PROBLEM #1: Short Answers:(55 points)

1. Write the change in a scalar field V due to a infinitesimal vectorial displacement dl. dV= $\vec{\nabla} V \cdot \vec{dl}$

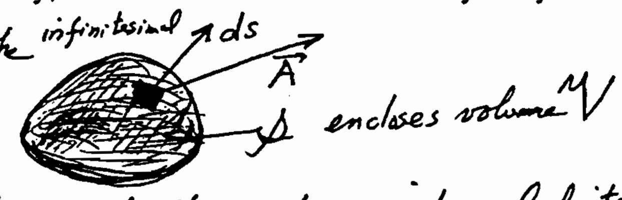
2. What is the spatial rate of change of a scalar field V in the direction dl? dV/|dl| = $\vec{\nabla} V \cdot \vec{a}_\ell = \vec{\nabla} V \cdot \frac{\vec{dl}}{|\vec{dl}|}$

3. Define the gradiant of a scalar field is a vector whose direction represents the direction of maximal spatial rate of increase of the scalar field and whose magnitude equals that maximum spatial rate of increase

4-5. Write the divergence theorem mathematically; make a clear sketch and define your symbols.

$$\int_{\mathcal{V}} \vec{\nabla} \cdot \vec{A}\, dv = \oint_{S} \vec{A} \cdot \vec{ds}$$

$\vec{ds}$ is the surface element vector

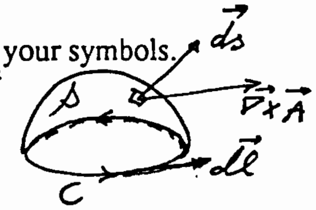


6. State the divergence theorem in words:
Net outward flux of a vector field equals the volume integral of its divergence over the enclosed volume.

7-8. Write the Stokes' theorem mathematically. Make a clear sketch. Define your symbols.

$$\oint_C \vec{A} \cdot \vec{dl} = \int_S (\vec{\nabla} \times \vec{A}) \cdot \vec{ds}$$

$\vec{dl}$ = infinitesimal line element vector
$\vec{ds}$ = " " surface "



9-10. State postulates of electrostatic in free space in differential form; define your symbols:

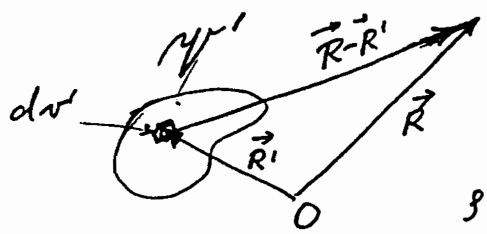
$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ ρ = total volume charge density
ϵ_0 = permittivity of free space
$\vec{\nabla} \times \vec{E} = 0$ $\vec{E}$ = electric field

11. Relate the Electrostatic Potential to the electric field in integral form: $V(\vec{R}_1) - V(\vec{R}_2) = -\int_{\vec{R}_1}^{\vec{R}_2} \vec{E} \cdot \vec{dl}$

12. Write the Poisson equation for the electrostatic potential: $\nabla^2 V = -\rho / \epsilon_0$ ρ = total vol. charge density

13-14. The electrostatic potential for a continuous distribution of charges is given by

$$V(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\rho(\mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|} dv'$$



$\vec{R}$ = observation point vector
$\vec{R}'$ = source point vector
$\rho\, dv'$ = infinitesimal charge
$\mathcal{V}'$ volume containing charge

Make a sketch and define <u>all</u> the symbols.

(Note: This is the solution to Poisson's Equ.)

