<td>DS-1C</td>
<td>48</td>
<td>Wire pair</td>
<td>Bipolar RZ</td>
<td>1</td>
<td>—</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>T1D</td>
<td>3.152</td>
<td>DS-1C</td>
<td>48</td>
<td>Wire pair</td>
<td>Duobinary NRZ</td>
<td>1</td>
<td>—</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>T1G</td>
<td>6.443</td>
<td>DS-2</td>
<td>96</td>
<td>Wire pair</td>
<td>4-level NRZ</td>
<td>1</td>
<td>200</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>T2</td>
<td>6.312</td>
<td>DS-2</td>
<td>96</td>
<td>Wire pair*</td>
<td>B6ZS b RZ</td>
<td>2.3</td>
<td>500</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>T3</td>
<td>44.736</td>
<td>DS-3</td>
<td>672</td>
<td>Coax.</td>
<td>B3ZS b RZ</td>
<td>e</td>
<td>e</td>
<td>e</td>
</tr>
<tr>
<td>T4</td>
<td>274.176</td>
<td>DS-4</td>
<td>4032</td>
<td>Coax.</td>
<td>Polar NRZ</td>
<td>1</td>
<td>500</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>T5</td>
<td>560.160</td>
<td>DS-5</td>
<td>8064</td>
<td>Coax.</td>
<td>Polar NRZ</td>
<td>1</td>
<td>500</td>
<td>$4 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

* Special two-wire cable is required for 12,000-ft repeater spacing. Because T2 cannot use standard exchange cables, it is not as popular as T1.

b BnZS denotes binary n-zero substitution, where a string of n zeros in the bipolar line code is replaced with a special three-level code word so that synchronization can be maintained [Fike and Friend, 1984; Bic, Duponteil, and Imbeaux, 1991].

c Used in central telephone office for building multiplex levels; not used for transmission from office to office.

---

Sec. 5-9 Binary Modulated Bandpass Signaling

![Diagram of binary modulated bandpass signaling]

Figure 5-19 Bandpass digitally modulated signals.
Regenerative Repeaters

(a) \[ \begin{array}{c}
A_1 \\
T_1
\end{array} \]

(b) Random component

(c) Periodic component

Repeater is placed at periodic (space) intervals along a channel. Detect incoming signal & regenerate clean pulses. Eliminates accumulation of noise & signal distortion. \( R_c = \frac{1}{T_c} \)

Periodic signal (closed signal at \( R_p \) pulses/sec) — use a resonant circuit tuned to closed frequency, to generate timing information. Bipolar signal (\( R_2 \)). Becomes an ON-OFF signal when rectified.

becomes an ON-OFF (\( R_2 \)) signal.

On off = polar + periodic

Figure 7.2 An on-off signal is the sum of a polar signal and a clock frequency periodic signal.
Transparent line code

is a line code in which the accuracy of timing information
is not affected by the bit pattern.

(Ex: polar scheme) on-off

and bipolar schemes are nontransparent

7.2 Line Coding

Derived properties of line codes

1. Transmission BW

2. Power efficiency

3. Error detection and correction capability

4. Favorable PSD. Should be zero at \( w = 0 \) (dc).

5. Adequate timing content

6. Transparency
7.2 Line Coding

Desirable properties

1. Transmission BW: Should be small
2. Power efficiency: For a given BW and a specified detection error probability transmitted power should be small.
4. Favorable PSD: should be zero at $w = 0$ (dc). DC paths provided by cable pairs between repeater sites are used to transmit the power required to operate the repeaters.
5. Adequate timing content: possible to extract timing or clock info from the signal (line code)
6. Transparency: possible to transmit a digital signal correctly regardless of bit pattern of 1's and 0's.
PSD of Various Line codes

Pulse Train

Impulse Train

Transmission rate = \frac{1}{T_c} (\text{Pulses/sec})

Basic pulse \( p(t) \) \( \leftrightarrow \) \( P(i\omega) \)

\( k \)-th pulse \( a_k p(t) \)

on-off, polar, bipolar line codes

\( a_k = 0, 1 \) or \(-1\), (random)

Time autocorrelation of \( x(t) \)

\[ R_x(T) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x(t-T) \, dt \]
Rectangular Pulse Train

\[ X(k) \]

Strength of

\( k \)-th impulse = \( a_K \)

\( k \)-th pulse height = \( h_k \)

\[ a_K = e^{h_k} \]

Figure 7.4 Deviation of PSD of a random binary signal.

For \( \gamma < \varepsilon \) (see Fig. 7.4 b)

\[ R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \sum_{k} h_k^2 (\varepsilon - \tau) \]

\[ = \lim_{T \to \infty} \frac{1}{T} \sum_{k} a_k^2 \left( \frac{\varepsilon - \tau}{\varepsilon^2} \right) = \frac{R_0}{\varepsilon T} \left( 1 - \frac{\gamma}{\varepsilon} \right) \]

where \( R_0 = \lim_{T \to \infty} \left( \frac{T_0}{T} \right) \sum_{k} a_k^2 \) (7.2)
As \( T \to \infty \), \( N \to \infty \)

\( N = \frac{T}{T_c} \) \hspace{1cm} (7.3)

\[
R_0 = \lim_{N \to \infty} \left( \frac{1}{N} \sum_{k} \alpha_k^2 \right) \\
= \alpha_k^2, \text{ Time Average} \hspace{1cm} (7.5)
\]

\[R_x(\gamma) = R_x(-\gamma), \hspace{1cm} (7.6)\]

\[
R_x(\gamma) = \frac{R_0}{\varepsilon T_c} \left[ 1 - \frac{|\gamma|}{\varepsilon} \right], \hspace{1cm} 1/\varepsilon < \gamma < \varepsilon
\]

(See Fig. 7.4d, center one)

For \( \varepsilon \leq T \leq T_c - \varepsilon \), \( R_x(\gamma) = 0 \).

