EXAM #2 EE 5361 / EE4330
INSTRUCTOR: Dr. K.R.Rao

04/06/2004 (3.30 p.m.- 4.50 p.m.)

1. Closed books and closed notes.
2. You can only use the four-page cheat sheet handout.
3. Please show all the steps in your work.
4. You may work problems in any order. At the end please arrange as 1,2,3,4,5.
5. Please print your name and last four digits of your ID.
6. Write on one side of the paper only.
7. No cheating.
8. Problems carry weights as indicated. \((EQUAL \ WEIGTHS)\)
1) A periodic square wave \( m(t) \) with a period of 1ms frequency modulates a carrier of frequency 100KHz and a maximum frequency deviation \( \Delta f = 2 \)KHz. The peak carrier amplitude is 10 volts.

*Identify all relevant points.*

The FM system is as shown below

Sketch the waveforms at points \( b, c, d, e \).
2] Consider the FM modulator with an output

\[ x_c(t) = 100 \cos (2\pi f(1000)t + \Phi(t)) \]

The modulator operates with a peak frequency deviation \( \Delta f = 40 \text{Hz} \); the input message signal is given as

\[ m(t) = 5 \cos (2\pi f(8)t) \]

The modulator is followed by a BPF centered at 1000Hz and having a bandwidth of 56 Hz.

The FM system is as shown

What is the power of the signal at the output of the filter?

*Hint: Use the table for Bessel's functions given.*
3] A NBFM to WBFM converter is implemented as shown.

A NBFM is such that the carrier frequency is 100 KHz. The peak frequency deviation is 50 Hz and the message bandwidth is 500 Hz. The WBFM signal is to have a carrier frequency of 85 MHz and a frequency deviation of 5.

a) Determine the multiplication factor ‘n’.
b) What are the two possible oscillator frequencies?
c) Determine the center frequency and bandwidth of the filter.
[4] Sketch the relevant frequency modulated signal for the following message signal. Specify the maximum and minimum frequencies of the signal and the frequencies at each of the intervals shown. Given \( k_f = 2\pi \times 10^5 \text{ rad/V} \), \( f_0 = 100 \text{ MHz} \). Explain the variation.

![Graph showing frequency modulation](image)

[5] An angle modulated signal is given by, \( \Phi_{EM}(t) = 10\cos(15000t) \). The carrier frequency, \( \omega_c = 10000 \text{ rad/sec} \).

(a) If this were a PM signal with \( k_p = 500 \), determine \( m(t) \).

(b) If this were an FM signal with \( k_f = 2000 \), determine \( m(t) \).
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#1 The FM system is as shown.

At (b) we have a FM signal given as

\[ f(t) = 100 + 2 \sin(2 \pi f_0 t) \]

where

\[ f_0 = \frac{100 - 2}{102} = 0.98 \text{ kHz} \]

At (d) we have an envelope detector.
At C the differentiated signal will have both frequency and amplitude modulation. The amplitude is given as $A\left[w + k_t m(t)\right]$. The signal at C is given as

At D the envelope detector output will look as shown.
#2 The modulation index $\beta$ of the FM signal

$$\beta = \frac{\Delta f}{f_m} = \frac{40}{8} = 5$$

The FM signal can be written as

$$\chi_c(t) = A_c \sum_{n=-\infty}^{\infty} \mathcal{J}(\beta) \cos(\omega_c + n\omega_m)t$$

The spectra would be:

![Graph showing the frequency spectrum with peaks at 9.81, 9.76, 9.84, 9.92, 10.00, 10.08, 10.16, 10.24, 10.32, 10.40, 39.1, 36.5, 32.8, 17.8, 4.7, and 2.61 Hz.]}
The power ratio is

\[ P_r = J_0^2 (5) + 2 \left[ J_1^2 (5) + J_2^2 (5) + J_3^2 (5) \right] \]

\[ = (0.178)^2 + 2 \left[ (0.328)^2 + (0.047)^2 + (0.365)^2 \right] \]

\[ = 0.518 \cdot \]

