Student Name: _____________________
Student ID #: ______________________
(Last 4 digits only)

EE4330 Section 001
Fundamentals of Telecommunications Systems

INSTRUCTOR: Dr. K. R. Rao

Spring 2006, Final
Thursday, 11 April 2006
2:00 – 3:30 pm (1 Hour and 30 minutes)

INSTRUCTIONS:

1. Closed books and closed notes.
2. Any additional information required is attached to the test, you can only use the four-page cheat sheet handout.
3. Choose only one answer from the options given, and show all your work.
4. Please print your name and last four digits of your ID.
(Q1) Antenna transmits the FM signal with power 10 kW. Free space loss in signal power is 16 dB. At the receiver, the received FM signal is amplified with gain 7 dB. Find the signal power in dB, after the amplifier.

A. 24 dB  
B. 31 dB  
C. 40 dB  
D. 51 dB

(Q2) For T1 carrier system, called DS1 (digital signal level 1), 1 frame consists of 24 multiplexed PCM channels and 1 framing bit. The bit rate of one PCM channel before multiplexing is 64 kbps (8 bits per sample with sampling frequency 8 kHz). Calculate the bit rate of T1 signal.

A. 512 kbps  
B. 1,536 kbps  
C. 1,544 kbps  
D. 2,048 kbps

(Q3) Given a diagram of DPCM (differential pulse code modulation), select block (i) and (ii).

\[ \sum^m[k] \rightarrow (i) \rightarrow \sum \rightarrow (ii) \rightarrow \text{To channel} \]

(i) ,  (ii)

A. Quantizer , Predictor  
B. Quantizer , Differentiator  
C. Lowpass filter , Predictor  
D. Lowpass filter, Differentiator

(Q4) In delta modulation (DM), a signal \(\sin(\omega t)\) is sampled at a rate 64 kHz. Signal bandwidth is 4 kHz. Find the SNR (signal-to-noise ratio). Given the granular-noise power of DM, \(N_0 = \sigma^2 B/(3f_s)\), where \(\sigma\) is the step size, \(B\) is signal bandwidth and \(f_s\) is sampling frequency.

A. \(12/\sigma^2\)  
B. \(24/\sigma^2\)  
C. \(48/\sigma^2\)  
D. \(96/\sigma^2\)
(Q5) Line coding is the process of coding digital data into electrical pulses to transmit the data over the channel (transmission line). If the timing extraction problem due to long zero sequences is a primary concern, which line code(s) can be used?

A. Polar  
B. On-off  
C. Bipolar  
D. None of the above

(Q6) In case of using the cable as a transmission line, the repeaters with operated dc power are used. Which line code(s) can be used?

A. On-off  
B. Polar  
C. Bipolar  
D. Both polar and bipolar

(Q7) Find the correlation \( R_p(\tau) \) of the pulse, \( p(t) = h_k \text{rect}(\frac{t}{\varepsilon}) \)

\[
A. h_k^2 (1 + \frac{\varepsilon}{\varepsilon}), \quad |\tau| \leq \varepsilon \\
B. h_k^2 (1 - \frac{\varepsilon}{\varepsilon}), \quad |\tau| \leq \varepsilon \\
C. h_k^2 (1 + \frac{\varepsilon}{\varepsilon}), \quad |\tau| \leq \varepsilon \\
D. h_k^2 (1 - \frac{\varepsilon}{\varepsilon})\varepsilon, \quad |\tau| \leq \varepsilon
\]

(Q8) The correlation of an impulse train \( x(t) = \sum_{n=-\infty}^{\infty} a_n \delta(t - nT_b) \) is given by \( R_x(\tau) = \sum_{n=-\infty}^{\infty} \frac{r_n}{T_b} \delta(\tau - nT_b) \), where \( T_b \) and \( r_n \) are constants, \( r_n = r_n^\alpha \). Find the PSD of \( x(t) \) from the Fourier transform of \( R_x(\tau) \).