15. Give symbols and units for the Electric Field Intensity: $\vec{E}$ (V/m)

16. Give symbols and units for the Electric Flux Density: $\vec{D}$ (C/m^2)

17. Give symbols and units for the Electric Polarization: $\vec{P}$ (C/m^2)

18-19. Give general relation between the above 3 quantities: $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ by definition

20. What is the relation between **E** and **P** for a linear isotropic homogeneous dielectric material? $\vec{P} = \epsilon_0 \chi_e \vec{E}$

21. What is the relation between **E** and **D** for a linear isotropic homogeneous dielectric material? $\vec{D} = \varepsilon \vec{E}$

22. Does the above equation hold for the case when there is permanent polarization? No

23. State Gauss's Law for the electric field in words:
Net outward flux of the electric field through a closed surface equals the total charge enclosed divided by ϵ_0

24 State Gauss's Law for the electric flux density in words.
Net outward flux of the electric flux density through a closed surface equals the net free charge enclosed

25. Give an integral expression for the stored electrostatic energy in terms of ρ and V.

$$W_e = \frac{1}{2} \int_{v'} \rho V \, dv'$$

26. Give an integral expression for the stored electrostatic energy in terms of **E** and **D**. $W_e = \frac{1}{2} \int_{v} \vec{E} \cdot \vec{D} \, dv'$

27. Give an expression for the polarization volume charge density ρ_p in terms of **P**. $\rho_p = -\vec{\nabla} \cdot \vec{P}$

28. Give an expression for the polarization surface charge density ρ_{ps} in terms of **P**. $\rho_{ps} = \vec{P} \cdot \vec{a}_n$

29-30. For a sphere with uniform polarization vector $\mathbf{P}=P_0\, \mathbf{a}_z$ find the polarization surface charge density.

$a_R \equiv \hat{R}$ $\quad a_z = \hat{z}$

$$\left.\begin{array}{l} \vec{a}_n = \vec{a}_R \\ \vec{P} = P_0 \vec{a}_z \end{array}\right\} \rightarrow \rho_{ps} = P_0 \underset{\hat{z}}{\vec{a}_z} \cdot \underset{\hat{R}}{\vec{a}_R} = \boxed{P_0 \cos\theta}$$

31. find the volume polarization charge density for the same sphere above

$$P = P_0 \vec{a}_z = \text{Constant} \quad \therefore \quad \vec{\nabla} \cdot \vec{P} = 0 \quad \Rightarrow \quad \boxed{\rho_p = 0}$$

32-33. Give the formula for the electrostatic potential of a dipole **p** centered at the origin oriented in the z direction

$$V(R,\theta,\phi) = \frac{p \cos\theta}{4\pi\epsilon_0 R^2}$$

(units: $\frac{1}{4\pi\epsilon_0} \frac{C\,m}{m^2} = E \cdot m = \frac{V}{m} \cdot m = V$)

34-35 What is the total electromagnetic force on a moving charged particle, define your symbols:

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ $\quad \vec{E}$ = electric field $\quad \vec{v}$ = velocity
$\vec{B}$ = magnetic flux density $\quad q$ = charge

36-37. State the postulates of Magnetostatic in free space in differential form; define your symbols

$\vec{\nabla} \cdot \vec{B} = 0$ $\quad \vec{B}$ = magnetic flux density $\quad \mu_0$ = permittivity of free space
$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ $\quad \vec{J}$ = total current density

38. What is the relation between the magnetic flux density and the vector magnetic potential?

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

39. What is the general relation between the vector magnetic potential and the current desity in differential form?

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0 \vec{J}$$

40. What does this reduce to for the for the choice $\nabla \cdot \mathbf{A} = 0$? $\nabla^2\vec{A} = -\mu_0\vec{J}$

41-42. Write the solution to the above Poisson's equation in integral form (similar to 13-14).

$$\vec{A}(\vec{R}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{R}')}{|\vec{R}-\vec{R}'|}$$

43-44. Give the expression for the vector magnetic potential due to a magnetic dipole moment **m**. for large R. Define your symbols.

$\vec{a}_R$ = unit vector from origin to observation

$$\vec{A}(\vec{R}) = \frac{\mu_0\, \vec{m}\times\vec{a}_R}{4\pi\, R^2}$$

45. What is the magnetic dipole moment **m** of a current I circularating in a loop of area A perpendicular to the x axis? $\vec{m} = IA\,\vec{a}_x$ $\vec{a}_x = \hat{x}$

46-47. Write the expression for magnetization surface current density $\mathbf{J}_{ms}$ in terms of the magnetization vector **M**. give units for both quantities. $\vec{a}_n = \hat{n}$

$$\vec{J}_{ms} = \vec{M}\times\vec{a}_n$$

A/m A/m

48. Write the expression for magnetization current density $\mathbf{J}_m$ in terms of **M**. $\vec{J}_M = \vec{\nabla}\times\vec{M}$

49-50. For a spherical permanent magnet at origin with uniform magnetization vector $\mathbf{M}=M_0\,\mathbf{a}_z$ find the magnetization surface current density $\mathbf{J}_{ms}$.

