1. Closed books and closed notes.
2. You can only use the four-page cheat sheet handout.
3. Please show all the steps in your work.
4. You may work problems in any order. At the end please arrange as 1,2,3,4,5.
5. Please print your name and last four digits of your ID.
6. Write on one side of the paper only.
7. No cheating.
8. Problems carry weights as indicated. \( \text{E Q U A L \ W E I G H T S} \)

1. A signal band-limited to 1MHz is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 levels using a \( \mu \)-law compander with \( \mu = 256 \). The output SNR of a \( \mu \)-law compander is,

\[
\frac{S_0}{N_0} = \frac{3L^2}{\left[ \ln(\mu+1) \right]^2} \quad \mu^2 \gg \frac{m_p^2}{\hat{m}^2(t)}
\]

Where \( L \) = number of quantization levels
\( m_p \) = peak signal amplitude
\( \hat{m}^2(t) \) = mean square value of signal

(a) Determine the signal-to-noise ratio in dB.
(b) The SNR (the received signal quality) found in part (a) was unsatisfactory. It must be increased at least by 10 dB. Would you be able to obtain the required SNR without increasing the transmission bandwidth if it was found that a sampling rate 20% above the Nyquist rate is adequate? If so, explain how? What is the maximum SNR that can be realized this way?

2. Find PSDs for polar, ON-OFF and bipolar signaling where \( p(t) \) is a full width rectangular pulse, \( p(t) = \text{rect}(t/T_b) \). Sketch roughly these PSDs and find their essential bandwidth.
3. Five telemetry signals each of bandwidth 1KHz are transmitted by Time Division Multiplexing using binary PCM. The maximum tolerable quantization error in sample amplitudes is 0.2% of the peak signal amplitude $m_p$. The signals must be sampled at least 20% above the Nyquist rate. Determine the maximum possible data rate (bits per second) that must be transmitted and the minimum bandwidth required to transmit this signal. The signals range from $-m_p$ to $m_p$.  

[HINT: Note that quantization step size $\Delta \nu = 2m_p/L$ where $L$ is the number of quantization levels].

4. A signal $g(t) = \text{sinc}(20\pi t)$ is sampled using uniformly spaced impulses at the rate of 25 Hz.

(a) Sketch the spectrum of the sampled signal. Identify all the important points.

(b) Can you recover the signal $g(t)$ from the sampled signal? Explain.

(c) If the sampled signal is passed through an ideal low pass filter of bandwidth 25Hz, sketch the spectrum of the output signal. Identify all the important points.
5.(a) For a polar line code using a half width pulses p(t), sketch the signal y(t) for the random binary sequence 1110001101001010.

(c) Determine R₀, R₁ and R₂ for this code. Assume 1’s and 0’s to be equally likely and occur at random. Justify your answers.
Figure 7.1  Some line codes. (a) On-off (RZ). (b) Polar
(RZ). (c) Bipolar (RZ). (d) On-off (NRZ). (e) Polar (NRZ).

ON-OFF (RZ)
POLAR (RZ)
BIPOLAR (RZ)
ON-OFF (NRZ)
POLAR (NRZ)

Figure 7.3  A random PAM signal and its generation from a PAM impulse sequence.
\[ R_n = \lim_{T \to \infty} \frac{T}{T} \sum_k a_k a_{k+n} \]
\[ = \lim_{T \to \infty} \frac{1}{N} \sum_k a_k a_{k+n} \]
\[ = a_k a_{k+n} \]

\[ S_x(\omega) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-j\omega T_b} \]

\[ S_z(\omega) = \frac{1}{T_b} \left( R_0 + 2 \sum_{n=1}^{\infty} R_n \cos n \omega T_b \right) \]

\[ S_y(\omega) = \frac{|P(\omega)|^2}{T_b} S_x(\omega) \]
\[ = \frac{|P(\omega)|^2}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-j\omega T_b} \]
\[ = \frac{|P(\omega)|^2}{T_b} \left( R_0 + 2 \sum_{n=1}^{\infty} R_n \cos n \omega T_b \right) \]

where N is the number of pulses in time T.