A. \( \frac{a}{T_b} + \sum_{n=1}^{\infty} \frac{2\alpha}{T_b} \cos(\pi nT_b) \)
B. \( \frac{a}{T_b} + \sum_{n=1}^{\infty} \frac{2\alpha}{T_b} \cos(\pi n) \)
C. \( \frac{a}{T_b} + \sum_{n=1}^{\infty} \frac{\alpha}{T_b} \cos(\pi nT_b) \)
D. \( \frac{a}{T_b} + \sum_{n=1}^{\infty} \frac{\alpha}{T_b} \cos(\pi n \frac{T_b}{T}) \)
(Q9) Given signal full width triangular pulse, \( p(t) = \Lambda \left( \frac{t}{T_b} \right) = 1 - \frac{|t|}{T_b} \), where \(|t| \leq T_b/2\)

Find the Fourier transform of this single pulse. (Define, \( \text{sinc}(x) = \frac{\sin(x)}{x} \))

\[
\begin{align*}
\text{p}(t) & \quad \text{t} \\
-\frac{T_b}{2} & \quad 0 & \quad \frac{T_b}{2}
\end{align*}
\]

A. \( \frac{T_b}{2} \text{sinc}[\omega T_b/2] \)
B. \( \frac{T_b}{4} \text{sinc}^2[\omega T_b/4] \)
C. \( \frac{T_b}{4} \text{sinc}^2[\omega T_b/4] \)
D. \( \frac{T_b}{4} \text{sinc}^2[\omega T_b/2] \)

(Q10) From (Q9), find the PSD of the polar line code using this full width binary pulse (bit 1 is represented by \( p(t) \) and bit 0 is represented by \( -p(t) \)).

**Hint:** PSD of a line code has the form
\[
S_y(\omega) = \frac{|P(\omega)|^2}{T_b} \left( R_0 + \sum_{n=1}^{\infty} 2R_n \cos(\omega n T_b) \right),
\]
where \( T_b \) is the time duration between the center of each pulse, \( P(\omega) \) is the spectrum of a single pulse \( p(t) \), \( R_n \) for different types of line codes is given in the table,

<table>
<thead>
<tr>
<th>( R_0 )</th>
<th>On-off (1,0)</th>
<th>Polar (1,-1)</th>
<th>Bipolar (1,-1,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( R_f )</td>
<td>1/4</td>
<td>0</td>
<td>( -\frac{1}{4} )</td>
</tr>
<tr>
<td>( R_n ) for ( n \geq 2 )</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

A. \( \frac{T_b}{4} \text{sinc}^2[\omega T_b/2] \)
B. \( \frac{T_b}{4} \text{sinc}^4[\omega T_b/4] \)
C. \( \frac{T_b}{16} \text{sinc}^4[\omega T_b/4] \)
D. \( \frac{T_b}{16} \text{sinc}^4[\omega T_b/2] \)
(Q11) In order to transmit the line code signal, where the channel bandwidth is $R_b$ Hz. Given the PSD of three line codes using half-width rectangular pulses in Figure 1, what type(s) of line code can be used?

A. Split phase  
B. Bipolar  
C. Polar  
D. All above

(Q12) Differential code with half-width rectangular pulses is applied to code a binary signal. In this line coding scheme, “a 1 is transmitted by a pulse identical to that used for the previous bit and a 0 is transmitted by a pulse negative of that used for the previous bit.” Given the 16 input digits:

<table>
<thead>
<tr>
<th>Coded digits</th>
<th>Waveforms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1</td>
<td>[Waveform image]</td>
</tr>
</tbody>
</table>

What is the transmitted waveform using differential coding?

A. Coded digits → 1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1  
B. Coded digits → 1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1  
C. Coded digits → 1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1  
D. Coded digits → 1 1 1 0 0 0 0 1 0 1 1 0 0 1 0 1
(Q13) Given a Bipolar line code PSD of a half-width rectangular pulse, $S_y(\omega)$, and an On-off line code PSD of half-width rectangular pulse, $S_x(\omega)$.
Find the PSD ratio $S_y(\omega)/S_x(\omega)$ in dB of the two line code at $\omega = \pi/T_b$.

$$S_y(\omega) = (T_b/4) \text{sinc}^2(\omega T_b/4) \sin^2(\omega T_b/2)$$

$$S_x(\omega) = (T_b/16) \text{sinc}^2(\omega T_b/4) \left[ 1 + \frac{\pi}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{T_b}\right) \right]$$

A. 0 dB
B. 1 dB
C. 4 dB
D. 6 dB

(Q14) Given a signal message $m(t) = U \cos(\omega t) + V \sin(\omega t)$.
We can rewrite $m(t) = M \cos(\omega t - \theta)$. Determine $M$ and $\theta$.

$$M, \quad \theta$$

A. $\sqrt{U^2 + V^2}$, $\tan^{-1}\left(\frac{V}{U}\right)$
B. $\sqrt{U^2 + V^2}/2$, $\tan^{-1}\left(\frac{V}{U}\right)$
C. $U + V$, $\tan^{-1}\left(\frac{V}{U}\right)$
D. $U + V/2$, $\tan^{-1}\left(\frac{V}{U}\right)$

(Q15) Which multiplexing system(s) is (are) vulnerable to the distortion where both phase and amplitude response are non-ideal?

