Several different sets of parameters can be used to describe a transistor. We may use impedance, admittance, hybrid or s-parameters, although others are possible as well. In this class we will use primarily hybrid parameters.

Hybrid Parameters

\[ V_{BE} = h_{ie} I_b + h_{be} V_{CE} \]
\[ I_C = h_{fe} I_b + h_{oe} V_{CE} \]

is the set that is appropriate to the common-emitter configuration.

Definitions:

\[ h_{ie} = \frac{\partial V_{BE}}{\partial I_b} = \frac{V_{BE}}{I_b} \bigg|_{V_{CE} = 0} = \frac{kT(\beta+1)}{q|I_e|} + r_b \]

From p-n junction equation:

\[ = \frac{kT}{q|I_d|} + r_b \]

Physical resistance of base region.
Notice that $h_{ie}$ is the CE input resistance with collector voltage held constant at the $Q$ point.

$$h_{re} = \frac{dV_{BE}}{dV_{CE}} = \frac{V_{BE}}{V_{CE}} \bigg|_{i_b=0}$$  Result of base-narrowing effect.

This parameter is the CE voltage feedback ratio.

$$h_{fe} = \frac{di_c}{di_b} = \frac{i_c}{i_b} \bigg|_{v_a=0}$$

This is the CE small-signal current gain.

$$h_{re} = \frac{di_c}{dV_{CE}} = \frac{i_c}{V_{CE}} \bigg|_{i_b=0}$$  Result of base-narrowing effect.

This is the CE output impedance.

Show examples
Hybrid-$\pi$ Model

This model is somewhat superior wrt. high-frequency effects. However, it has slightly greater complexity.

\[ V_{be} = \frac{r_b}{r_m} I_b + V_{\pi} \]

\[ I_C = V_{ee} \left( g_U + g_m \right) + V_{\pi} \left( g_m - g_U \right) \]

Also,

\[ V_{\pi} = r_{\pi} \left[ I_b + \left( V_{ee} - V_{\pi} \right) g_m \right] \]

We can combine these 3 expressions to obtain

\[ V_{be} = I_b \left( \frac{r_b + r_{\pi}}{1 + r_{\pi} g_m} \right) + V_{ee} \left( \frac{g_m - g_U}{1 + r_{\pi} g_m} \right) \]

\[ I_C = I_b \left( \frac{r_{\pi} \left( g_m - g_U \right)}{1 + r_{\pi} g_m} \right) + V_{ee} \left( g_U + \frac{g_m - g_U}{1 + r_{\pi} g_m} \right) \]
Comparing these last 2 equations with the form shown at the onset we find

\[ h_{ie} = r_b + \frac{r_\pi}{1 + r_\pi g_m} \approx r_b + r_\pi \]

\[ h_{fe} = \frac{r_\pi (g_m - g_m)}{1 + r_\pi g_m} \approx r_\pi g_m \]

\[ h_{re} = \frac{r_\pi g_m}{1 + r_\pi g_m} \approx r_\pi g_m \approx 0 \]

\[ h_{oe} = g_0 + g_m + \frac{r_\pi g_m (g_m - g_m)}{1 + r_\pi g_m} \approx g_0 + r_\pi g_m g_m \approx g_0 \]

Mutual conductance or transconductance

\[ g_m = \frac{\partial i_c}{\partial v_{be}} \bigg|_{v_{ce}=0} = \frac{g_m |i_c|}{kT} \]

Using the approximations shown above

\[ r_\pi = \frac{h_{fe}}{g_m} \]

\[ r_b = h_{ie} - r_\pi \]

\[ g_0 = h_{oe} - g_m h_{re} \]

\[ r_m = \frac{r_\pi}{h_{re}} \]
Frequency-dependent small-signal bipolar model

At frequencies less than 100-200MHz, lumped circuit models currently predict device behavior. Beyond that, frequency devices are more commonly described in terms of scattering parameters.

All pn junctions have associated voltage-depletion capacitances in the basic IC transistor structure. These include:

1. base-emitter junction, operated in forward bias
2. collector-base junction, operated in reverse bias
3. collector-substrate junction (if present) is always in reverse bias.
Due to the typical doping profiles, the CB junction behaves as an abrupt junction. The distributed depletion-region capacitance is modeled by

\[
C_m = \frac{C_m0}{(1 + \frac{V_{cm}}{V_0})^{\frac{1}{2}}}
\]

Value is about 8 \( \mu \)F for 2N3904A, 0.5 to 0.7 V

Similarly, collector-to-substrate

\[
C_{cs} = \frac{C_{cs0}}{(1 + \frac{V_{cs}}{V_0})^{\frac{1}{2}}}
\]

Typically 0.5 to several \( \mu \)F

Base-to-emitter

\[
C_{be} = C_b + C_{je} \approx C_b
\]

Typically a few \( \mu \)F

\[
C_b \approx \frac{\alpha_n q I_c}{kT}
\]

\[
\frac{q I_b}{I_c} = \text{base transit time}
\]
\[ V_\pi = I_s \left( \frac{1}{j\omega(c_\pi + c_m)} \right) r_\pi I_s \]

\[ = \frac{r_\pi}{1 + j\omega r_\pi (c_\pi + c_m)} I_s \]

\[ I_s \approx g_m V_\pi \quad \beta_0 = g_m r_\pi \]

\[ S_\omega = \frac{I_o}{I_s} (j\omega) = \frac{\beta_0}{1 + j\omega \frac{g_m}{g_m} (c_\pi + c_m)} = \frac{1}{w_p} \]

\[ |\beta(\omega = \omega_p)| = 1 \Rightarrow \]

\[ \omega_p = \frac{g_m}{c_\pi + c_m} \quad f_p = \frac{\omega_p}{2\pi} \approx 300 \text{MHz} \quad \text{for a 2N2222} \]