For \( T_c \leq T \leq T_c + \varepsilon \)

\[
R_1 = \lim_{T \to \infty} \frac{T_c}{T} \sum_k \frac{a_k a_{k+1}}{T} \\
= \lim_{N \to \infty} \frac{1}{N} \sum_k \frac{a_k a_{k+1}}{T} \\
= \alpha_k a_{k+1}
\]
Generalizing

\[ R_n = \lim_{T \to \infty} \frac{T_n}{T} \sum_{k} a_k a_{k+n} \]
\[ = \lim_{N \to \infty} \frac{1}{N} \sum_{k} a_k a_{k+n} = a_n a_{k+n} \]

\[ \lim_{\tau \to 0} R_x(\tau) \to R_x(\tau) \]

\[ R_x(\tau) = \frac{1}{T_n} \sum_{n=-\infty}^{\infty} R_n \delta(\tau-nT_n) \]  \hspace{1cm} (7.7)

PSD of \( S_x(\omega) = \mathcal{F}[R_x(\tau)] \)

\[ S_x(\omega) = \frac{1}{T_n} \sum_{n=-\infty}^{\infty} R_n e^{-j\omega nT_n} \]  \hspace{1cm} (7.8)

All \( R_n = R_{-n} \) \hspace{1cm} \( e^{j\theta} + e^{-j\theta} = 2\cos \theta \)

\[ S_x(\omega) = \frac{1}{T_n} \left( R_0 + 2 \sum_{n=1}^{\infty} R_n \cos \omega n T_n \right) \]  \hspace{1cm} (7.9)

\[ S_y(\omega) = |H(\omega)|^2 S_x(\omega) = |P(\omega)|^2 S_x(\omega) \]  \hspace{1cm} (7.10a)
\[ S_y(w) = \frac{|P(w)|^2}{T_L} \left( R_0 + 2 \sum_{n=1}^{\infty} R_n \cos n \omega T_L \right) \quad (7.9) \]

**PSDN of various line codes**

**Polar signal**

\[ \begin{align*}
\text{bit signal} \quad & \quad P(x) \\
1 \quad & \quad (\text{see Fig.}) \quad (7.16) \\
0 \quad & \quad -P(x)
\end{align*} \]

\( a_k \) in 1 or -1, equally likely

\[ R_0 = \lim_{N \to \infty} \frac{1}{N} \sum_{k} a_k^2 = \lim_{N \to \infty} \frac{1}{N} N = 1 \quad (7.11a) \]

\[ R_1 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (1) + \frac{N}{2} (-1) \right] = 0 \quad (7.11b) \]

Similarly \( R_n = 0 \), \( n \geq 1 \)

\[ S_y(w) = \frac{|P(w)|^2}{T_L} R_0 \]

\[ = \frac{|P(w)|^2}{T_L} \quad (7.12) \]
PSD of Polar Signal

Let \( p(t) \) be

\[
\text{rect}\left(\frac{2t}{T_v}\right)
\]

\[
P(\omega) = \frac{T_v}{2} \ \text{sinc}\left(\frac{\omega T_v}{4}\right) \quad (7.13)
\]

\[
S_y(\omega) = \frac{T_v}{4} \ \text{sinc}^2\left(\frac{\omega T_v}{4}\right) \quad (7.14)
\]

Essential BW is zero-crossing i.e., \( \frac{4\pi}{T_v} \ \text{rad/sec} \) or \( \frac{2}{T_v} \ \text{Hz} \). \( T_v(1/T_v) = R_v \)

= closed freq., Essential BW = 2 \( R_v \) Hz
\[
\text{Pulse width } = \tau
\]

Consider essential BW as \( T \) zero crossing i.e., \( \omega = \frac{2\pi}{\tau} \)
or \( f = \frac{1}{T} \). Reciprocal relationship

\[\text{(2 pulses per Hz)} \quad \text{(min BW req'd for } R_L \text{ pulses/Hz)} = \frac{R_L}{2} \quad \text{H}^2\]

Essential BW of Polar Signaling

\(2 R_L \ H^2\) is 4 times the Nyquist BW required to transmit \( R_L \) pulses/sec. Polar Signaling in not BW efficient. See

7-11a.

Advantages & disadvantages

see paragraphs 2 & 3 (P. 303)
Polar signalling (contd.)

For full-width pulse, i.e., pulse width $= T_v$

$$p(t) = \text{rect} \left( \frac{t}{T_v} \right)$$

i.e., NRZ

I zero crossing

Essential BW is $R_v \frac{1}{2}$. This is still twice the theoretical BW.

$$\text{rect} \left( \frac{t}{T_v} \right) \leftrightarrow T_v \left| \text{nic} \left( \frac{\omega T_v}{2} \right) \right|^2 = P(\omega)$$

$$S_y(\omega) = \left| \frac{P(\omega)}{T_v} \right| = T_v \left| \text{nic}^2 \left( \frac{\omega T_v}{2} \right) \right|$$

$$\left| \text{nic} \left( \frac{\omega T_v}{2} \right) \right| = 0$$

I zero crossing

$$\left( \omega \frac{T_v}{2} \right) = \pi, \omega = \frac{2\pi}{T_v} \Rightarrow R_v$$

(See Eq. 7.14)

$$\frac{T_v}{2} \text{ BW (peak, for } R_v \text{ pulses/sec)}$$

$$= \frac{R_v}{2} \text{ Hz, (Two pulses/Hz)}$$

See Sec. 6.1.3 on p. 260
Advantages and disadvantages of polar signalling

Disadvantages
1. It is not BW efficient
2. It has no error detection/correction capability
3. It has nonzero PSD at dc \((\omega = 0)\) See Fig. 7.5. This rules out ac coupling during transmission. AC coupling allows transformers and blocking capacitors to aid in impedance matching and bias removal. AC coupling also allows dc powering of regenerative repeaters over the cable pairs.

