Transistor Amplifiers

Section 3

Outline

Single Transistor Connections

CE  BJT
CS  NMOS
CB  BJT
CG  NMOS
CC (Emitter Follower)  BJT
CD (Source Follower)

Two Transistor Amplifiers

Darlington Pair
Cascode

Emitter Coupled Pair (Differential Amplifiers)

Common Base – Gate Output Resistance To find the output resistance for the common base connection, terminate the input side with the signal source impedance, put a current source, $i_o$ on the collector side and determine the resulting output voltage, $V_o$. The output resistance is $R_{out} = \frac{V_o}{i_o}$. Both the common gate and common source output resistance can be found at the same time. For the BJT, when $r_o = \infty$ and $r_b = 0$, the input resistance is

$$r_e = \frac{\alpha_0}{g_m} \Delta R_{ideal}$$
$$G_m = g_m$$

and for the MOSFET,

$$\frac{1}{g_m + g_{mb}} \Delta R_{ideal}$$
$$G_m = g_m + g_{mb}$$

In addition, the necessary bias resistances, $R_C$ for bipolar amplifiers and $R_D$ for MOSFET amplifiers will be designated as $R$. The node equations that need to be solved are as follows.

$$\begin{cases}
V_1 \left( \frac{1}{R_S} + \frac{1}{r_o} + \frac{1}{R_{ideal}} \right) - \frac{V_o}{r_o} = 0 \quad (1) \\
\frac{V_o}{r_o} - \frac{V_1}{r_o} - G_m V_1 = i_t \quad (2)
\end{cases}$$
where $i_t$ is a test current source placed across the output terminals. Equation 1 gives a value for $V_1$ in terms of the output voltage.

$$V_1 = \frac{V_o}{r_o} \frac{1}{1/R_s + 1/r_o + 1/R_{ideal}}$$

Equation 2 then gives a ratio of the output voltage to the current source at the output side.

$$\frac{V_o}{i_t} = \frac{1}{r_o} - \frac{(G_m + 1/r_o)/r_o}{1/R_s + 1/r_o + 1/R_{ideal}}$$

$$\frac{V_o}{i_t} = \frac{1}{r_o} - r_o (1/R_s + 1/r_o + 1/R_{ideal})$$

$$\frac{V_o}{i_t} = \frac{1}{r_o} (1/R_s + 1/r_o + 1/R_{ideal}) - G_m - 1/r_o$$

The output resistance is found by incorporating $R$.

$$R_{out} = \frac{V_o}{i_t} || R$$

$$R_{out} = R || \frac{r_o (r_o + R_s r_o / R_{ideal} + R_s)}{1 + R_s / R_{ideal} - G_m R_S}$$

The output resistance for the bipolar amplifier is found by replacing $R_{ideal}$ and $R$. The denominator for $R_{out}$ is

$$Den. = 1 + \frac{R_S g_m}{\alpha_o} - g_m R_S$$

$$= 1 + R_S g_m \left( \frac{1}{\alpha_o} - 1 \right)$$

$$= 1 + R_S g_m \left( \frac{1 + \beta}{\beta} - 1 \right)$$

$$= 1 + R_S g_m \left( \frac{1}{\beta} \right)$$

$$= 1 + R_S g_m \left( \frac{1}{g_m r_\pi} \right)$$

$$R_{out} = R_C || \left[ \frac{r_o + R_s (1 + r_o g_m / \alpha_o)}{1 + R_s / r_\pi} \right]$$

For the MOSFET,

$$R_{out} = R_D || \left[ \frac{r_o + R_s r_o (g_m + g_mb) + R_s}{1 + R_s (g_m + g_mb) - R_s (g_m + g_mb)} \right]$$

$$R_{out} = R_D || [r_o [1 + R_s (g_m + g_mb)] + R_s]$$

$$\approx R_D || [r_o [1 + R_s (g_m + g_mb)]]$$
Clearly, when \( r_o = \infty \), \( R_{out} = R_C \) or \( R_D \).

**Common Base – Gate Input Resistance**

The hybrid \( \pi \) equivalent circuit is again used to find the input resistance. This time the impedance is found from the perspective of the source resistance, \( R_S \), so of course \( R_S \) is not included. The bias resistor that controls the collector (drain) current as well as the actual external load resistance, \( R_L \), affects the input resistance. Hence, these are included with the circuit model. The same values for \( R_{ideal} \) and \( R \) as in the output resistance case are used. The current node equations for this circuit are:

\[
\begin{align*}
\frac{V_1}{R_{ideal}} + \frac{V_i}{r_o} - \frac{V_o}{r_o} &= i_t \\
\frac{V_o}{R_L} + \frac{V_o}{r_o} - \frac{V_i}{r_o} - G_m V_1 &= 0
\end{align*}
\]

