Abstract—In this paper, a new statistical model is proposed for modeling the nature images in the transform domain. We demonstrate that the von Mises distribution (VM) fits accurately the behaviors of relative phases in the complex directional wavelet subband from different nature images. Moreover, a new image feature based on the VM model is proposed for texture image retrieval application. The VM based feature yields higher retrieval accuracy compared to the energy features and the relative phase features. In addition to magnitude information typically used in many other feature extraction methods, the VM based phase information is also incorporated to further improve the performance.

I. INTRODUCTION

Many applications in image processing such as image compression, image denoising or image retrieval can benefit from a statistical model to characterize the image in the transform domain. A clean, precise probability model which can describe sufficiently typical images becomes essential. In this paper, the von Mises model for relative phases of the complex directional wavelet coefficients is proposed for image modeling and its application in the transform image retrieval is also.

There are many works on the statistics of decomposition coefficients of the wavelet transform [1]. The wavelet coefficients within a subband were often assumed to be independent and identically distributed. With this assumption, the wavelet coefficients can be modeled by the marginal model whose distribution is a two-parameter generalized Gaussian density (GGD). The GGD is a suitable distribution for the peaky and heavy-tailed non-Gaussian statistic of typical image wavelet decomposition. A number of researchers have successfully developed joint statistical models in wavelet domain [2][3]. The Hidden Markov Tree was introduced in [2] to model the wavelet decomposition. A bivariate probability density function has been proposed to model the statistical dependencies between a wavelet coefficient and its parent [4]. In [3], the authors developed a model for the neighborhoods of oriented pyramid coefficients based on a Gaussian scale mixture (GSM) which is the product of a Gaussian random vector and an independent random scalar multiplier. This model can account for both marginal and pairwise joint distributions of wavelet coefficients.

In most of the above statistical model, though the phase holds the crucial information about image structures and features [5], only the real part or the magnitude of the transform coefficients is modeled and used for image processing applications. Some previous works employ phase information of the complex wavelet for some applications. The image features such as edges and shadows are determined by analyzing the phase of the harmonic components [6] or computing the phase congruency [7]. Some other applications exploit the local phase information across scales of the complex wavelet such as the description of image texture [8], the detection of blurred images [9] and object recognition [10]. The investigation of local phase in the same orientation and the same scale is based on the dual-tree complex wavelet transform [11] and the complex directional filter bank (CDFB) [12]. Therefore an accurate statistical model of the phase of the complex wavelet coefficients can be beneficial to the developments in the image processing community. In particular, in this paper we propose a model of the phase difference of neighboring complex wavelet coefficients called relative phase (RP), which has been successfully applied in order to classify texture images [12]. In simulations, we show that the proposed model further improves the classification rate.

The paper is organized as follows. The probability model for RPs in a complex directional wavelet subband is described in Section II. The procedure to retrieval texture images and the experimental results are presented in Section III. We conclude the paper in Section IV.

II. RELATIVE PHASES MODELED USING VON MISES DISTRIBUTION

Our objective in this section is to find a statistical model which is able to accurately capture the phase information in the complex directional wavelet domain, and is also able to sufficiently describe the nature images.

A. Relative phases in complex directional wavelet domain

The RP of the coefficients within a complex directional subband at a spatial location \((i, j)\) is given as [12]

\[
\theta_{sk}(i, j) = \angle y_{sk}(i, j) - \angle y_{sk}(i, j + 1),
\]

or

\[
\theta_{ak}(i, j) = \angle y_{ak}(i, j) - \angle y_{ak}(i + 1, j),
\]

where \(y_{sk}(i, j)\) is the coefficient at position \((i, j)\) at scale \(s \in \{1, 2, ..., S\}\) and orientation \(k \in \{1, 2, ..., K\}\) and \(\angle\) denotes the phase.

Fig. 1. The phase histogram at a particular complex wavelet subband for texture image 'Bark.0000'. (a) the phases of curvelet coefficients, (b) the RP phases.

Fig. 1(a) shows the uniform distribution of the modified curvelet (See Sec III) phases of the texture 'Bark.0000' in the VisTex collection at the finest scale \(s = 1\) and orientation \(k = 4\).
This uniform distribution of curvelet phases can not inform us any information of the image, while the distribution of the curvelet RPs has a particular shape as in Fig. 1(b). It has been shown in [12] that the RP has a linear relationship with the angle of dominant orientation within a subband. Hence the RP can be used to characterize the orientations of edges in an image.

