1) You are working as a consultant for a communication system design firm and you are asked to
design a communication system, which uses analog modulation. You are asked to investigate the
following analog modulation schemes.
In each case the message signal \( g(t) \) is strictly band limited to 7kHz, (i.e. \( B=7kHz \)) and the carrier
signal is a cosine wave with unit amplitude and carrier frequency \( f_c = 11kHz \) (i.e. carrier signal is
\( \cos(2\pi \times 11000t) \)). The frequency spectrum of the message signal is shown in figure 1.1.

(a) The first analog communication scheme uses Single Side Band-Suppressed Carrier (SSB-SC)
    modulation. Sketch the frequency spectrum of the SSB-SC modulated signal. Use upper side band.
(b) The second analog modulation scheme uses Double side Band with carrier, commonly referred to
    as AM. What is the Fourier transform of the AM signal? Sketch the frequency spectrum of the
    AM modulated signal.
(c) Assume that \( g(t) \) ranges from -2 to +2. Can you recover the message signal \( g(t) \) from the
    modulated signal in part (b) using an envelope detector? Why/Why not? Justify.

Solution:
(a) Single Side Band Suppressed Carrier (SSB-SC)
   \[ \Phi_{USB}(\omega) = G_c(\omega - \omega_c) + G_c(\omega + \omega_c) \]
   The frequency spectrum of the SSB-SC signal is given in figure S1.1

(b) AM: The AM signal is given as:
    \[ = (1 + g(t)) \cos \omega_c t \]
    \[ = \cos \omega_c t + g(t) \cos \omega_c t \]
The Fourier transform of the AM signal is given below:
\[ \omega_c = 2\pi \nu_k \]

\[ = \pi \left[ \delta(\omega - \omega_c) + \delta(\omega + \omega_c) \right] + \frac{1}{2} \left[ G(\omega - \omega_c) + G(\omega + \omega_c) \right] \]

The frequency spectrum of the AM signal is given in figure S1.2

(c) The peak value of \( g(t) \) is 2 and the peak value of the carrier = \( A = 1 \)
   For envelope detection, \( A + g(t) \geq 0 \), \( \forall t \)
   Positive peak of \( g(t) = 2 \).
   Therefore, \( A + g(t) = 1 + 2 \geq 0 \)
   Negative peak of \( g(t) = -2 \).
   Therefore, \( A + g(t) = 1 - 2 < 0 \).
   Therefore, envelope detection is not possible.

2) Consider a Vestigial Side Band Suppressed Carrier modulation with the transmit filter whose
   frequency spectrum is given in figure 2.1. The message signal is a base band signal, band limited
to 7kHz. The frequency spectrum of the message signal is given in figure 2.2. The carrier signal is
a cosine wave with unit amplitude and carrier frequency \( f_c = 11kHz \) (i.e. carrier signal is
\( \cos(2\pi \times 11000t) \)).
(a) Find (sketch) the frequency spectrum of the equalizer filter to be used at the receiver. Sketch all intermediate steps explaining your end result.
(b) What is the bandwidth of the modulated signal?
(c) What is the percentage increase in bandwidth as compared to SSB modulation?

Solution:
(a) The equalizer filter is given as:

\[ H_e(\omega) = \begin{cases} 
1 & |\omega| \leq 2\pi B \\
0 & |\omega| > 2\pi B 
\end{cases} \]

\( H_e(\omega + \omega_c) + H_e(\omega - \omega_c) \) is given in figure S2.1

We now need to find \( H_e(\omega) \), and hence we need to invert or take the reciprocal of the spectrum in the figure S2.1. This is shown in figure S2.2 below. (Figure not drawn to scale)
(b) Bandwidth:

The VSB signal is given by:

$$\Phi_{VSB}(\omega) = [G(\omega + \omega_c) + G(\omega - \omega_c)]H_1(\omega)$$

The first term in the product is a DSB signal and is represented in figure S2.3. The second term is the VSB filter, which is represented in figure S2.4. The product of the two terms will result in the VSB signal and will have a bandwidth of 8 KHz.

Therefore, $BW_{VSB} = 8$ KHz

(c) BW of SSB signal = BW of message signal = 7 KHz = $2\pi(7)$ rad/sec

BW of VSB signal = 8 KHz = $2\pi(8)$ rad/sec

Therefore, \% increase in bandwidth = \frac{8 - 7}{7} \times 100\% = 14\%

3) Consider the system shown in figure 3.1. Assume that the average value of $m(t)$ is zero and that the maximum value of $|m(t)|$ is $M$. The frequency spectrum of $m(t)$ is shown in figure 3.2. Also assume that the square law device is defined by

$$y(t) = 4x(t) + 2x^2(t)$$

![Figure 3.1](image-url)
(a) Write the equation for \( y(t) \), and draw its frequency spectrum.