From the second equation a relationship between \( V_o \) and \( V_1 \) is found.

\[
V_o = \frac{G_m V_1}{1 + \frac{R}{R_L} + \frac{R_{ideal}}{r_o}}
\]

The first equation, by substitution, gives an expression in terms of the input variables.

\[
\frac{i_t}{V_1} = \frac{1}{R_{in}} = \frac{1}{R_{ideal}} + \frac{1}{r_o} \left[ 1 - \frac{G_m r_o R_{ideal}}{r_o + R_{ideal}} \right]
\]

\[
R_{in} = \frac{r_o + R_{ideal} R_L}{R_{ideal} + 1 + \frac{R_{ideal} R_L}{r_o}} - \frac{R_{ideal} R_L}{r_o} R_L - \frac{R_{ideal} R_L}{r_o}
\]

Making the substitutions for the bipolar transistor gives the following.

\[
R_{in} = \frac{r_o + R_{C} R_L}{1 + \frac{r_o g_m}{\alpha_o} + \frac{R_{C} R_L g_m}{\alpha_o - g_m R_{C} R_L}}
\]

The last two terms in the denominator nearly cancel since \( \alpha \approx 1 \). The input resistance is then

\[
R_{in} = \frac{r_o + R_{C} R_L}{1 + r_o g_m / \alpha_o}
\]

\[
R_{in} \approx \frac{\alpha_o R_{C} R_L}{g_m} + \frac{\alpha_0 R_{C} R_L}{r_o g_m}
\]

For the MOSFET, the value of \( R_{in} \) can also be found with the appropriate substitutions.

\[
R_{in} = \frac{r_o + R_{D} R_L}{r_o (g_m + g_{mb}) + R_{D} R_L (g_m + g_{mb}) + 1 - (g_m + g_{mb}) R_{D} R_L}
\]

\[
= \frac{r_o + R_{D} R_L}{r_o (g_m + g_{mb}) + 1}
\]

\[
\approx \frac{1}{g_m + g_{mb}} + \frac{R_{D} R_L}{r_o (g_m + g_{mb})}
\]
This last expression is based on \(1 << r_o(g_m + g_{mb})\). When \(r_o = \infty\), the input resistance, \(R_{in}\), is basically the reciprocal of the transconductance. Roughly speaking, the input resistance is small and the output resistance is high.

**Cascode CE-CB Output Resistance**

The equivalent circuit for this circuit is found in the text, but the form shown in class is in a little more natural layout for analysis. Recall that for the common base circuit,

\[
  r_e = \frac{\alpha}{g_{m2}} = \frac{1}{g_m(1 + 1/\beta^2)}
\]

To find the output resistance, short the input voltage source and apply a test current, \(i_t\), at the output port. This turns off the \(g_{m1}V_1\) controlled current source. The node equations follow.

\[
\begin{align*}
  0 &= \frac{v_e}{r_{01}} + \frac{v_e}{r_{02}} + \frac{v_e}{r_0} \\
  i_t &= -\frac{v_e}{r_{02}} + \frac{V_{out}}{r_{02}} - g_{m2}v_e
\end{align*}
\]

From (1.),

\[
  v_e = \frac{v_{out}/r_{02}}{1/r_{01} + 1/r_3 + 1/r_{02}}
\]

and from (2.),

\[
\begin{align*}
  i_t &= -\frac{v_e}{r_{02}} + \frac{V_{out}}{r_{02}} \left( \frac{1}{r_{02}} + g_{m2} \right) + \frac{v_{out}}{r_{02}} \\
  i_t &= v_{out} \left[ \frac{1}{r_{02}} - \left( \frac{1}{r_{02}} \right) \frac{1/r_{02} + g_{m2}}{1/r_{01} + 1/r_{02} + 1/r_e} \right] \\
  i_t &= \frac{V_{out}}{r_{02}} \left[ 1 - \frac{1 + g_{m2}r_{02}}{1 + r_{02}/r_{01} + r_{02}g_{m2}(1 + 1/\beta^2)} \right] \\
  &= \frac{V_{out}}{r_{02}} \left[ \frac{1 + r_{02}/r_{01} + r_{02}g_{m2}(1 + 1/\beta^2) - 1 - g_{m2}r_{02}}{1 + r_{02}/r_{01} + r_{02}g_{m2}(1 + 1/\beta^2)} \right]
\end{align*}
\]
\[
\frac{i_t}{V_{out}} = \frac{1}{r_{02}} \frac{r_{02}/r_{01} + r_{02}g_{m2}/\beta_2}{1 + r_{02}/r_{01} + r_{02}g_{m2}(1 + 1/\beta_2)}
\]

\[
\frac{V_{out}}{i_t} = R_o = \frac{1}{1/r_{02} + g_{m2}/\beta_2}
\]

\[
R_o = r_{02} + \frac{1 + r_{02}g_{m2}}{1/r_{01} + g_{m2}/\beta_2}
\]

\[
= r_{02} \left[ 1 + \frac{r_{01}/r_{02} + r_{01}g_{m2}}{1 + g_{m2}r_{01}/\beta_2} \right]
\]

\[
\approx r_{02}\beta_2
\]