Advantages
1. It is the most efficient scheme from the power requirement viewpoint. For a given power, the detection-error probability is the smallest possible (Sec. 7.6).
2. It is also transparent as there is always a pulse (+ or -) regardless of the bit stream. Rectification of a polar signal yields a periodic signal of clock frequency, useful for timing extraction.
Achieving dc null by pulse shaping

Figure 7.6 Split-phase (Manchester or twinned-binary) signal. (a) Basic pulse \( p(t) \) for Manchester signaling. 
(b) PSD of Manchester signaling. 

PSD of a Random binary signal (Fig. 7.4) 

\[ S_y(w) = \left| P(w) \right|^2 \frac{R_0 + 2 \sum_{n=1}^{\infty} R_n \cos n\omega_T}{T_L} \]  

To achieve dc null i.e., \( S_y(0) = 0 \), select \( p(t) \) such that \( P(0) = 0 \), \( p(t) \leftrightarrow P(w) \)

\[ P(w) = \int_{-\infty}^{\infty} p(t) e^{-j\omega t} \, dt \]

\[ P(0) = \int_{-\infty}^{\infty} p(t) \, dt \]

one possibility is shown in Fig. 7.6a.
Problem 7.2.2 \( p.349 \)

Devise \( S_y(w) \).

Random binary data sequence

\[ 100110 \ldots \]

Manchester line code

\[ P(t) = \text{rect} \left( \frac{t + T_c/4}{T_c/2} \right) - \text{rect} \left( \frac{t - T_c/4}{T_c/2} \right) \]

\[ P(w) = \frac{T_c}{2} \sin \left( \frac{w T_c}{4} \right) \left[ e^{-j \omega T_c/4} - e^{j \omega T_c/4} \right] \]

\[ = j T_c \sin \left( \frac{w T_c}{4} \right) \sin \left( \frac{\omega T_c}{4} \right) \]

\[ \frac{\text{PSD}}{S_y(w)} = \left| P(w) \right|^2 / T_c \quad (7.12) \]

\[ = T_c \sin^4 \left( \frac{\omega T_c}{4} \right) \left( \frac{\omega T_c}{4} \right)^2 \]

\[ \sin \left( \frac{\omega T_c}{4} \right) = \sin \pi = 0 \text{, BW in} \]

\[ \omega = \left( 4 \pi / T_c \right) f = \frac{3}{T_c} = 2 R_c \text{ Hz} \]
PSD of on-off signaling

See Fig. 7.1a/p. 295

\[ R_0 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (1) + \frac{N}{2} (0) \right] = \frac{1}{2} \]

\( R_n \), compute product \( a_n a_{n+k} \)

Equally likely to be 1x1, 1x0, 0x1 or 0x0.

\[ R_n = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{4} (1) + \frac{3N}{4} (0) \right] \]

\[ = \frac{1}{4}, \quad n \geq 1 \quad (7.8) \]

\[ S_X(w) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} R_n \exp(-jnwT_c) \]

\[ S_X(w) = \frac{1}{2T_c} + \frac{1}{4T_c} \sum_{n=-\infty}^{\infty} \exp(-jnwT_c) \quad n \neq 0 \quad (7.17a) \]
on-off signaling (constant)

\[ S_x(w) = \frac{1}{4T_L} + \frac{1}{4T_L} \sum_{n=-\infty}^{\infty} e^{-jnw_0T_L} \]

\[ \delta_{T_0}(t) \quad \leftrightarrow \quad \omega_0 \delta_w(w) \]

(Fig. 3.24/P.97) \( \omega_0 \delta_w(w) \)

periodic

Impulse Train \( \leftrightarrow \) Impulse Train

\[ \delta_{T_0}(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{jnw_0T_0}, \quad w_0 = \frac{2\pi}{T_0} \]

\[ \delta_{T_0}(t) \leftrightarrow \omega_0 \sum_{n=-\infty}^{\infty} \delta(w - n\omega_0) \]

footnote

= \( \omega_0 \delta_w(w) \), (3.42)

\[ \sum_{n=-\infty}^{\infty} \delta(t - nT_L) = \frac{1}{T_L} \sum_{n=-\infty}^{\infty} e^{-jn\omega_0T_L} \]

\[ (\omega_L = \frac{2\pi}{T_L}) \]

\[ \sum_{n=-\infty}^{\infty} e^{-jn\omega_0T_L} = \frac{2\pi}{T_L} \sum_{n=-\infty}^{\infty} \delta(w - \frac{2\pi n}{T_L}) \]
on-off Signaling (cont'd.)

\[ S_x(\omega) = \frac{1}{4T_L} + \frac{2\pi}{4T_L^2} \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{2\pi n}{T_L}) \]

\[ S_y(\omega) = \frac{|P(\omega)|^2}{4T_L} \left[ 1 + \frac{2\pi}{T_L} \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{2\pi n}{T_L}) \right] \]

For half-width rectangular pulse \[ S_y(\omega) = \frac{T_L}{16} \sin^2 \left( \frac{\omega T_L}{4} \right) \]

compared with Eq. (7.14), Polav Signal.

on-off signal = polav signal + periodic signal (Fig. 7.2).
**On-off Signaling (contd.)**

For half width rectangular pulse

\[
P(w) = \frac{T_e}{2} \ \text{sin} \left( \frac{\omega T_e}{4} \right) \tag{7.13}
\]

\[
S_y(w) = \frac{|P(w)|^2}{4-T_e} \left[ 1 + \frac{3\pi}{T_e} \sum_{n=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi n}{T_e} \right) \right]
\]

Substitute (7.13) in (7.18b) \tag{7.18b}

\[
S_y(w) = \frac{T_e}{16} \ \text{sin}^2 \left( \frac{\omega T_e}{4} \right) \left[ 1 + \frac{3\pi}{T_e} \sum_{n=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi n}{T_e} \right) \right]
\]