$\vec{a}_n = \vec{a}_R$

$\vec{M} = M_0\vec{a}_z$

$\therefore \vec{J}_{ms} = M_0\,\vec{a}_z\times\vec{a}_R = M_0\,\vec{a}_\phi$

51. find the magnetization volume current density $\mathbf{J}_m$ for the same sphere above $\vec{\nabla}\times\vec{M} = M_0\,\vec{\nabla}\times\vec{a}_z = 0$

52. Which statement is true for the above sphere?

a) **B**=0 inside but not outside ✗ b) **H**=0 inside but not outside ✗

c) **H**=0 everywhere ✗ d) **B**=0 everywhere ✗ $\boxed{\text{c) None of the above}}$ ✓

53. What is the general relation between **H**, **B**, and **M**? $\vec{B} = \mu_0\vec{H} + \mu_0\vec{M}$ or $\vec{H} = \frac{1}{\mu_0}(\vec{B}-\mu_0\vec{M})$ or $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

54. What is the relation between **B** and **M** for a linear isotropic Magnetizable material?

55. What is the relation between **B** and **H** for such a material?

$$\vec{B} = \mu\vec{H}$$

$$\vec{B} = \mu_0(1+\chi_m)\vec{H}$$

$\vec{M} = \chi_m\vec{H}$

$\therefore \vec{H} = \frac{1}{\chi_m}\vec{M}$

$\vec{B} = \mu_0\frac{(1+\chi_m)}{\chi_m}\vec{M}$

$$\boxed{\vec{M} = \left(\frac{\chi_m}{\mu_0(1+\chi_m)}\right)\vec{B}}$$

EE 3318 EXAM #3 Dr. K. Alavi Nov. 30, 1988 NAME: ____________________

Problem 1. Short Answer Questions :

1. Consider current due to motion of one type of charge carriers, at velocity **u**. write the relationship between current density **J**, volume charge density ρ, and **u**. Give units for all 3 quantities:

$\frac{A}{m^2} \leftarrow \vec{J} = \rho \vec{u} \rightarrow m/s$ $\qquad \rho$ = mobile Charge density

$\rho \rightarrow C/m^3$

2. Write the relation between current I through a surface S and current density **J**:

$$I = \int_S \vec{J} \cdot \vec{ds}$$

3. Write the constitutive relation (relating **J** to **E**) for a ~~homogeneous~~ Linear Isotropic medium with conductivity σ : $\vec{J} = \sigma \vec{E}$

4. Write the equation of continuity relating spatial variation of current density to time variation of volume charge density in point form:

Conservation of Charge $\qquad \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

5. Write the equation governing **J** in steady state: $\vec{\nabla} \cdot \vec{J} = 0$

6-7. Write the boundary conditions for steady state current density **J** in the absence of nonconserving energy sources between two media with σ_1 and σ_2 :

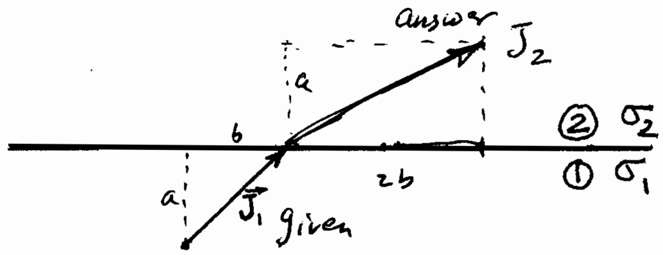
6. normal components: $J_{1n} = J_{2n}$

7. tangential components: $\cancel{\frac{1}{\sigma_1} J_{1t} = \frac{1}{\sigma_2} J_{2t}}$ or $J_{2t} = \frac{\sigma_2}{\sigma_1} J_{1t}$

