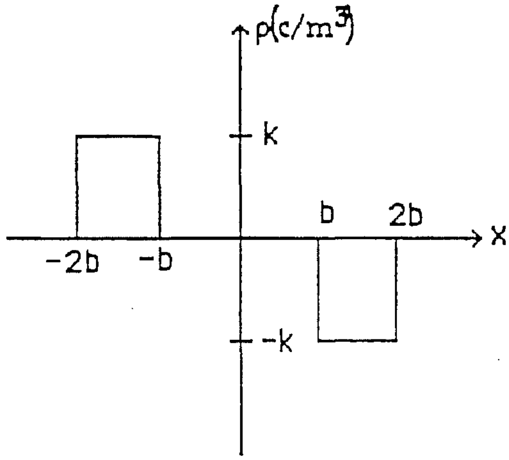
Problem #2. (25 points)

Consider two large uniformly charged slabs (thickness b) parallel to yz plane separated by distance 2b from each other. The slab on the left has charge density k(C/m^3) and the one on the right, -k (C/m^3) (k > 0 and constant) the charge distribution is sketched below. Assume that the slabs are large enough so that E is in the x direction and only depends on x.



1. Calculate E_x as a function of x
2. Carefully plot E_x (x).
3. Assume V= 0 at x = -∞ , calculate V(x) .
4. Carefully plot V(x).

1.) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ 2 ; $\vec{E} = E_x(x,y,z)\vec{a}_x + E_y(x,y,z)\vec{a}_y + E_z(x,y,z)\vec{a}_z$

From symmetry of problem $\vec{E} = E_x(x)\,\vec{a}_x$ 1 no y, z dependence
no y, z components

∴ 1. → $\frac{\partial E_x(x)}{\partial x} = \frac{\rho}{\varepsilon_0}$ 1 no y, z dep. →

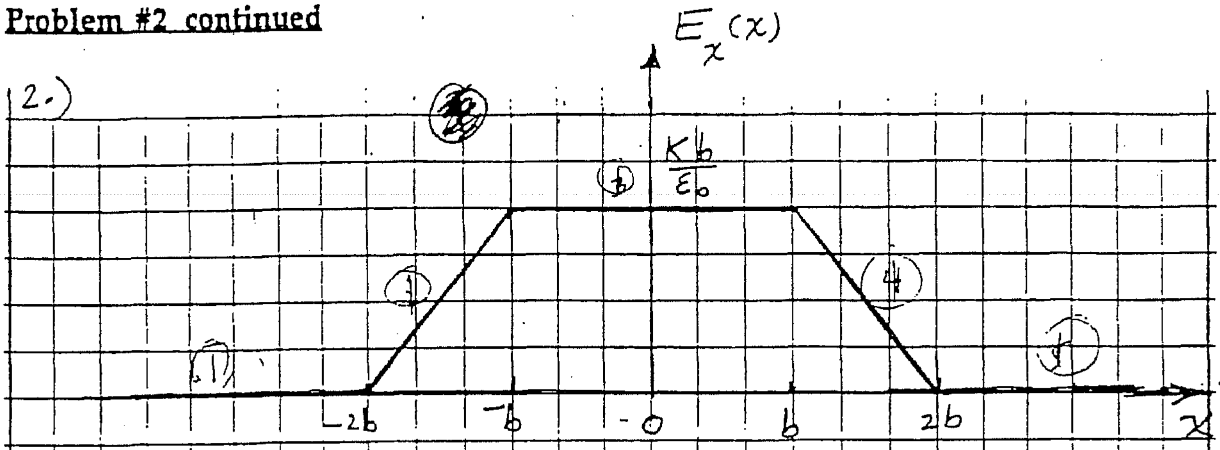
$\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_0}$. also from symmetry of ρ, at x = -∞ $E_x = 0$ 1

integrate once

$$E_x(x) = \int_{-\infty}^{x} \frac{1}{\varepsilon_0}\rho(x')\,dx' = \begin{cases} 0 & x \le -2b \\ \frac{k}{\varepsilon_0}(x+2b) & -2b \le x \le -b \\ \frac{kb}{\varepsilon_0} & -b \le x \le b \\ \frac{k}{\varepsilon_0}(-x+2b) & b \le x \le 2b \\ 0 & 2b \le x \end{cases}$$

Problem #2 continued

2.)



3) $V(x) - V(-\infty) = -\int_{-\infty}^{x} E_x(x)\,dx$ (2)

$$V(x) - 0 = \begin{cases} 0 & x < -2b \quad (1) \\ -\frac{K}{\varepsilon_0}\left(\frac{1}{2}x^2 + 2bx + 2b^2\right) & -2b \le x \le -b \quad (1) \\ -\frac{Kb}{\varepsilon_0}\left(+x + \frac{3b}{2}\right) & -b \le x \le b \quad (1) \\ -\frac{K}{\varepsilon_0}\left(-\frac{1}{2}x^2 + 2bx + b^2\right) & b \le x < 2b \quad (1) \\ -\frac{3Kb^2}{\varepsilon_0} & 2b \le x \quad (1) \end{cases}$$

4)