Continuous spectrum + discrete spectrum

\[
\delta \left( \omega - \frac{2\pi n}{T_e} \right) \text{ spectra at } \omega = o, \pm \frac{2\pi}{T_e}, \pm \frac{4\pi}{T_e}, \pm \frac{6\pi}{T_e}, \ldots
\]

or at \( f = 0, \pm \frac{1}{T_e}, \pm \frac{2}{T_e}, \pm \frac{3}{T_e}, \ldots \)

clock frequency \( R_e = 1/T_e \)

Continuous spectrum is \( \frac{T_e}{16} \ \text{sin}^2 \left( \frac{\omega T_e}{4} \right) \)

(For polar signal \( S_y(w) = \left( \frac{T_e}{4} \right) \ \text{sin}^2 \left( \frac{\omega T_e}{4} \right) \)) \tag{7.14}

See Fig.7.2

**On-off Signal = Polar Signal + Periodic Signal**
EXAMPLE 2.12 Find the exponential Fourier series and sketch the corresponding spectra for the impulse train $\delta_T(t)$ shown in Fig. 2.27.

**Periodic Impulse Train**

$$z(t) = \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - n T_0)$$

![Diagram of periodic impulse train](image)

**Figure 2.27** Impulse train and its exponential Fourier spectra.

The exponential Fourier series is given by

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} D_n e^{j n \omega_0 t}$$

where $\omega_0 = \frac{2 \pi}{T_0}$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta_T(t) e^{-j n \omega_0 t} dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) e^{-j n \omega_0 t} dt = \frac{1}{T_0} e^{-j n \omega_0 t}$$

(8.88)

(8.89)

$$\int e^{j n \omega_0 t} dt = \int_{-\infty}^{\infty} \frac{2 \pi}{2 \pi} \delta(\omega - n \omega_0) e^{j \omega t} d\omega$$

$$\mathcal{F}^{-1} \left[ 2 \pi \delta(\omega - n \omega_0) \right] = \frac{2 \pi}{2 \pi} \int_{-\infty}^{\infty} \delta(\omega - n \omega_0) e^{j \omega t} d\omega$$

See Footnote on p. 305

$$\mathcal{F} \left[ \delta_T(t) \right] = \frac{2 \pi}{T_0} \sum_{n=-\infty}^{\infty} \delta(\omega - n \omega_0)$$

Fourier transform of a periodic impulse train $\delta_T(t)$ is a periodic impulse train in $\omega$. 
**On-off Signaling (Cont'd)**

Noise immunity depends on difference of pulses representing '1' and '0'. For the same noise immunity:

- Polar signal pulse amplitudes: '1' and '−1'
- On-off signal '2' and '0'
- Pulse amplitude '1' is '−1', energy = $E$
- "2" energy = $4E$

Bit rate transmission = $1/T_b$

- $R_b$ (bits/sec)
- Polar signal power ($E/T_b$) = $ER_b$ (Joules/sec)

On-off signal power = \( \frac{2E}{T_b} = 2ER_b \)

Twice that required for polar signal.

It has also all other disadvantages as those of polar signal.
Bipolar Signaling (Pseudoternary or AMI) used in PCM

[Pseudoternary or Alternate Mark Inverted (AMI)]
signaling scheme currently used in PCM.

Uses 3 symbols, \( p(t), 0 \) & \(-p(t)\) (Fig. 7.1c)

\[
R_0 = \lim_{N \to \infty} \frac{1}{N} \sum_{k} a_k^2 = \frac{1}{2}
\]

\[
R_1 = \frac{-1}{4}, \quad R_2 = 0, \quad \text{See next page}
\]

\[
R_n = \lim_{N \to \infty} \frac{1}{N} \sum_{k} a_k a_{k+n} = 0, \quad n > 1
\]

\[
\frac{PSD}{S_y(w)} = \frac{|P(w)|^2}{T_c} \left[ R_0 + 2 \sum_{n=1}^{8} R_n \cos(n\omega T_c) \right]
\]
Bipolar Signaling (cont'd.)

\[ R_0 = \lim_{N \to \infty} \frac{1}{N} \sum_k a_k^2 \]

On the average half of \( a_k \)'s are 0 & the remaining half are either +1 or -1.

\[ R_0 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (1)^2 + \frac{N}{2} (0) \right] = \frac{1}{2} \]

\[ R_1 = \lim_{N \to \infty} \frac{1}{N} \sum_k a_k a_{k+1} \]

4 possible equally likely sequences of 2 bits:
11, 10, 01, 00.

\( a_k a_{k+1} = 0 \), \( \frac{3N}{4} \) combinations
\( a_k a_{k+1} = -1 \), \( \frac{N}{4} \) combinations

\[ R_1 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{4} (-1) + \frac{3N}{4} (0) \right] = -\frac{1}{4} \]

\[ R_2 = \lim_{N \to \infty} \frac{1}{N} \sum_k a_k a_{k+2} \]

Consider all possible combinations of 3 bits in sequence (equally likely):
111, 101, 110, 100, 011, 010, 001, 000

\( \frac{N}{8} \), \( \frac{N}{8} \), \( \frac{N}{8} \), \( \frac{N}{8} \) (\( B N/4 \)) combinations

\[ R_2 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{8} (1) + \frac{N}{8} (1) + \frac{3N}{4} (0) \right] = 0 \]
Bipolar Signaling (cont'd.)

\[ S_y(w) = \frac{1}{2 T_b} \left[ 1 - \cos \omega T_b \right] \]

\[
1 - \cos \theta = 1 - \cos \left( \frac{\theta}{2} + \frac{\theta}{2} \right) \\
= \left( \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right) - \left[ \cos^2 \left( \frac{\theta}{2} \right) - \sin^2 \left( \frac{\theta}{2} \right) \right] \\
= 2 \sin^2 \left( \frac{\theta}{2} \right) \\
\]

\[ S_y(w) = \left[ \frac{1}{T_b} \right] \sin^2 \left( \frac{w T_b}{2} \right) \]

\[ \sin (\omega T_b/2) = 0, \; \omega = \left( \frac{2\pi}{T_b} \right) \]

\[ \sin^2 \pi = 0, \; \therefore f = \left( \frac{1}{T_b} \right) = R_b \]