8. Using above results draw J_2 in the diagram for $\sigma_2 = 2\sigma_1$. Draw to scale.

$J_{2n} = J_{1n}$

$J_{2t} = \frac{\sigma_2}{\sigma_1} J_{1t} \rightarrow J_{2t} = 2 J_{1t}$



9. Write the total force **F** on a particle with charge q moving with velocity **u** in the presence of both an electric field **E** and a magnetic field with flux density **B** :

F= $q\vec{E} + q\vec{u} \times \vec{B}$

10-13. Write down the fundamental postulates of magnetostatic in free space:

a) point form: 10. $\nabla \cdot B = 0$ $\qquad$ 11. $\cancel{\oint_S B/ds \neq 0}$ $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

b) integral form: 12. $\oint_S B \cdot ds = 0$ $\qquad$ 13. $\oint B \cdot dl = \mu_0 I = \mu_0 \int \vec{J} \cdot$

14. Write the relation between **B** and vector magnetic potential **A**: $\vec{B} = \nabla \times A$

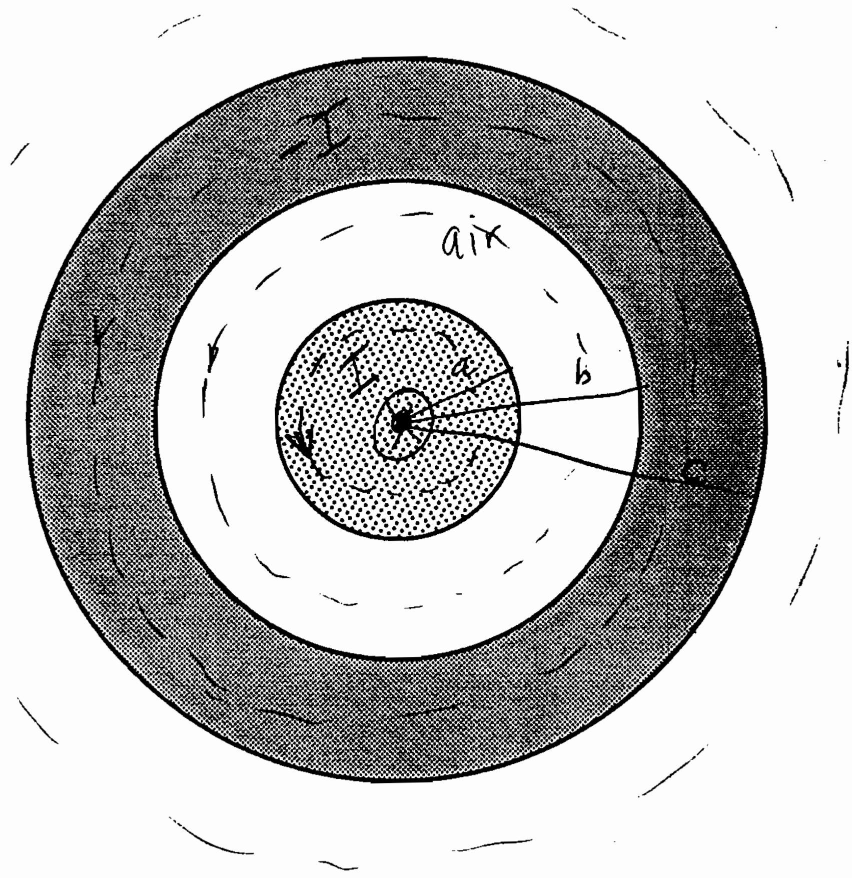
15. If we choose $\nabla \cdot$**A**=0 then what would ∇^2 **A** equal to? ∇^2 **A**= $-\mu_0 \vec{J}$

Problem #1: (25 points)

Consider 3 concentric long cylinders with radii a, b, and c. The inner cylinder is a conductor carrying a uniformly distributed I. the inner region (r between a and b) is air. The region between r=b and c is another conductor carrying uniformly distributed current -I ~~current density~~.