B. Von Mises distribution

An angular random variable $\theta$ has the von Mises distribution with the parameters $\mu$ and $\nu$, $VM(\mu, \nu)$ [13] if its probability density function (PDF) has the form

$$p(\theta; \mu, \nu) = \frac{1}{2\pi I_0(\nu)} e^{\nu \cos(\theta - \mu)}, \quad (3)$$

where $I_0(\nu)$ denotes the modified Bessel function of the first kind and the zero-th order which can be defined by $I_0(\nu) = \int_0^\pi e^{\nu \cos \theta} d\theta$. The parameter $\mu$ is the mean direction and the parameter $\nu$ is known as the concentration parameter. Note that $VM(\mu, \nu)$ and $VM(\mu + \pi, -\nu)$ are the same distribution. For our model, we set the values of $\nu$ to be non-negative, and hence the range of $\theta$ is $[-\pi, \pi]$.

The von Mises distribution fits well with the marginal distribution of the RPs in complex subbands. Fig. 2 shows an empirical histogram of RPs in a particular complex wavelet subband for three different images. Below each plot are the estimated parameter values, and the relative entropy $\Delta H$ (KLD) between the histogram (with 32 bins) and the model, as a fraction of the histogram entropy $H$.

\[ \begin{align*}
(a) & \text{Lena (512 } \times 512) \\
& [\mu, \nu] = [2.35, 0.71] \\
& \Delta H/H = 0.0016
\end{align*} \]

\[ \begin{align*}
(b) & \text{House (256 } \times 256) \\
& [\mu, \nu] = [-0.18367, 1.94] \\
& \Delta H/H = 0.0048
\end{align*} \]

\[ \begin{align*}
(c) & \text{Leaves.0011 (128 } \times 128) \\
& [\mu, \nu] = [-2.07, 1.67] \\
& \Delta H/H = 0.0079
\end{align*} \]

Fig. 2. VMs fitted to the empirical histograms. Below each plot are the estimated parameter values, and the relative entropy $\Delta H$ (KLD) between the histogram (with 32 bins) and the model, as a fraction of the histogram entropy $H$.

III. Applications

In this section, the von Mises model is applied to texture image retrieval. A comparison of the VM model, and the RP-and-magnitude-based extraction schemes [12] using curvelet transform in texture retrieval is presented. We also include two other multiresolution directional decompositions in feature extraction, namely Gabor decomposition [14] and CDFB [12]. The latter is very similar to the curvelet transform in terms of directionality of the filters and one-side support in the frequency domain.

A. An implementation of the curvelet transform

In this section we briefly described a modified multiresolution and multidirectional discrete transform that will be used in later experiments. Our transform borrows the ideas from two recently introduced discrete transforms, which are the pyramidal dual-tree directional filter bank (PDCTFDB) [15] and the fast discrete curvelet transform [16]. Our transform is essentially a filter bank implementation in the frequency domain.

We define in the frequency plane a set of $N$ 2-D directional filters $\phi_i(z), i = 1, \ldots, N$ and a lowpass filter $\phi_0(z)$ in such a way that the directional subbands and the lowpass subband can be decimated without aliasing. The decimation ratio for the lowpass band is $diag\{2, 2\}$, and the decimation ratios for the first and second $N/2$ directional filters are $diag\{M/2, 2\}$ and $diag\{2, M/2\}$, respectively, where $M$ is the power of two number and is linearly proportional to $N$. The defined filters in the frequency domain are real-valued functions and satisfy the perfect reconstruction conditions, taking into account the decimation ratios: $\frac{1}{2} \phi_0(\omega) + \frac{1}{M} \sum_{i=1}^N \phi_i(2\omega) + \phi_i(-2\omega) = 1$.

Similar to the PDCTFDB, our directional filters have one-side support in the frequency domain, making the subband coefficients complex. The same transform with different values of $M$ and $N$ is applied iteratively at the lowpass subband to create a multiresolution decomposition. In the reconstruction procedure, the final complex components are simply discarded. We can interpret this as a dual-tree FB structure [17].