BW of bipolar signal = \( R_b \) Hz

For half-width rectangular pulse see

\[ S_y(w) = \frac{T_b}{4} \sin^2 \left( \frac{w T_b}{4} \right) \sin^2 \left( \frac{w T_b}{2} \right) \]
Bipolar Signaling (Cont'd) 7-20

\[(BW \text{ in } R_0) = \frac{1}{2} (BW \text{ of Polar or on-off signal})\]

Advantages:
1. dc null \((PSD=0 \text{ at } \omega=0)\)
2. BW is not excessive
3. Single error detection capability

Disadvantages:
1. Requires double the power for a polar signal. (3dB more)
2. Not transparent
   Long strings of '0's (no pulses)
   Output of resonant circuit starts decaying, causing an error in timing information
Bipolar Signal (contd.)

1) HDB (high density bipolar) signal
2) Binary with 8 zero substitution (B8ZS)

Techniques to prevent long string of "zeros" from allowing the extracted clock signals to decay. High density bipolar signaling

Add pulses when the string of "zeros" exceeds \( N \).

\[ \text{HDBN, } N = 1, 2, 3, \ldots \]

I S: HDB3, \( N = 3 \)

\[ \text{HDBN code: when a run of } N+1 \text{ "zeros" occurs replace} \]
Bipolar Signal (Cont'd.)

These 'errors' by one of the special $N + 1$ bit sequences. These include some '1's to increase the timing content of the signal. Ex.: HDB3 coding special sequences 0000V & BOOV $B = 1$ follows bipolar rule $V = 1$ violates "" 

HDB3 signaling retains error-detecting capability despite deliberate bipolar violations.
HDB Signaling (cont'd.)

Choice of sequence 000V and 200V is made in such a way that consecutive V pulse alternate signs in order to maintain dc null in the PSD and avoid dc warden, i.e., 200V is used when there is an even number of 1's following the last special sequence and 000V is used when there is an odd # of 1's following the last sequence. Decoder has to check two things: 1) Bipolar violations & 2) # of 0's preceding each violation, to determine if the previous 1 is also a substitution.
Binary with 8-Zero Substitution (B8ZS) Signaling

BNZS Code: If there are N zeros that occur consecutively, they are substituted by one of two special sequences containing some 1’s to increase timing content. There are deliberate bipolar violations.

B8ZS used in DS1 signals: Replaces any string of 8 zeros with a sequence of 1’s and 0’s containing two bipolar violations, e.g., 000VBOVB

B6ZS used in DS2 signals: Replaces any string of 6 zeros with 0VBOVB

B3ZS used in DS3 signals: This is more complex than B8ZS & B6ZS. B0V or 00V is used such that the # of B pulses between consecutive V pulses is odd. As BNZS codes involve bipolar violations, they must be replaced by zero strings at the receiver.
Devise PSDs for Polar, on-off and bipolar signaling.

For full width rect pulse \( p(t) = \text{rect}(\frac{t}{T_b}) \),

\[
P(\omega) = T_b \text{sinc}(\frac{aT_b}{2})
\]

\[
S_y(\omega) = \left| P(\omega) \right|^2 / T_b
\]

For polar signaling [see Eq. (7.12)]

\[
S_y(\omega) = \left( \frac{P(\omega)}{T_b} \right)^2 \text{sinc}^2 \left( \frac{aT_b}{2} \right)
\]

For on-off case [see Eq. (7.18b)]

\[
S_y(\omega) = \left( \frac{P(\omega)}{T_b} \right)^2 \left[ 1 + \frac{2\pi}{T_b} \sum_{m=-\infty}^{\infty} \delta \left( \omega - \frac{2m}{T_b} \right) \right]
\]

But \( \text{sinc}^2 \left( \frac{aT_b}{2} \right) = 0 \) for all \( n \neq 0 \), and 1 for \( n = 0 \). Hence,

\[
\omega T_b = n\pi
\]

For bipolar case [Eq. (7.20b)]

\[
S_y(\omega) = \left( \frac{P(\omega)}{T_b} \right)^2 \left( \frac{aT_b}{2} \right)^2 \delta(\omega)
\]

\[
S_y(\omega) = T_b \text{sinc}^2 \left( \frac{aT_b}{2} \right) \gamma \text{sinc}^2 \left( \frac{aT_b}{2} \right)
\]

The PSDs of the three cases are shown in Fig. S7.2.1. From these spectra, we find the bandwidths for all three cases to be \( R_b \) Hz.

The bandwidths for the three cases, when half-width pulses are used, are as follows:

Polar and on-off: \( 2R_b \) Hz; bipolar: \( R_b \) Hz.

Clearly, for polar and on-off cases the bandwidth is halved when full-width pulses are used. However, for the bipolar case, the bandwidth remains unchanged. The pulse shape has only a minor influence in the bipolar case because the term \( \text{sinc}^2 \left( \frac{aT_b}{2} \right) \) in \( S_y(\omega) \) determines its bandwidth.

For half width rectangular pulse

\[
P(t) = \text{rect}(\frac{t}{T_b})
\]

\[
P(\omega) = \left( \frac{T_b}{2} \right) \text{sinc}(\omega \frac{T_b}{2})
\]

\[
\text{I Zero crossing}
\]

\[
\omega T_b = n\pi, \quad f = \frac{n}{T_b}
\]

\[
\text{I Zero crossing}
\]

\[
\omega T_b = \frac{R_b}{2}, \quad f = \frac{1}{T_b}
\]

\[
\text{I Zero crossing}
\]

\[
\omega T_b = \frac{R_b}{2}, \quad f = \frac{1}{T_b}
\]
Differential Code

1 is transmitted by a pulse identical to that used for the previous bit and a 0 is transmitted by a pulse negative of that used for the previous bit.