CE-CB Voltage Gain

Using the same basic equivalent circuit, the voltage gain can be found. Again write the two nodal equations for the circuit where the input voltage is \(V_1\) and the output is open circuited.

\[
0 = \frac{V_c}{r_{01}} + \frac{V_c}{g_{m1}V_1} - \frac{V_c}{r_{02}} - \frac{V_{out}}{r_{02}}
\]

where

\[
r_e = \frac{\alpha_0}{g_{m2}} = \frac{1}{g_{m2}(1 + 1/\beta_2)}
\]

Also,

\[
V_{out} = g_{m2}V_eR_{02} + V_c = V_e(1 + g_{m2}r_{02})
\]

\[
v_e = \frac{V_{out}}{1 + g_{m2}r_{02}}
\]

Therefore

\[
-g_{m1}V_1 = \frac{V_{out}}{1 + g_{m2}r_{02}} \left[ \frac{1}{r_{01}} + \frac{1}{r_{02}} + \frac{1}{r_e} \right] - \frac{V_{out}}{r_{02}}
\]

\[
= \frac{V_{out}}{1 + g_{m2}r_{02}} \left[ \frac{1}{r_{01}} + \frac{1}{r_e} - \frac{g_{m2}r_{02}}{r_{02}} \right]
\]

\[
A_v = \frac{V_{out}}{V_1} = \frac{-g_{m1}(g_{m2}r_{02} + 1)}{1/r_{01} + g_{m2}(1 + 1/\beta_2) - g_{m2}}
\]

\[
= \frac{-g_{m1}(g_{m2}r_{02} + 1)}{1/r_{01} + g_{m2}/\beta_2}
\]

If \(g_{m2}r_{02} \gg 1\) and neglecting \(r_{01}\)

\[
A_v = -g_{m1}r_{02}\beta_2
\]

\[
= -G_mR_o
\]

Comparison of \(R_o\) previously found and \(A_v\) implies the effective transconductance is \(G_m = g_{m1}\). This makes sense, since the gain of the common base circuit is \(\approx 1\).

Large Signal Differential Amplifier For large signal analysis, the possibility that the input differential voltage, \(V_{id} \gg V_t\), where \(V_t\) is the thermal voltage, must be considered.
In this case the output current is found to be the following.

\[ I_{out} = I_{c1} - I_{c2} = \alpha F I_{EE} \tanh \left( \frac{V_{id}}{2V_t} \right) \]

In this expression, \( I_{EE} \) is the total DC emitter current drawn through both sides of the differential pair and \( V_{id} \) is the differential input voltage applied to the two bases of the transistors on each side of the circuit. As \( V_{id} \) becomes large relative to \( V_t \), the tanh function “saturates” out to \( \pm 1 \). When \( V_{id} \) is near 0, the differential amplifier has a linear response. The transconductance of the differential amplifier is found to be approximately \( g_m \).

**General Analysis of Differential Amplifiers** Analysis of differential amplifiers can be greatly simplified if the circuit is split into two parts and the problem done twice. The symmetry of the circuit allows bisecting the circuit into its common mode and differential mode parts. This idea is similar to dividing a coupled transmission line problem into its even and odd mode excitation parts. In any case, the differential mode voltage is simply the difference between the two input voltages.

\[ V_{id} = V_{i1} - V_{i2} \]

Also, the common mode voltage is given as half the sum of the two input voltages.

\[ V_{ic} = \frac{V_{i1} + V_{i2}}{2} \]

Inverting these two equations gives the two input voltages. The output voltage can also be found since they would be proportional to the input voltages.

\[ V_{i1} = V_{ic} + \frac{1}{2} V_{id} \]
\[ V_{i2} = V_{ic} - \frac{1}{2} V_{id} \]
\[ V_o = V_{oc} \pm \frac{1}{2} V_{od} \]

There is no loss in generality, since any set of the two input voltages can be expressed uniquely in terms of the differential and common mode voltages.

If each input to a symmetrical circuit is excited by the same voltage, \( V_c \), then the circuit is said to be excited in its common mode. If the two halves of the circuit are split in two, then the horizontal wires between the two halves must carry zero current. Since each side is symmetrical, each side of the wire from the left and right side have equal voltages. If there is no current flowing in these horizontal wires, they can all be open circuited without affecting either half of the circuit in any way. The analysis of only half of the circuit need now be done.