(b) Describe the filter that yields an AM signal for \( g(t) \). Give the necessary filter type. Draw the frequency response of the filter assuming it to be ideal. Clearly mark the frequencies of interest.

(c) What value of \( M \) yields a modulation index of 0.17?

Solution:

(a) \[ y(t) = 4x(t) + 2x^2(t) \]
\[ x(t) = m(t) + \cos \omega_c t \]
\[ y(t) = 4[m(t) + \cos \omega_c t] + 2[m^2(t) + \cos^2 \omega_c t] \]
\[ = 4m(t) + 4\cos \omega_c t + 2m^2(t) + \cos^2 \omega_c t + 2m(t)\cos \omega_c t \]
\[ = 4m(t) + 4\cos \omega_c t + 2m^2(t) + 2\cos^2 \omega_c t + 4\cos \omega_c t \]
\[ = 4m(t) + 4\cos \omega_c t + 2m^2(t) + 4\cos \omega_c t + 1 + \cos 2\omega_c t \]

Fourier Transform of the above signal is:

\[ Y(\omega) = 4M(\omega) + 2[M(\omega - \omega_c) + M(\omega + \omega_c)] + 4\pi[M(\omega) * M(\omega)] + 4\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] \]

\[ + 2\pi \delta(\omega) + \pi[\delta(\omega - 2\omega_c) + \delta(\omega + 2\omega_c)] \]

The Frequency Spectrum of the signal is given in figure S3.1.

(b) \( g(t) \) should be an AM signal i.e., \( g(t) \) should be of the form \( [A + m(t)]\cos \omega_c t \)

The output of the square law device, \( y(t) \) is given below:

\[ y(t) = 4m(t) + 4\cos \omega_c t + 2m^2(t) + 2\cos^2 \omega_c t + 4m(t)\cos \omega_c t \]

\[ = 4[1 + m(t)]\cos \omega_c t + 4m(t) + 2m^2(t) + 1 + \cos 2\omega_c t \]
The frequency spectrum of this signal is given in figure S3.1. From the figure it is observed that a band pass filter centered at \( \omega_c \), with bandwidth \( 4\pi B \) is needed to obtain \( g(t) \) at the output.

The frequency response of the band pass filter assuming it to be ideal is given in figure S3.2.

\[
\begin{align*}
M &= \mu = 0.1 \\
\lambda &= \lambda \mu \\
A &= 1 \\
\therefore M &= 0.1
\end{align*}
\]

4) In this problem we examine the efficiency of AM modulation for the case in which the message signal does not have symmetrical maximum and minimum values. Two message signals are shown in figure below. Each is periodic with period \( T \) and \( \tau \) is chosen such that the dc value of \( m(t) \) is zero. Calculate the efficiency for each \( m(t) \) for \( A=1 \).

Hint: For the DC value of the signal to be zero, \( \tau = (S/6)T \)

Fig 4.1

Solution:

\[
\eta = \frac{\text{Useful Power}}{\text{Total Power}} = \frac{P_s}{P_s + P_c} = \frac{\overline{m^2(t)}/2}{A^2/2 + \overline{m^2(t)}/2} \times 100\% = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}} \times 100\%
\]

Now, \( A=1 \),
case (i)

\[ m^2(t) = \frac{(-1)^2(t) + (5)^2(T-t)}{T} = \frac{1 \times \frac{5}{6} T + 25 \times \left( T - \frac{5}{6} T \right)}{T} = 5 \]

\[ \eta = \frac{5}{1+5} \times 100\% = 83.3\% \]

case (ii)

\[ m^2(t) = \frac{(1)^2(t) + (-5)^2(T-t)}{T} = \frac{1 \times \frac{5}{6} T + 25 \times \left( T - \frac{5}{6} T \right)}{T} = 5 \]

\[ \eta = \frac{5}{1+5} \times 100\% = 83.3\% \]

Efficiency remains the same in both cases.

\[ \mu^2(t) \]

\[ \begin{array}{c}
1 \\
\hline
0 \quad \frac{5}{6} T \quad T \\
25 \\
\end{array} \]

\[ \mu^2(T) = \frac{1}{T} \left( \frac{5}{6} T + \frac{25}{6} T \right) = 5 \]

\[ \eta = \frac{5}{5+1} = \frac{5}{6} = 83.3\% \]