a) Determine the magnetic flux density vector **B**, everywhere.
b) let b=3a, and c=4a. Plot the magnitude of B as a function of r.

uniformly distributed current I in −z direction   Air   uniformly distributed current I in + z direction



V

$$\oint B \cdot dl = \mu_0 I$$

$0 < r < a$

$$I_{enclosed} = I\left(\frac{\pi r^2}{\pi a^2}\right) = I\left(\frac{r^2}{a^2}\right)$$

$$2\pi r \, \bar{B}_1 = \mu_0 I\left(\frac{r^2}{a^2}\right)$$

$$B_1 = \frac{\mu_0 I\left(\frac{r^2}{a^2}\right)}{2\pi r}$$

Problem 1 Continued

$$\boxed{\bar{B}_1 = \frac{\mu_0 I r}{2\pi a^2} \vec{a_\phi}} \qquad 0 < r < a$$

For $a < r < b$

$$2\pi r B_2 = \mu_0 I$$

$$\boxed{\bar{B}_2 = \frac{\mu_0 I}{2\pi r} \bar{a}_\phi} \qquad a < r < b$$

For $b < r < c$

$$2\pi r B_3 = \mu_0 I_{enclosed} = \mu_0 \left(I - \left[\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2} \right] I \right).$$

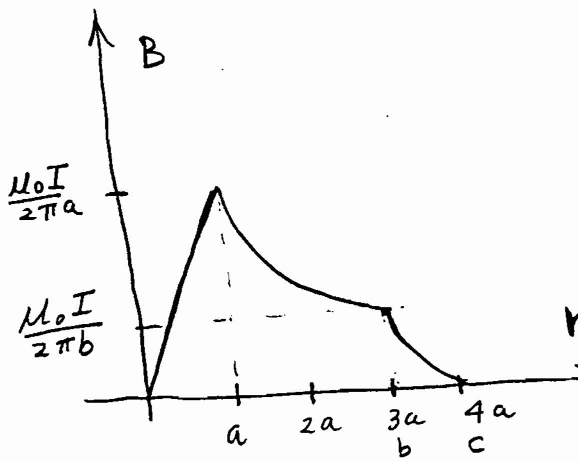
$$\boxed{\bar{B}_3 = \frac{\mu_0 I}{2\pi r} \left[1 - \left(\frac{r^2 - b^2}{c^2 - b^2} \right) \right] \bar{a}_\phi} \qquad b < r < c$$

For $r > c$

$$I_{enclosed} = 0$$

$$B_4 2\pi r_4 = \mu_0 I = 0$$

$$\boxed{B_4 = 0} \quad r > c$$



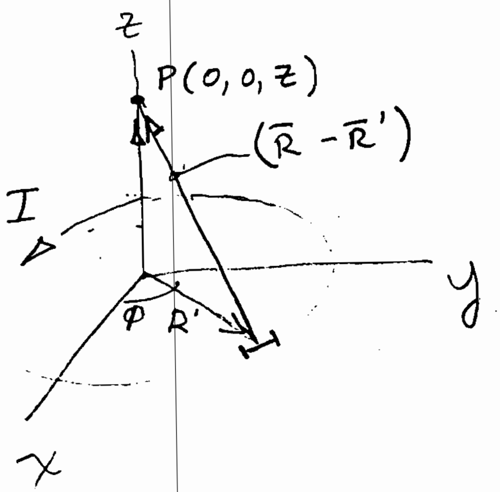
$b = 3a$
$c = 4a$

Problem #2 (25 points)
Consider a circular loop of radius b centered at the origin and lying in the x-y plane. This loop carries a current I (counterclockwise)

a) Steate Biot-Savart law in English and mathematically. Define all your symbols.

b) Calculate the magnetic flux density vector **B** at a general point on the z axis.

c) Calculate the magnetic vector potential **A** at the same point.