A. FDM
B. TDM
C. Both systems
D. None of above

(Q16) Given non-ideal channel transfer function, $H(\omega) = (1 + \cos\omega)e^{-j\omega\tau}$,
$|\alpha| < 2\pi B$, $H(\omega) = 0$, $|\alpha| > 2\pi B$. Input pulse $g(t)$ is band limited to $B$ Hz.
Due to the channel distortion, determine the bandwidth (in Hz) of output signal $y(t)$.

Hint: consider the channel as a linear system and use the relationship among output, input and system transfer function.

A. $1/(2B)$
B. $1/B$
C. $B$
D. $2B$
(Q17) What method that uses to reduce the distortion caused by multi-path effects?
A. Bandpass filter
B. Uniform quantizer
C. Low noise amplifier
D. Tapped delay equalizer

(Q18) A signal input $x(t) = 100 \text{sinc}(100t)$ is passing through a nonlinear channel. Find the bandwidth in rad/sec of the input signal $x(t)$.
A. 50 rad/sec
B. 100 rad/sec
C. 200 rad/sec
D. 400 rad/sec

(Q19) From (18), the output of the nonlinear channel is given by $y(t) = x(t) + 0.01x^2(t)$. Find the spectrum of the output, $Y(\omega)$ and its bandwidth in rad/sec.

\[ Y(\omega) = \pi \text{rect}(\omega/200) + \pi \Lambda(\omega/400), \quad 200 \text{ rad/sec} \]
A. $\pi \text{rect}(\omega/200) + \pi \Lambda(\omega/400)$, 200 rad/sec
B. $\pi \text{rect}(\omega/200) + \pi \Lambda(\omega/400)$, 400 rad/sec
C. $\text{rect}(\omega/200) + 2 \Lambda(\omega/400)$, 400 rad/sec
D. $\text{rect}(\omega/200) + \Lambda(\omega/400)$, 200 rad/sec

(Q20) Find the energy of signal $g(t) = \text{sinc}(\pi t)$.

Hint: use Parseval’s theorem,
\[ E_g = \int_{-\infty}^{\infty} g^2(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega, \quad (\text{Define}, \text{sinc}(x) = \frac{\sin(x)}{x}) \]
A. 1
B. 2
C. 3
D. 4

END OF THE TEST QUESTIONS
1. B  \[ \text{Tx Power} = 10 \log 10 \text{kw} = 40 \text{ dB} \]

\[ R_x \text{ Power after amplifier} = 40 - 16 + 7 = 31 \text{ dB} \]

2. C  \[ \text{Bit rate} = \left( \frac{8 \text{ bits/channel} \times 24 \text{ channels}}{1 \text{ frame bit}} \right) \times 8 \text{ kHz} \]

\[ = 193 \text{ bits} \times 8 \text{ kHZ} \]

\[ = 1544 \text{ kbps} \]

3. A

4. B  \[ \text{SNR} = \frac{\text{Signal Power}}{N_0} = \frac{1/2}{\frac{\Delta^2 \cdot 4k}{3 \cdot 64k}} = \frac{24}{\sigma^2} \]

5. A

6. C

4. D  \[ p(t) \text{ is energy signal (Finite Energy, 0 Power)} \]

\[ R_p(\tau) = \int_{-\epsilon}^{\epsilon} p(t) p(t-\tau) \, dt \]

\[ \tau \in [-\epsilon, 0] \quad R_p(\tau) = \int_{-\epsilon}^{\tau} h_k \cdot h_k \, dt = h_k^2 (\epsilon + \tau) \]

\[ \tau \in [0, \epsilon] \quad R_p(\tau) = \int_{\tau}^{\epsilon} h_k \cdot h_k \, dt = h_k^2 (\epsilon - \tau) \]

For other \( \tau \), \( R_p(\tau) = 0 \)
8. A \( \text{PSD of } X(t), S_x(\omega) = \text{FT} (R_x(\tau)) = \)
\[
\int \sum_{n=-\infty}^{\infty} \frac{r_n}{T_b} \delta(\tau - nT_b) e^{-j\omega \tau} d\tau, \quad \text{let } \tau - nT_b = \alpha \\
= \sum_{n=-\infty}^{\infty} \frac{r_n}{T_b} \int \delta(\alpha) e^{-j\omega \alpha} d\alpha \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \leftarrow \delta(\alpha) \uparrow \downarrow d=0, \delta(1) = 1 \rightarrow \int \ldots = 1 \\
\text{with } r_n = r_{-n} \\
= \frac{r_0}{T_b} + \sum_{n=1}^{\infty} \frac{r_n}{T_b} (e^{-j\omega n T_b} + e^{j\omega n T_b}) \\
= \frac{r_0}{T_b} + \sum_{n=1}^{\infty} \frac{2r_n}{T_b} \cos(\omega n T_b) \quad \checkmark \\
\]