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Our discrete transform also has some similarities to the wrapping-based fast discrete curvelet transform (FDCT) [16] in the sense that both are defined based on windowing in the DFT domain. The main difference is that in the wrapping-based FDCT, the redundancy of the transform is reduced by wrapping the frequency domain of the subbands, while in our implementation, the redundancy is reduced by decimating the subbands by diagonal integer matrices. By this construction, our curvelet basis functions are located on a uniform integer grid at each resolution, while the basis of the FDCT is located on a non-integer grid. Fig. 3 is an example of the zoneplate image decomposed by our discrete curvelet transform. The decomposition has three directional scales, with \( N = 6, 12, 24 \) and \( M = 4, 8 \) and 16 at three resolutions. The Fig. 3(c) shows the magnitude of the complex coefficients in the transform domain.

**B. Texture feature extraction**

Each image in the database is decomposed using the following three decompositions: the curvelet, the 2-D Gabor transform and the CDFB. The Gabor wavelet and the curvelet are applied with four scales and six orientations per scale, while the CDFB has three scales of eight orientations. For each subband, the mean and standard deviation of the absolute values of the coefficients are calculated as in [14].

To construct the RP based feature vectors, the RP matrix of each complex subband is created as in (1). The circular mean and the standard deviation of the absolute values of the coefficients are calculated as

\[
m_k(y) = \tan^{-1}\left(\frac{\sum_{i,j} \sin \theta(i,j)}{\sum_{i,j} \cos \theta(i,j)}\right), \quad \text{and} \quad \sigma_k(y) = \sqrt{\frac{\left(\sum_{i,j} \sin \theta(i,j)\right)^2 + \left(\sum_{i,j} \cos \theta(i,j)\right)^2}{L}},
\]

where \( \theta(i,j) \) is the \((i,j)\)th element in the RP matrix and \( L \) is the number of elements.

For each RP matrix, the two parameters \( \mu \) and \( \nu \) of the VM model are estimated by fitting the RP histogram and the VM density function. These parameters are used to form the VM model based feature vector. For the first experiment (results shown in Table I), the feature vectors are formed from the six finest subbands of the curvelet transform. The feature vector length is twelve because each subband is represented by two parameters.

In the second experiment (results shown in Table II), the Gabor feature is formed as in [14] and the CDFB-RP feature vector is formed as in [12]. In order to obtain a feature vector which has the same dimension as that of the Gabor and the CDFB-RP, the Cur-RP feature vector is formed by twenty-four means of the magnitudes of the curvelet coefficients (all subbands), eighteen circular means of RPs (from finest subbands) and six circular variances of RPs (from finest subbands). However, the magnitude-and-VM-based feature vector (Cur-VM) is created by twenty four features of the magnitudes, the VM model in six finest subbands, and eighteen means of the magnitudes of the rest subbands.

**C. Texture image database and feature database**

The texture database used in the experiment contains 40 images from the VisTex databases used in [18]. Each of these 512 \( \times \) 512 images is divided into sixteen 128 \( \times \) 128 non-overlapping sub-images, thus creating a database of 640 texture samples. Each original image is treated as a single class and therefore there are 16 samples from each of the 40 classes. To reduce the intensity correlation, all images are normalized to have zero mean and unit variance. For each image in the database, all the three decompositions are applied. The RP matrix of each subband is created as in (1), and their corresponding feature vectors are computed.

**D. Distance measure and query processing**

The query pattern can be any one of the texture patterns from the image database. Let \( f_x \) and \( f_y \) be two feature vectors obtained from the magnitude of coefficients or the RPs. The distance between them is defined by

\[
d(f_x, f_y) = \sum_k \left(\frac{m_k(x) - m_k(y)}{\alpha(m_k)} + \frac{\sigma_k(x) - \sigma_k(y)}{\alpha(\sigma_k)}\right),
\]

where \( \alpha(m_k) \) and \( \alpha(\sigma_k) \) are the standard deviations of \( m_k(\cdot) \) and \( \sigma_k(\cdot) \) of the entire database.

The distance between two VM feature vectors \( f_x \) and \( f_y \) is given by

\[
d(f_x, f_y) = \sum_k D_{KL}(p(\cdot; \mu_k(x), \nu_k(x)) || p(\cdot; \mu_k(y), \nu_k(y))),
\]

where \( D_{KL} \) is defined in (5).