$\bar{dl}' = \bar{a}_\phi \, b \, d\phi$

$\bar{R} = z \bar{a}_z$

$\bar{R}' = b \bar{a}_r$

$= b\cos\phi \, \bar{a}_x + b\sin$

(a) Biot-Savart Law

$$\bar{B} = \frac{\mu_0 I}{4\pi} \oint_{c'} \frac{\bar{dl}' \times (\bar{R} - \bar{R}')}{|\bar{R} - \bar{R}'|^3}$$

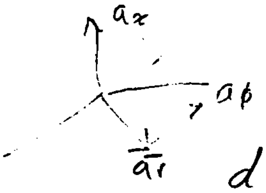
The magnetic flux density $\bar{B}$ at a point in space due to a current I in a closed circuit is equal to the closed line integral around that circuit of the cross product of the differential current element and the relative distance from the differential current element to the point in question divided by the relative distance cubed.

$$\bar{R} - \bar{R}' = z\bar{a}_z - b\bar{a}_r$$

Problem #2 Continued

$$\bar{R} - \bar{R}' = z\bar{a}_z - (b\cos\phi\, a_x + b\sin\phi\, \bar{a}_y)$$

$$= -b\cos\phi\,\bar{a}_x - b\sin\phi\,\bar{a}_y + z\bar{a}_z$$



$$d\bar{l}' \times (\bar{R} - \bar{R}') = \bar{a}_\phi\, b\,d\phi \times (z\bar{a}_z - b\bar{a}_r)$$

$$= bz\,d\phi\,\bar{a}_r + b^2 d\phi\,\bar{a}_z$$

The $\bar{a}_r$ component will drop out due to symmetry; all that remains is the $\bar{a}_z$ term

$$|\bar{R} - \bar{R}'|^3 = \left(\sqrt{z^2 + b^2}\right)^3 = (z^2 + b^2)^{3/2}$$

$$\text{so, } \bar{B} = \bar{a}_z \frac{\mu_0 I}{4\pi} \oint \frac{b^2\,d\phi}{(z^2 + b^2)^{3/2}} = \bar{a}_z \frac{\mu_0 I b^2}{4\pi (z^2 + b^2)^{3/2}}$$

$$\boxed{\bar{B} = \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}}\,\bar{a}_z}$$

$$\bar{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\bar{l}'}{R} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{b\,d\phi\,\bar{a}_\phi'}{(z^2 + b^2)^{1/2}}$$

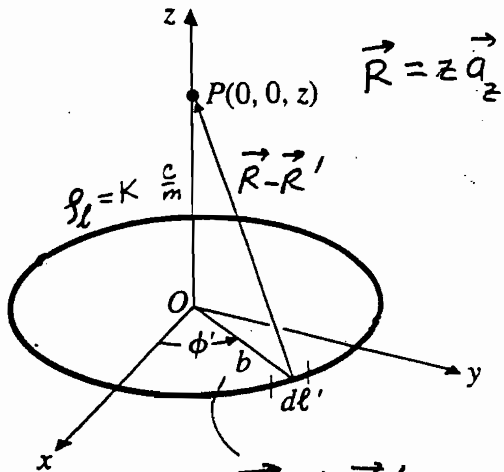
$$\bar{A} = \frac{\mu_0 I b}{(z^2 + b^2)^{1/2}} \int_0^{2\pi} (-\hat{a}_x \sin\phi' + \hat{a}_y \cos\phi)\,d\phi'$$

$$\boxed{\bar{A} = 0}$$

Problem #2 (20 points)

Consider a circular loop of charge centered at the origin and lying in the x-y plane. This loop carries a constant linear charge density $\rho_l = K$ (C/m) (see figure). Calculate the electrostatic potential V and the Electrostatic field E at the point P on the z axis indicated in the figure.

assume $V(\infty) = 0$



$\vec{R} = z\vec{a}_z$

$\vec{R}' = b\vec{a}_r'$

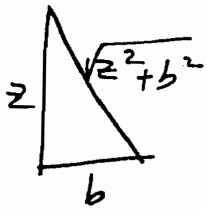
$|\vec{R}-\vec{R}'| = \sqrt{b^2+z^2}$

then

$$V(P) = \frac{1}{4\pi\epsilon_0}\int_{\phi'=0}^{2\pi} \frac{1}{|\vec{R}-\vec{R}'|}\rho_l(\phi)\,dl'$$

$$= \frac{1}{4\pi\epsilon_0}\int_{\phi'=0}^{2\pi} \frac{1}{\sqrt{(z^2+b^2)}} K b\, d\phi'$$