7.2-3 For differential code (Fig. 7.17) \( R_0 \) for differential code

\[
R_0 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (1) + \frac{N}{2} (-1) \right] = 1
\]

To compute \( R_1 \), we observe that there are four possible 2-bit sequences 11, 00, 01, and 10, which are equally likely. The product \( a_k a_{k+1} \) for the first two combinations is 1 and is \(-1\) for the last two combinations. Hence,

\[
R_1 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (1) + \frac{N}{2} (-1) \right] = 0
\]

Similarly, we can show that \( R_n = 0 \) for \( n > 1 \). Hence,

\[
S_Y(\omega) = \frac{|P(\omega)|^2}{T_0} = \left( \frac{T_0}{4} \right) \sin^2 \left( \frac{\omega T_0}{4} \right)
\]

Similarly, \( a_k \sim a_{k+n} \) for \( n > 0 \) is equally likely 1 or \(-1\). Hence \( R_n = 0 \), \( n \geq 1 \)

\[
S_Y(\omega) = \frac{|P(\omega)|^2}{T_0} \left( R_0 + 2 \sum_{n=1}^{\infty} R_n \cos n \omega T_0 \right)
\]

For half width rectangular pulse

\[
\rho(t) = \text{rect} \left( \frac{t}{T_0} \right)
\]

\[
\rho(\omega) = \frac{T_0}{2} \sin \left( \omega T_0/4 \right)
\]
7.2-4

**Duobinary line coding**

0 is transmitted by no pulse

1 is transmitted by pulse $p(t)$ or $-p(t)$ as follows:

1 is encoded by the same pulse as that used for the previous 1 if there is an even # of 0's between them.

1 is encoded by a pulse of opposite polarity if there is an odd # of 0's between them.

---

7.2-4  (a) Fig. S7.2-4 shows the duobinary pulse train $y(t)$ for the sequence 1110001101001010.

(b) To compute $R_0$, we observe that on the average, half the pulses have $a_k = 0$ and the remaining half have $a_k = 1$ or $-1$. Hence,

$$R_0 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (a_1)^2 + \frac{N}{2} (0) \right] = \frac{1}{2}$$

To determine $R_1$, we need to compute $a_k a_{k+1}$. There are four possible equally likely sequences of two bits: 11, 10, 01, 00. Since bit 0 is encoded by no pulse ($a_k = 0$), the product of $a_k a_{k+1} = 0$ for the last three of these sequences. This means on the average $\frac{3N}{4}$ combinations have $a_k a_{k+1} = 0$ and only $\frac{N}{4}$ combinations.
have nonzero $a_k a_{k+1}$. Because of the duobinary rule, the bit sequence 11 can only be encoded by two consecutive pulses of the same polarity (both positive or both negative).

This means $a_k$ and $a_{k+1}$ are 1 and 1 or -1 and -1 respectively. In either case $a_k a_{k+1} = 1$. Thus, these $\frac{N}{4}$ combinations have $a_k a_{k+1} = 1$. Therefore,

$$R_1 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{4} (1) + \frac{3N}{4} (0) \right] = \frac{1}{4}$$

To compute $R_2$ in a similar way, we need to observe the product $a_k a_{k+2}$. For this we need to observe all possible combinations of three bits in sequence. There are eight equally likely combinations: 111, 101, 010, 000, 110, 100, 011, 001, and 000. The last six combinations have either the first and/or the last bit 0.

Hence, $a_k a_{k+2} = 0$ for all these six combinations. The first two combinations are the only ones which yield nonzero $a_k a_{k+2}$. Using the duobinary rule, the first combination is encoded by three pulses of the same polarity (all positive or negative). Thus $a_k$ and $a_{k+2}$ are 1 and 1 or -1 and -1, respectively, yielding $a_k a_{k+2} = 1$. Similarly, because of the duobinary rule, the first and the third pulses in the second bit combination 101 are of opposite polarity yielding $a_k a_{k+2} = -1$. Thus on the average, $a_k a_{k+2} = 1$ for $\frac{N}{8}$ terms, -1 for $\frac{N}{8}$ terms, and 0 for $\frac{3N}{4}$ terms. Hence,

$$R_2 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{8} (1) + \frac{N}{8} (-1) + \frac{3N}{4} (0) \right] = 0$$

In a similar way we can show that $R_n = 0 \quad n > 1$, and from Eq. (7.10c), we obtain

$$S_y(\omega) = \frac{|P(\omega)|^2}{2T_b} \left( 1 + \cos \omega T_b \right)$$

$$S_y(\omega) = \left[ \frac{|P(\omega)|^2}{2T_b} \left( \frac{1}{2} + \frac{1}{2} \cos \omega T_b \right) \right] \left[ \frac{|P(\omega)|^2}{2T_b} \left( \frac{1}{2} \cos \omega T_b \right) \right]$$

From Fig. S7.2-4 we observe that the bandwidth is approximately $R_b/2$ Hz. 

$$f = \frac{R_b}{2}$$
Binary Line Coding

Ref: Couch, page 161
Unipolar NRZ (Non return to zero)

Most common line code used in mobile communications.

A volts = logic 1  0 volts = logic 0

Assume the occurrence of logic 1 and logic 0 are equally likely, thus the probability density function is equal to 0.5

The encoding power is

\[ \int A^2 p(x) dx = 0.5 A^2 \text{ watts} \]

into a one ohm resistance.

This signal is unbalanced in that a dc component can accumulate if there are a large number of logic 1’s compared to logic 0’s. If this condition exists, a dc blocker and restorer is necessary in the receiver.

The signal can be represented by the relation:

\[ s(t) = \sum_{n=-\infty}^{\infty} a_n f \prod \left( \frac{t - nT}{T} \right) \]

where \( a_n = A \) when logic 1 and 0 when logic 0 and \( T \) is the time interval for each bit.