9. B \( p(t) = \Lambda \left( \frac{t}{T_b} \right) \quad \text{FT} \quad P(\omega) = \frac{T_b}{2} \text{sinc}^2 \left( \omega \frac{T_b}{4} \right) \quad \checkmark \)

10. B \( \text{Full-width triangular pulse line code PSD, } S_y(\omega) = \)
\[
= \left| \frac{T_b}{2} \text{sinc}^2 \left( \frac{\omega T_b}{4} \right) \right|^2 \left( 1 + 0 \right) \\
= \frac{T_b}{4} \text{sinc}^4 \left( \frac{\omega T_b}{4} \right) \quad \checkmark \\
\]

11. B

12. D
13. \[ \frac{S_y(w)}{S_x(w)} \bigg|_{w = \frac{\pi}{T_b}} = \frac{T_b}{4} \sin^2 \left(\frac{\pi}{T_b} \cdot \frac{T_b}{4}\right) \sin^2 \left(\frac{\pi}{T_b} \cdot \frac{T_b}{2}\right) = 4 \]

\[ \frac{S_y}{S_x} (\text{dB}) = 10 \log 4 = 6.02 \text{ dB} \]

14. \[ m(t) = \sqrt{U^2 + V^2} \left[ \frac{U}{\sqrt{U^2 + V^2}} \cos(ut) + \frac{V}{\sqrt{U^2 + V^2}} \sin(ut) \right] \]

\[ \cos \theta \]

\[ \sin \theta \]

Using \[ \cos(A-B) = \cos A \cos B - \sin A \sin B \]

\[ B = \tan^{-1} \left( \frac{\sin B}{\cos B} \right) \]

\[ M(t) = \sqrt{U^2 + V^2} \cos \left( wt - \tan^{-1} \frac{V}{U} \right) \]

15. B

16. C \[ Y(w) = H(w) X(w) = X(w) \cdot \left(1 + \cos w\right) e^{-j \omega t} \]

\[ \text{Product} = Bw = B \text{ Hz} \]

17. D

18. B \[ x(t) = 100 \text{sinc}(100t) \]

\[ = 100 \text{sinc}(\frac{100}{\pi} \pi t) \leftrightarrow \text{FT with scaling property} \]

\[ = \pi \text{rect} \left( \frac{\pi t}{100} \right) \]

\[ Bw = 100 \text{ rad/sec} \]
19. A

\[ y_d(t) = x(t) + 0.01 \cdot x^2(t) \]

\[ \downarrow \text{FT} \]

\[ Y(w) = X(w) + 0.01 \cdot X(w) \ast X(w) \]

From (19), \( X(w) = \pi \text{rect} \left( \frac{w}{200} \right) \),

Find \( X(w) \ast X(w) \), we can use similar approach as (7) or by graphical way,

\[
\begin{align*}
\pi & \quad \ast \quad \pi \\
-100 & \quad 100 \\
\end{align*}
\]

\[
\begin{align*}
\pi & \quad \ast \quad \pi \\
-100 & \quad 100 \\
\end{align*}
\]

value at center \( X(w) \ast X(w) \bigg|_{w=0} = \frac{1}{2\pi} \int_{-100}^{100} \pi \cdot \pi \, dw \)

\[
= \frac{\pi^2 \cdot 200}{2\pi} = 100 \pi
\]

Then \( X(w) \ast X(w) = 100 \pi \Lambda \left( \frac{w}{400} \right) = 100 \pi \left( 1 - \frac{|w|}{200} \right), |w| \leq 200 \)

Therefore, \( Y(w) = \pi \text{rect} \left( \frac{w}{200} \right) + \pi \Lambda \left( \frac{w}{400} \right) = 1 \)

20. A With Parseval's Theorem

\[
E_g = \int_{-\omega}^{\omega} \text{sinc}^2 \pi t \, dt = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \text{rect} \left( \frac{w}{2\pi} \right) \right|^2 \, dw
\]

\[
= \frac{2\pi}{2\pi} \left| \text{rect} \left( \frac{w}{2\pi} \right) \right|_{-\pi}^{\pi} = 1
\]

\[
= 1
\]