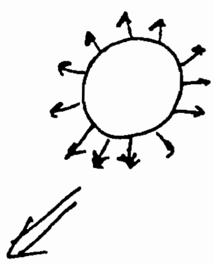
$$= \frac{1}{4\pi\epsilon_0}\frac{Kb}{\sqrt{z^2+b^2}}\cdot 2\pi = \frac{1}{2\epsilon_0}\frac{Kb}{\sqrt{z^2+b^2}}$$



$$E(P) = \frac{1}{4\pi\epsilon_0}\int_{\phi'=0}^{2\pi} \frac{(\vec{R}-\vec{R}')}{|\vec{R}-\vec{R}'|^3}\rho_l(\phi')\,dl'$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{z\vec{a}_z - b\vec{a}_r'}{(z^2+b^2)^{3/2}} K b\, d\phi'$$

$$= \frac{1}{4\pi\epsilon_0}\int_{\phi'=0}^{2\pi} \frac{z\vec{a}_z\, K b\, d\phi'}{(z^2+b^2)^{3/2}} + \frac{-b^2K}{(z^2+b^2)^{3/2}}\underbrace{\int_{\phi'=0}^{2\pi} \vec{a}_r'\, d\phi'}_{0}$$



$$\boxed{E(P) = \frac{1}{2\epsilon_0}\frac{zKb}{(z^2+b^2)^{3/2}}\vec{a}_z}$$

also $E(P) = -\frac{dV}{dz}\vec{a}_z = -\frac{-2z\,Kb \times 1/2}{2\epsilon_0\left(\sqrt{z^2+b^2}\right)^3}\vec{a}_z$

4 20

PROBLEM #~~3~~: (~~15~~ points) Consider the Magnetic vector potential **A** potential given by the following expressions (K>0):

$$A(R,\theta,\phi)=\frac{\mu_0 K}{4\pi} R \sin\theta\, a_\phi \quad \text{for } R\leq b$$

$$A(R,\theta,\phi)=\frac{\mu_0 Kb^3}{4\pi}\frac{\sin\theta}{R^2}\, a_\phi \quad \text{for } R\geq b$$

$a_x = \hat{x}$ $\quad a_\theta = \hat{\theta}$

$a_R = \hat{R}$

$a_z = \hat{z}$

1- Write the relation between the magnetic flux density **B** and the magnetic vector potential.

$\bar{B} = \nabla \times \bar{A}$

2-7. calculate **B**(R, θ, ϕ) for both regions:

$$\bar{B} = \nabla \times \left(\frac{\mu_0 K}{4\pi} R \sin\theta\, \bar{a}_\phi\right)$$

$$\nabla \times \bar{A} = \bar{a}_R \frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta)\right] + \bar{a}_\theta \frac{1}{R}\left[-\frac{\partial}{\partial R}(R A_\phi)\right]$$

$$= \frac{\mu_0 K}{4\pi}\left[\bar{a}_R(\cos\theta + \cos\theta) + \bar{a}_\theta(-\sin\theta - \sin\theta)\right]$$

$$B(R,\theta,\phi)=\frac{\mu_0 K}{4\pi}\left(2\cos\theta\ a_R + -2\sin\theta\ a_\theta\right) \quad \text{for } R\leq b$$

$$B(R,\theta,\phi)=\frac{\mu_0 Kb^3}{4\pi}\left(\frac{2\cos\theta}{R^3}\ a_R + \left(\frac{\sin\theta}{R^3}\right) a_\theta\right) \quad \text{for } R\geq b$$

8-9. Evaluate **B**(R, θ, ϕ) at R=b from the above two formulas:

from R<b formula----> $B(R=b, \theta, \phi)= \frac{\mu_0 K}{4\pi}\left(2\cos\theta\ \hat{a}_R - 2\sin\theta\ \hat{a}_\theta\right)$

from R>b formula----> $B(R=b, \theta, \phi)= \frac{\mu_0 K}{4\pi}\left(2\cos\theta\ \hat{a}_R + \sin\theta\ \hat{a}_\theta\right)$