For each query image, \( N \) nearest neighbors are selected, and the number of these textures belonging to the same class as the query texture, except for itself, is counted. This number (less than or equal to fifteen) is divided by fifteen is defined as the retrieval rate. The performance of the entire class is obtained by averaging this rate over the sixteen members which belong to the same class of texture. The average of all classes is the overall performance of the method.

**E. Experimental results**

Table I summarizes the overall retrieval rates using the curvelet transform with various features extracted from the finest subbands. If only the top 15 texture images that are nearest to the query texture are
TABLE I
AVERAGE RETRIEVAL ACCURACY OF 40 TEXTURE IMAGES USING CURVELET TRANSFORM WITH VARIOUS FEATURES EXTRACTED FROM SIX FINEST SUBBANDS. (Mag denotes the magnitude feature)

<table>
<thead>
<tr>
<th>Feature length</th>
<th>Mag</th>
<th>RP</th>
<th>VM</th>
<th>Mag-RP</th>
<th>Mag-VM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feature type</td>
<td>w, σ</td>
<td>m_c, σ_c</td>
<td>μ, ν</td>
<td>w, σ</td>
<td>m_c, σ_c</td>
</tr>
<tr>
<td>N = 15</td>
<td>62.07</td>
<td>60.29</td>
<td>68.90</td>
<td>74.68</td>
<td>77.75</td>
</tr>
<tr>
<td>N = 20</td>
<td>68.0</td>
<td>66.82</td>
<td>75.17</td>
<td>80.57</td>
<td>81.96</td>
</tr>
<tr>
<td>N = 25</td>
<td>71.6</td>
<td>71.11</td>
<td>79.07</td>
<td>83.57</td>
<td>84.40</td>
</tr>
<tr>
<td>N = 30</td>
<td>74.6</td>
<td>74.93</td>
<td>81.52</td>
<td>85.74</td>
<td>86.19</td>
</tr>
</tbody>
</table>

TABLE II
AVERAGE RETRIEVAL ACCURACY OF 40 TEXTURE IMAGES IN THE VISTEX DATABASE USING VARIOUS TRANSFORMS

<table>
<thead>
<tr>
<th>Feature length</th>
<th>Gabor</th>
<th>CDFB-RP</th>
<th>Cur-RP</th>
<th>Cur-VM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redundant ratio</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>42</td>
</tr>
<tr>
<td>N = 15</td>
<td>80.81</td>
<td>82.26</td>
<td>82.80</td>
<td>84.83</td>
</tr>
</tbody>
</table>

considered, and only 12 features are used, the VM feature gives the best overall retrieval performance of 68.90 %, while the magnitude feature and the RP feature are at 62.07 % and 60.29 %, respectively. Fig. 4 shows the overall performances for the case of N from 15 to 65. It is clear that the feature vector based on the VM model is consistently better than the magnitude feature and the RP feature. This confirms that the behavior of the RPs is captured accurately by the von Mises distribution. If the magnitude feature and the VM feature are combined, the overall retrieval accuracy of the Mag-VM feature is also higher than the Mag-RP feature which is formed by the magnitude and the RP features (m_c and σ_c) as proposed in [12] for the case of N from 15 to 30 as shown in Table I.

In the second experiment, we compare our VM-model based features using the modified discrete curvelet transform with the Gabor and CDFB. If only the top 15 texture images nearest to the query texture are considered, the Cur-VM gives the best overall retrieval performance of 84.83 % as shown in Table II. The CDFB-RP [12] and the Cur-RP are at 82.26 % and 82.80 %, while the magnitude based Gabor [14] is at 80.81 %. It is clear that the information gained from the VM phase model raises the performance of the Cur-VM significantly higher than those of other features.

IV. CONCLUSION

A new statistical model is proposed for modeling the phase distribution in the transform domain. From experiments, the von Mises distribution captures the behaviors of RPs in the complex directional wavelet subbands of different nature images. Moreover, a new image feature based on the VM model is proposed for texture image retrieval application. The VM model describes nicely the directional information from texture images because higher image retrieval accuracy is achieved by using the VM model instead of using the magnitude [14] or the RP parameters [12]. In addition to magnitude information typically used in many other feature extraction methods, the VM-based phase information is incorporated to further improve the performance. In this paper, the phase information has been explicitly and successfully modeled and utilized for the application of texture image retrieval. It would be interesting to see if phase information can be beneficial in other image processing applications.

REFERENCES