1. Closed books and closed notes.
2. You can only use the four-page cheat sheet handout.
3. Please show all the steps in your work.
4. You can work problems in any order. At the end please rearrange as 1, 2, 3, 4
5. No one can leave the classroom between 4:50 and 5:00 pm. We will collect your solutions at the end.
6. Please print your name, last four digits of your ID and indicate your course number (4330 or 5361).
7. Show your work on one side of the paper only.
8. No cheating. No talking.
10. Please turn off all cell phones.

The carrier frequency of a certain VSB signal is \( f_c = 40 \) KHz and the baseband signal bandwidth is 9 KHz. The shaping filter \( H(f) \) at the input, which cuts off the lower sideband gradually over 3 KHz is shown below. Find the equalizer filter \( H_e(f) \) at the receiver output.

(Sketch of \( |H_e(f)| \) and the relevant points should be marked in the frequency spectrum.)
2. A message signal \( m(t) = 50 \cos(200 \pi t) \) modulates a carrier \( c(t) = 25 \cos 500\pi t \).

(a) What is the modulation index of the modulated AM signal. Can the message signal be recovered from the AM signal using envelope detection? Justify.

(b) What should the carrier amplitude be changed to, in order to obtain a power efficiency of 30%?

(c) In part (a), if \( m(t) \) cannot be recovered by envelope detection, then what other scheme can be employed to demodulate the signal? Design the demodulator. Justify your answer.

3. The magnitude frequency spectra of the baseband signals \( m_1(t) \) and \( m_2(t) \) are given below:

The scheme used to modulate these signals and simultaneously transmit them over the same channel is shown below.

(a) For the modulator shown below, sketch the frequency spectra at points a, b, c and d. Identify all relevant points.

(b) What should be the channel bandwidth (in Hz) in order to transmit the modulated signal without any distortion?

\[
\omega_1 = 10^9 \text{ rad/s} \quad \omega_2 = 5 \times 10^9 \text{ rad/s} \quad \omega_3 = 3 \times 10^9 \text{ rad/s}
\]

\[
= 10^9 \text{ rad/s}
\]
4.) It is required to design a DSB-SC modulator to generate a modulated signal \( km(t)\cos1000\pi t \), where \( m(t) \) is a signal band limited to 100Hz, and has a magnitude frequency spectrum shown below.

The only carrier generator available gives an output of \( \cos^31000\pi t \). The block diagram of the modulator is shown below:

(a) What kind of filter would you choose to obtain the required modulated signal at point b? Sketch the frequency response of the filter. (Assume the filter to be ideal)

(b) Determine the signal spectra at points a and b and indicate the frequency bands occupied by these spectra.

(c) Would this scheme work if the carrier generator output were \( \cos^2500\pi t \) instead of \( \cos^31000\pi t \). Justify by showing the frequency spectra.

Hints:

\[
\cos^3 \theta = \frac{1}{4}(3\cos \theta + \cos 3\theta)
\]

\[
\cos^3 \theta = \frac{1}{2}(\cos 2\theta + 1)
\]
2. A message signal \( m(t) = 50 \cos(200\pi t) \) modulates a carrier \( c(t) = 25 \cos(500\pi t) \).
   (a) What is the modulation index of the modulated AM signal. Can the message signal be recovered from the AM signal using envelope detection? Justify.
   (b) What should the carrier amplitude be changed to, in order to obtain a power efficiency of 30%?
   (c) In part (a), if \( m(t) \) cannot be recovered by envelope detection, then what other scheme can be employed to demodulate the signal? Design the demodulator. Justify your answer.

Solution:
(a) \( c(t) = 25 \cos(500\pi t) \)
\[ m(t) = 50 \cos(200\pi t) \]
\[ \mu = \frac{m_p}{A} = \frac{50}{25} = 2 \]

For viability of demodulation of AM by envelope detection, \( 0 < \mu < 1 \). Since the modulation index here is greater than 1, the signal is clearly over-modulated and hence cannot be recovered by envelope detection.