10. Is the normal component (i.e.R component) continuous?

Yes

11. Do you expect the result? What law of magnetostatic supports this?

Yes. Since $\nabla \cdot \bar{B} = 0$, the normal components must be contin

12. Is the tangential component (θ component) continous?

No

13. True or false: the above result suggests there is a surface current at R=b

True. Since $\mu_1 = \mu_2 = \mu_0$ only a surface current explains discont

14-15. Show that for R<b, **B** is constant. (Hint: write a_z in spherical coordinate system and compare with the above result).

$\bar{a}_z = \bar{a}_R \cos\theta - \bar{a}_\theta \sin\theta$ $\qquad B = \frac{\mu_0 K}{4\pi} 2\hat{a}_z$

$B = \frac{\mu_0 K}{4\pi} 2\hat{a}_z = \text{Constant}$

Bonus

PROBLEM #: (20 points) Consider the electrostatic potential given by the following (K>0)

$a_R \equiv \hat{R} \quad a_\theta = \hat{\theta} \quad a_z = \hat{z}$

$$V(R, \theta, \phi) = \frac{K}{4\pi\varepsilon_o} R \cos\theta \quad \text{For } R \leq b$$

$$V(R, \theta, \phi) = \frac{Kb^3}{4\pi\varepsilon_o} \frac{\cos\theta}{R^2} \quad \text{For } R \geq b$$

1-7. Find E(R, θ, φ) for both regions:

we start with $E = -\vec{\nabla} V$. Then calculate grad for both regions

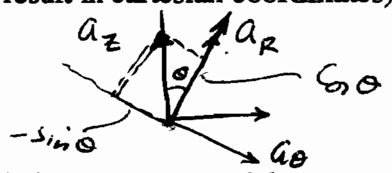
$$E(R, \theta, \phi) = \frac{K}{4\pi\varepsilon_o} (-\cos\theta \; a_R + (+\sin\theta) \; a_\theta) \quad \text{For } R < b \quad ①$$

$$E(R, \theta, \phi) = \frac{Kb^3}{4\pi\varepsilon_o R^3} (2\cos\theta \; a_R + \sin\theta \; a_\theta) \quad \text{For } R > b \quad ②$$

8-10. Show that for R<b, **E** is constant. (Hint: write the above result in cartesian coordinates)

recall $\vec{a}_z = \cos\theta \, \vec{a}_R - \sin\theta \, \vec{a}_\theta$

$\therefore E(R, \theta, \phi) = \frac{-K}{4\pi\epsilon_0} \vec{a}_z = \text{Constant}$



11-13. Evaluate your formulas at the spherical surface of radius b, is the the tangential component of the electric field continuous? If yes is this to be expected? What law of electrostatic supports this?

@ $R=b$ the second formula ② gives $\frac{K b^3}{4\pi\epsilon_0 b^3}$ for the coefficient

$\therefore$ ① $\rightarrow E = \frac{K}{4\pi\epsilon_0} (-\cos\theta \, a_R + \sin\theta \, a_\theta)$ ② $\Rightarrow E = \frac{K}{4\pi\epsilon_0} (2\cos\theta \, a_R + \sin\theta \, a_\theta)$

14-16. Calculate the discontinuity in $E_R(R, \theta, \phi)$ at the surface of sphere of radius b. Now give expression for surface charge density at this surface

We see a_θ components are equal! E_t is Continuous

but $E_R(R=b+0) - E_R(R=b-0) = \frac{3K}{4\pi\epsilon_0} \cos\theta$

17-18. Calculate the volume charge density inside the sphere of radius b. $\rho/\epsilon_0 = \nabla \cdot \vec{E}$

but E = Constant inside the Sphere $\therefore \boxed{\rho = 0}$

$\therefore \nabla \cdot \vec{E} = 0$

19-20. The potential outside the sphere is exactly that of an electric dipole centered at the origin oriented along the z axis