Power at output of modulator

\[ = \frac{1}{2} A_e^2 = \frac{1}{2} [100]^2 \]

\[ = 5000 \text{ Watts} \]

Power at filter output

\[ = 5000 \times 0.518 \]

\[ = 2590 \text{ Watts} \]
#3

(a) Deviation ratio of the NBFM signal

\[ \beta_1 = \frac{A_f}{f_m} = \frac{50}{500} = 0.1 \]

The multiplication factor 'n' is

\[ n = \frac{D_2}{D_1} = \frac{5}{0.1} = 50 \]

(b) The carrier frequency at output of multiplier is

\[ 50 \times 100 \text{KHz} = 5 \text{MHz} \]

The output carrier frequency = 85 MHz
The possible local oscillator frequencies are

\[
85 - 5 = 80 \text{ MHz} \\
85 + 5 = 90 \text{ MHz}
\]

(c) Center frequency of BPF = 85 MHz

The bandwidth of the filter given by Carson's Rule

\[
B = 2(\beta + 1)B = 2(\Delta f + B) = 2[5 + 1] \times 500
\]

\[
= 6000 \text{ Hz}
\]
**Solution Q#4**

\[ k_f = 2\pi \times 10^5 \text{ rad/s} \]

\[ f_c = 100 \text{ MHz} \]

(i) for \( t = 0, 1 \)

\[ f_c = f_c + \frac{k_f m(t)}{2\pi} = 100 \times 10^6 + \frac{2\pi \times 10^5 \times 0}{2\pi} = 100 \text{ MHz} \quad t = 0 \]

\[ = 100 \times 10^6 + \frac{2\pi \times 10^5 \times 2}{2\pi} = 100.2 \text{ MHz} \quad t = 1 \]

Since the signal increases linearly from 0 to 2V at times \( t = 0 \) to \( t = 1 \), the frequency of the frequency modulated signal also increases linearly from 100 MHz to 100.2 MHz during that time period.

(ii) \( t = 1, 2 \)

\[ f_c = f_c + \frac{k_f m(t)}{2\pi} = 100 \times 10^6 + \frac{2\pi \times 10^5 \times 4}{2\pi} = 100.4 \text{ MHz}, \quad 1 \leq t \leq 2 \]

The message signal doesn't change amplitude during this interval, hence the frequency of the FM modulated
The signal remains constant at 100.4 MHz during this interval.

(iii) \( t = (2, 3) \)

\[
f_2 = f_c + \frac{k_m(m-1)}{2\pi} = 100 \times 10^6 + \frac{2\pi \times 10^5 \times 3}{2\pi}
\]

\[= 100.3 \text{ MHz} \quad 2 \leq t \leq 3\]

(iv) \( t = (3, 4) \)

\[
f_3 = f_c + \frac{k_m(m-1)}{2\pi} = 100 \times 10^6 + \frac{2\pi \times 10^5 \times 2}{2\pi}
\]

\[= 100.2 \text{ MHz} \quad 3 \leq t \leq 4\]
\[ \psi_E = 10 \cos (15000t) \] -- (1)

\[ \omega_c = 10000 \text{ rad/s}, \quad k_p = 500, \quad k_f = 2000 \]

(a) PM signal is given by,
\[ \psi_{PM} = A \cos (\omega_c t + k_p m(t)) \] -- (2)

Comparing (1) and (2), we see that
\[ k_p m(t) = 5000t \]
\[ \Rightarrow m(t) = \frac{5000t}{500} = 10t \]

(b) FM signal is given by
\[ \psi_{FM} = A \cos (\omega_c t + k_f \int m(t) \, dt) \] -- (3)

Comparing (1) and (3), we get
\[ k_f \int m(t) \, dt = \frac{5000t}{2000} = 2.5t \]
\[ \therefore m(t) = \frac{d(2.5t)}{dt} \]
\[ = 2.5 \]