(b) Carrier efficiency required by the system = \( \eta = 30\% \)
For tone modulation, the power efficiency is given by:
\[ \eta = \frac{\mu^2}{2 + \mu^2} \times 100 \geq 30\% \]
\[ 100\mu^2 = 30 \times 2 + 30\mu^2 \]
\[ \mu^2 = \frac{60}{70} \]
\[ \mu = 0.9258 \]
\[ \mu = \frac{m_p}{A} \]
\[ A = \frac{m_p}{\mu} = \frac{50}{0.9258} \approx 54 \]

Hence, for 30% efficiency, the carrier amplitude should be 54.
The signal can be demodulated using synchronous demodulation using the scheme given below:

\[ (A + m(t)) \cos 500\pi t \rightarrow \text{LPF} \rightarrow C \rightarrow m(t) \]

\[ \cos 500\pi t \]

The input AM signal to the input of the demodulator is:

\[ [25 + 50\cos 200\pi t] \cos 500\pi t \]

\[ = 25\cos 500\pi t + 25\cos 700\pi t + 25\cos 300\pi t \]

At the demodulator, the incoming AM signal is multiplied by a locally generated carrier signal which is in frequency and phase synchronism with the carrier at the transmitter.

The output of the multiplier is:

\[ \cos 500\pi t [25\cos 500\pi t + 25\cos 700\pi t + 25\cos 300\pi t] \]

\[ = 25\cos^2 500\pi t + \frac{25}{2} [\cos 200\pi t + \cos 1200\pi t] + \frac{25}{2} [\cos 200\pi t + \cos 800\pi t] \]

\[ = 25\cos^2 500\pi t + 25\cos 200\pi t + \frac{25}{2} [\cos 800\pi t + \cos 1200\pi t] \]

\[ = \frac{25}{2} [1 + \cos 1000\pi t] + 25\cos 200\pi t + \frac{25}{2} [\cos 800\pi t + \cos 1200\pi t] \]

The signal is passed through a low pass filter with a bandwidth of 1000Hz. The filter suppresses the high frequency components of the signal and the output of the filter obtained is:

\[ \frac{25}{2} + 25\cos 200\pi t \]

It is observed that an unwanted dc term is present which can be removed using a blocking capacitor C. The output of the receiver thus designed is the message signal \( m(t) \).

The magnitude frequency spectra of the baseband signals \( m_1(t) \) and \( m_2(t) \) are given below:

\[ M_1(\omega) \]

\[ M_2(\omega) \]
The scheme used to modulate these signals and simultaneously transmit them over the same channel is shown below.

(a) For the modulator shown below, sketch the frequency spectra at points a, b, c and d. Identify all relevant points.

(b) What should be the channel bandwidth (in Hz) in order to transmit the modulated signal without any distortion?

\[ m_1(t) \quad \times \quad a \quad \sum \quad \times \quad \times \quad b \quad \times \quad c \quad \times \quad d \quad \text{Channel} \]

\[ 2 \cos \omega_1 t \]

\[ m_2(t) \]

\[ 2 \sin \omega_2 t \]

\[ 2 \cos \omega_3 t \]

\[ \omega_1 = 10 \text{ G rad/sec} \]

\[ \omega_2 = 5 \text{ G rad/sec} \]

\[ \omega_3 = 30 \text{ G rad/sec} \]

1 G rad/sec = 1 Giga radians/sec = \(1 \times 10^9\) radians/sec

**Solution:**

The frequency spectra at points a, b, c and d are shown below:

At point a:

Signal at point a: \(2 \cos (10G) t \cdot m_1(t)\)
At point b:
Signal at point b is: \(2\sin(5G)t \cdot m_2(t)\)

At point c:
Signal at point c: \(2\cos(10G)t \cdot m_1(t) + 2\sin(5G)t \cdot m_2(t)\)

At point d:
Signal at point d: \(2\cos(30G)t \cdot [2\cos(10G)t \cdot m_1(t) + 2\sin(5G)t \cdot m_2(t)]\)

At point d, only the positive frequencies are sketched in order to make the sketch simpler.