If the left side of the circuit is excited by a voltage, \( V_d \) and the right side of the circuit is excited by \(-V_d\), then the circuit has been excited in its differential mode. If the two
halves of the circuit are split in two, then the horizontal between the two halves must be at zero voltage, since they lie midway between equal but opposite voltages. In this case, all these horizontal lines can be shorted to ground without affecting either half of the circuit in any way. Each half of the circuit can now be analyzed independently for the differential mode excitation.

Where there are vertical resistors in the center between the two halves, they should be split up into two parallel resistors of twice the value of the original resistor. With these simple rules, the symmetrical differential mode circuit can be easily analyzed. Even if the circuit uses active loads, this method still gives an approximate solution.

**Device Mismatch Errors** Device mismatch can be represented as an ideally matched differential amplifier plus two offset sources: the offset voltage, $V_{os}$, and the offset current, $I_{os}$. Consider first the offset voltage. The $V_{os}$ is defined as the voltage applied between the differential amplifier inputs that would reduce the differential output voltage $V_{out}$ to 0. The offset current is the difference between the base currents of the two input transistors. If Q1 and Q2 are the two input transistors, then $V_{BE1}$ and $V_{BE2}$ are the two base to emitter voltages for Q1 and Q2. In the emitter of Q1 there is an emitter resistor $R_E$ and in the emitter of Q2 an emitter resistor $R_E + \Delta R_E$. There is a current $I_1$ in the Q1 emitter resistor and a current of $I_2$ in the emitter resistor of Q2. The current source drawing current from the emitters of both Q1 and Q2 has a value $I_0 = I_1 + I_2$. Applying $V_{os}$ guarantees the output voltage is 0. Using Kirchoff’s voltage law,

$$V_{os} - V_{BE1} - I_1 R_E + I_2 (R_E + \Delta R_E) + V_{BE2} = 0.$$

Since $I_1 = I_0 - I_2$,

$$V_{os} - V_{BE1} + V_{BE2} - I_0 R_E + 2 I_2 R_E + I_2 \Delta R_E = 0.$$

The differential amplifier is presumed to be almost balanced, so that

$$I_2 \approx \frac{I_0}{2}.$$

The offset voltage is

$$V_{os} = V_{BE1} - V_{BE2} - \frac{I_0 \Delta R_E}{2}$$

where $\Delta R_E$ is the mismatch in the emitter degeneration resistors. Note, here the base-emitter voltages are not exactly equal, so the more exact values for these voltages must be used.

$$V_{BE1} = V_t \ln \frac{I_{C1}}{I_{S1}} \quad \text{and} \quad V_{BE2} = V_t \ln \frac{I_{C2}}{I_{S2}}$$

The saturation currents are proportional to the area. Since $V_{out} = 0$ when $V_{os}$ is applied. The difference in the collector currents is associated with the mismatch in the collector resistors.

$$I_{C1} R_C = I_{C2} (R_C + \Delta R_C)$$

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\[
\frac{I_{C1}}{I_{C2}} = \frac{R_C + \Delta R_C}{R_C}
\]

Now a Taylor series expansion can be useful at this point.

\[
\ln \left(1 + \frac{\Delta x}{x}\right) = \frac{\Delta x}{x} - \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta x}{x}\right)^3 - \left(\frac{\Delta x}{x}\right)^4 + \ldots
\]

\[
\approx \frac{\Delta x}{x}
\]

when \(\frac{\Delta x}{x} \ll 1\).

In a fashion similar to the collector currents, the ratio of the saturation currents can be found. First, the difference between the two saturation currents are given as

\[
I_{S1} = I_{S2} + \Delta I_S \quad \text{and} \quad I_S = \frac{I_{S1} + I_{S2}}{2}.
\]

As a result

\[
\frac{I_{S1}}{I_{S2}} = \frac{I_S \left(1 + \frac{\Delta I_S}{2I_S}\right)}{I_S \left(1 - \frac{\Delta I_S}{2I_S}\right)} \approx 1 + \frac{\Delta I_S}{I_S}.
\]

Making use of the approximation for \(\ln(1 + \Delta x/x)\), the offset voltage can be found.

\[
V_{os} = V_t \left[\frac{\Delta R_C}{R_C} - \frac{\Delta I_S}{I_S} - \frac{I_0 R_E \Delta R_E}{2V_t R_E}\right]
\]

A similar procedure can be used for the offset current.

\[
I_{os} = \frac{I_0}{2\beta} \left[\frac{\Delta R_C}{R_C} - \frac{\Delta \beta}{\beta}\right]
\]