Note that each pulse has a duration for the entire period of \( T \).

The power spectral density (PSD) of the unipolar NRZ signal is:

\[ PSD = \frac{A^2 T}{4} \sin^2(\pi f T) \left[ 1 + \frac{1}{T} \sum_{n=\infty}^{\infty} \delta(f - \frac{n}{T}) \right] \]
**Polar NRZ**

This signal is represented by +A for logic 1 and −A for logic 0. In so doing it takes only one half the power to encode then in unipolar NRZ signaling. This signaling is usually called a balanced code.

Notice that each signal is maintained for the entire duration of the time interval.

The Power Spectrum Density is equal to:

\[ PSD = A^2 T \sin c^2 (\pi fT) \]

**Unipolar RZ**

In this signaling, the duration of the pulse is one half the time interval T

The Power Spectrum Density of the unipolar RZ signaling is:

\[ PSD = \frac{A^2 T}{16} \sin^2 (\pi fT) \left[ 1 + \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T}) \right] \]

The first sinc null bandwidth is twice that for the unipolar or polar NRZ since the pulse width is half as wide.

**Bipolar RZ**

The binary 1’s are represented by alternating values of A, whereas the binary 0’s are represented by \( a_n = 0 \).

The duration of a signal pulse is one half the time interval T as in the case of the unipolar RZ,

PSD for this signaling is:

\[ PSD = \frac{A^2 T}{4} \sin^2 (\pi fT) \sin^2 (\pi fT) \]
Manchester NRZ

The pulse shapes for the Manchester NRZ is:

\[ f(t) = \Pi\left(\frac{t+T/4}{T/2}\right) - \Pi\left(\frac{t-T/4}{T/2}\right) \]

and the power spectrum density is:

\[ PSD = A^2 T \left( \text{sinc}^2 \pi f \frac{T}{2} \right) \sin^2 \pi f \frac{T}{2} \]

Notice that the sinc null bandwidth of the Manchester code is twice that of the bipolar signaling. The Manchester code has a zero dc level for each succession of bits.

Below is another line code known as the Bipolar(AMI) coding. This line code is designed to eliminate dc energy contained in the signal. This is done by using three levels to encode binary data. A logic 0 is encoded with zero voltage and a logic 1 is alternately encoded with positive and negative voltage. Thus the average voltage level stays at zero.

This bipolar coding is used by T1 lines in the telephone network. However, T1 lines use a 50% duty cycle pulse to encode each logic one. The power spectrum of the bipolar code is given as:

\[ PSD = \frac{2p(1-p)}{T} |G(\omega)|^2 \frac{1 - \cos \omega T}{1 - 2(2p-1) \cos \omega T + (2p-1)^2} \]
where $p$ is the probability of a logic 1 and $G(\omega)$ is the spectrum of an individual pulse. For 50% duty cycle pulses the spectrum is:

$$G(\omega) = \frac{T}{2} \sin c(\omega \frac{T}{4})$$
Figure 7.27 (a) Carrier $\cos \omega t$. (b) Modulating signal $m(t)$. (c) ASK: modulated signal $m(t) \cos \omega t$.

Figure 7.28 (a) Modulating signal $m(t)$. (b) PSK: modulated signal $m(t) \cos \omega t$. (c) FSK: modulated signal.

Polar (NRZ)

$\begin{array}{c}
1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
\end{array}$

$m(t) \cos \omega t$

PSK

FSK interleaved ASK signals with modulating frequencies $\omega_0$ & $\omega_1$

ASK: Amplitude Shift Keying

on-off shift keying (OOK)

PSK: Phase Shift Keying

FSK: Frequency Shift Keying

Transmission of random bit stream 11000101 0001... using line codes (see Fig. 7.1 / p. 295)

FSK is sum of two interleaved ASK signals at two different modulating frequencies $\omega_0$ and $\omega_1$. 
Figure 7.1 Some line codes. (a) On-off (RZ). (b) Polar (RZ). (c) Bipolar (RZ). (d) On-off (NRZ). (e) Polar (NRZ).

ON-OFF (RZ)

POLAR (RZ)

BIPOLAR (RZ)

PSEUDOLERTARY (AMI)

ON-OFF (NRZ)

POLAR (NRZ)

LINE CODES

Figure 7.5 Power spectral density of a polar signal.

PSD of Polar Signal

Figure 7.7 Power spectral density of an on-off signal.

PSD of On-off Signal
PSD of ASK

(a)

PSD of PSK

(b)

PSD of FSK

(c)

Figure 7.29 PSD of: (a) ASK, (b) PSK, (c) FSK.

(see Fig. 7.7)

ASK = on-off line code modulates \( \cos \omega t \)

PSK = polar line code modulates \( \cos \omega t \)

(see Fig. 7.5)

FSK = sum of two ASK interleaved signals modulating \( \cos \omega_1 t \) and \( \cos \omega_2 t \)

BW of PSK is higher than that of ASK & PSK
Demodulation of ASK, PSK & FSK 7-32

Signals

ASK (Fig. 7.27) can be demodulated coherently (synchronously) or noncoherently (envelope detection). Envelope detection is followed in practice (simple and inexpensive).

PSK (Fig. 7.28) (No envelope detection)

Pulse $A \cos \omega_c t$ represents bit 1

Pulse $-A \cos \omega_c t$ represents bit 0

Hence coherent detection is followed.

DPSK differently coherent PSK

![Diagram](image)

**Figure 7.38** (a) Differential coding. (b) Differential PSK receiver.

Bit 1 is encoded by same pulse used to encode the previous data bit 1 or 0.

Bit 0 is encoded by the negative of the pulse used to encode the previous data bit 1 or 0.
FSK (Two interleaved signals modulating carrier frequencies \( w_{c0} \) & \( w_{c1} \))

(Fig. 7.28)  

Figure 7.31  (a) Noncoherent detection of FSK. (b) Coherent detection of FSK.