\(\text{c) Channel BW } \nu_{\text{req, } y} = (40G^4 + 400\pi) - (20G^4 - 400\pi)\)
\(= (20G^4 + 800\pi) \frac{\text{val}}{\text{sec}} = 3.183 G^4 \text{ Hz}\)
(c) Channel Bandwidth required = (40G+400π)-(20G-400π) = 20G+800π rad/sec
   = (20G+800π)/2π = 3.183 GHz

4) It is required to design a DSB-SC modulator to generate a modulated signal \( km(t)\cos 1000\pi t \), where \( m(t) \) is a signal band limited to 100Hz, and has a magnitude frequency spectrum shown below.

The only carrier generator available gives an output of \( \cos^3 1000\pi t \). The block diagram of the modulator is shown below:

(a) What kind of filter would you choose to obtain the required modulated signal at point b? Sketch the frequency response of the filter. (Assume the filter to be ideal)
(b) Determine the signal spectra at points a and b and indicate the frequency bands occupied by these spectra.
(c) Would this scheme work if the carrier generator output were \( \cos^3 500\pi t \) instead of \( \cos^3 1000\pi t \). Justify by showing the frequency spectra.

Hints:
\[
\cos^3 \theta = \frac{1}{4}(3\cos \theta + \cos 3\theta) \\
\cos^2 \theta = \frac{1}{2} (\cos 2\theta + 1)
\]
Solution:

(a) The signal at point a is:

\[ g_a(t) = \cos^3(1000\pi t) \]

\[ = m(t) \left[ \frac{3}{4} \cos(1000\pi t) + \frac{1}{4} \cos(3000\pi t) \right] \]

The term \((3/4)m(t)\cos(1000\pi t)\) is the desired modulated signal, whose spectrum is centered at \(\pm1000\pi\) rad/sec. The remaining term \((1/4)m(t)\cos(3000\pi t)\) is the unwanted term, which represents the modulated signal with carrier frequency \(3000\pi\) rad/sec with spectrum centered at \(\pm3000\pi\) rad/sec. The band pass filter centered at \(\pm1000\pi\) rad/sec allows passing the desired term, but suppresses the unwanted term. The frequency response of this filter assuming it to be ideal is given in figure 3-a-1 below.

![Figure 3-a-1](image)

(b) Figures 3-b-1 and 3-b-2 show the signal spectra at points a and b respectively.

At point a:

![Figure 3-b-1](image)
At point b:

(c) If the carrier generator output were \( \cos^2 500\pi \), the signal at point a is:

\[
m(t)\cos^2 500\pi = \frac{m(t)}{2} [1 + \cos 1000\pi t] = \frac{1}{2} m(t) + \frac{1}{2} m(t)\cos 1000\pi t
\]

This signal consists of the base band signal \( \frac{1}{2} m(t) \) and a modulated signal, \( \frac{1}{2} m(t)\cos 1000\pi t \), which is the required modulated signal. The filter will suppress the base band component and pass the modulated signal. Hence this scheme would certainly work if the output of the carrier generator were \( \cos^2 500\pi \). The frequency spectra is shown below:

![Diagram of frequency spectra]

\( H(f + fc) \)
From $H_d(f) = \begin{cases} 1 & f \leq B \\ \frac{H_0(f + f_c) + H_0(f - f_c)}{0} & f > B \end{cases}$

\[
\left( \frac{1}{H_d(f)} \right) = \left[ H_0(f + f_c) + H_0(f - f_c) \right]
\]

Suppress freq. spectra around $2\omega_c$, i.e., 80 KHz