- Bit 0 — modulates carrier freq. \( w_{c0} \)
- Bit 1 — \( w_{c1} \)

(Sample of the top half > sample of the bottom half) represents bit 0
(Sample of the bottom half > sample of the top half) represents bit 1

FSK coherent detection (see Fig. 7.31b)

PSK is superior to ASK & FSK in terms of noise immunity. PSK also requires less BW than FSK (Fig. 7.29).
QUADRATURE PHASE SHIFT KEYING (QPSK)

In Quadrature Phase Shift Keying, two bits are transmitted in a single modulation symbol. The phase of the carrier takes on one of four equally spaced values. Since there are two bits for each symbol, and they represent four phases each the code can break down in the following examples:

<table>
<thead>
<tr>
<th>Phase</th>
<th>Binary bits</th>
<th>or</th>
<th>Phase</th>
<th>Binary bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00</td>
<td></td>
<td>π/4</td>
<td>11</td>
</tr>
<tr>
<td>π/2</td>
<td>01</td>
<td></td>
<td>3π/4</td>
<td>10</td>
</tr>
<tr>
<td>π</td>
<td>10</td>
<td></td>
<td>5π/4</td>
<td>00</td>
</tr>
<tr>
<td>3π/2</td>
<td>11</td>
<td></td>
<td>7π/4</td>
<td>01</td>
</tr>
</tbody>
</table>

The QPSK signal for this set of symbol states can be defined as:

$$s(t) = \sqrt{\frac{2E_s}{T_s}} \sin[(i-1)\frac{\pi}{2}]\sin \omega t$$

Where $0 \leq t \leq T_s$ and $i = 1, 2, 3, 4$

Each symbol corresponds to two bits, so that the energy per symbol is twice the energy per bit. So $E_s = 2E_b$.

If, for the sake of implementation of transmission, two carriers are employed. $\phi_1(t)$ and $\phi_2(t)$, the QPSK signal can be expressed as:

$$s(t) = \sqrt{E_s} (\cos[(i-1)\frac{\pi}{2}]\phi_1(t) - \sin[(i-1)\frac{\pi}{2}]\phi_2(t))$$
If the transmission medium is infected with white Gaussian noise, it can be shown that the average probability of bit error in the presence of white Gaussian noise is:

\[ P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \]

where

\[ Q = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-x^2/2} dx \]

The bit error probability of QPSK is identical to the binary phase shift keying, BPSK, but twice as much data can be sent in the same bandwidth.

The power spectral density of a QPSK signal can be expressed as:

\[ \text{PSD} = E_b \text{sinc}^2 (\omega - \omega_c)T_b + E_b \text{sinc}^2 (-\omega - \omega_c)T_b \]

If the signals are rectangular pulses.

The PSD of a QPSK signal for rectangular and raised-cosine filtered pulses is shown below. The null-to-null RF bandwidth is equal to the bit rate, \( R_b \).
Summarizing the signals responding to the codes:

<table>
<thead>
<tr>
<th>Input bits</th>
<th>signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>$A \cos \omega_c t + 45^0$</td>
</tr>
<tr>
<td>10</td>
<td>$A \cos \omega_c t + 135^0$</td>
</tr>
<tr>
<td>00</td>
<td>$A \cos \omega_c t + 225^0$</td>
</tr>
<tr>
<td>01</td>
<td>$A \cos \omega_c t + 315^0$</td>
</tr>
</tbody>
</table>

Thus each signal element represents two bits instead of only one.

It is possible to transmit three bits at a time using eight different phase angles. Each angle can have more than one amplitude. A standard 9600bps modem uses 12 phase angles four of which has two amplitudes.

So, for speeds of 4800 bps and faster, a combination of phases and amplitudes is used.

Let $D =$ modulation rate in bauds
$R =$ the data rate in bits per sec.
$L =$ number of different signal elements ( symbols )
$l =$ number of bits per signal element.

Then $R = D \log_2 L = Dl$

So, for a signaling speed of 2400 bauds, the data rate is 9600 bps when $l = 4$ and $L = 16$

The transmission bandwidth for ASK and PSK is determined from the relation

$$B = ( 1 + r ) R, \text{ } R \text{ being the bit rate}$$

$r$ is a factor known as the roll factor of the filter characteristic.

For FSK, the bandwidth is: $B = 2\Delta f + ( 1 + r )R$

Where $\Delta f$ is the frequency deviation. Usually, for very high frequencies, the $\Delta f$ dominates.
One of the standards for FSK signaling on a coaxial cable multipoint local network uses $\Delta f = 1.25$ Mhz, $f_c = 5$ Mhz, and $R = 1$ Mbps. In this case $B = 2\Delta f = 2.5$ Mhz. With multilevel signaling, improvements in bandwidth can be made. View the formula:

$$B = \left(1 + r\right)R/I = \left(1 + r\right)R/\log L$$

Where $I$ is the number of bits encoded per signal element and $L$ is the number of different signal elements.

Example of phase angles for a 9600 bit per second transmission;

![Diagram of phase angles for FSK signaling]

**Table of signal Coefficients for a Two Bit Input QPSK Modulation**

<table>
<thead>
<tr>
<th>Data Values</th>
<th>$\cos \omega_c t$</th>
<th>$\sin \omega_c t$</th>
<th>Composite Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.707</td>
<td>-0.707</td>
<td>$\cos(\omega_c t + \pi/4)$</td>
</tr>
<tr>
<td>00</td>
<td>-0.707</td>
<td>-0.707</td>
<td>$\cos(\omega_c t + 3\pi/4)$</td>
</tr>
<tr>
<td>10</td>
<td>-0.707</td>
<td>0.707</td>
<td>$\cos(\omega_c t - 3\pi/4)$</td>
</tr>
<tr>
<td>11</td>
<td>0.707</td>
<td>0.707</td>
<td>$\cos(\omega_c t - \pi/4)$</td>
</tr>
</tbody>
</table>