(CE) EE3444

For our purposes we work in terms of $V_i, I_i, V_o, I_o$.

Consider the following quantities:

1. Voltage Amplification
2. Input Resistance
3. Current Amplification
4. Output Resistance

Consider the following node equation, assuming $r_o \to \infty$.

\[ V_o \left[ g_o + G_c + G_L \right] + g_m V_{ir} = 0 \]

On the other hand

\[ V_{ir} = \frac{r_{\pi} V_i}{r_o + r_{\pi}} \]

So

\[ V_o \left[ g_o + G_c + G_L \right] + \frac{g_m r_{\pi} V_i}{r_o + r_{\pi}} = 0 \]
\[
A_{oi} = \frac{V_o}{V_i} = \frac{-g_m \pi r_t}{(g_o + g_c + g_L)(r_t + r_b)} \\
\approx -g_m (\frac{R_e}{r_t}) \quad \text{assuming} \quad \begin{array}{l}
g_o << g_c + g_L \\
r_b << r_t
\end{array}
\]

**Note:**
It is reasonable to neglect \( r_m \) as long as \( \frac{I_m}{I_t} < 0.05 \). This leads to \((g_o + g_c + g_L) > 20 g_m h \).

(2) \((\text{Input})\) we find \( Z_{in} = \frac{V_i}{I_i} \)

So, \( I_i = \frac{V_i}{R_b} + \frac{V_i}{R_b(r_t + r_b)} = \frac{V_i R_b + V_i r_t + r_b}{R_b(r_t + r_b)} \)

So, \( Z_{in} = \frac{V_i}{I_i} = \frac{R_b(r_t + r_b)}{R_b + r_t + r_b} \)

(3) \((\text{Current Amplification})\)

\[
A_i = \frac{I_o}{I_i} = \left( \frac{V_o}{V_i} \right) \left( \frac{R_e}{R_i} \right) = A_{oi} \left( \frac{R_i}{R_e} \right) = \frac{-g_m \pi R_e}{(g_o + g_c + g_L)(R_e + r_t + r_b) R_L}
\]
(4) Output Impedance Defined as resistance seen looking back into the amplifier with \( V_i = 0 \).

\[
\begin{align*}
\left( R_o \right) & = \frac{V_o}{I_x} = \frac{1}{2o + G_c}
\end{align*}
\]

Very commonly a CE amplifier is modified to have AC feedback via an emitter resistor. Let's see what effect that has:

\[
\begin{align*}
V_o & = (G_c + G_L) + g_m V_{\pi} = 0 \\
V_e & = (V_{\pi} g_\pi + g_m V_{\pi}) R_E \\
V_i & = V_{\pi} g_\pi (r + r_{\pi}) + V_e \\
& = V_{\pi} \left[ G_c g_\pi + 1 + (g_\pi + g_m) R_E \right]
\end{align*}
\]
Combining the 3 equations leads to

$$A_{vi} = \frac{V_o}{V_i} = \frac{-g_m}{[g_m r_o + 1 + (g_m + g_m) R_e][G_e + G_C]}$$

$$\approx \frac{-g_m}{(1 + gm R_e)(G_e + G_C)} \approx -\frac{R_{em} R_L}{R_e}$$

when \( g_m R_e \gg 1 \).

Working through a little algebra

$$R_i = \frac{V_i}{I_i} = \frac{R_e \left[ r_o + r_\pi + (1 + gm r_\pi) R_e \right]}{R_e + r_o + r_\pi + (1 + gm r_\pi) R_e}$$

$$\approx R_{em} \left[ r_\pi + (1 + \beta) R_e \right] \text{ for } r_\pi \gg r_o$$

$$A_i = \frac{I_o}{I_i} = \frac{V_o}{V_i} \frac{R_i}{R_L} = \frac{-g_m G_e R_e}{(G_e + G_C)(R_e + r_o + r_\pi + (1 + \beta) R_e)}$$

If \( r_o \) is ignored then

$$R_e \approx R_{em}.$$

If \( r_o \) is not ignored then

$$R_e \approx R_{em} \left( 1 + \frac{\beta_o R_e}{R_o + r_o + R_e} \right)$$
**Voltage Amplification**

At the collector node:

\[ V_o G_c + V_o G_e + V_o g_o + g_m V_r = V_i \frac{g_m}{r_m + r_o} \]

But \[ V_r = \frac{-r_m V_i}{r_m + r_o} \]

Combining these 2 equations leads to:

\[ A_v = \frac{V_o}{V_i} = \left[ \frac{1}{(G_c + G_e + g_o)} \right] \left[ g_o + \frac{g_m r_m}{r_m + r_o} \right] \]
\[ EE3444 \]

\[ \approx \frac{2m}{G_c + G_L} = gm R_{e1} R_L \]

making the same assumptions as before.

Input Resistance

\[ \frac{V_i}{r_{in}} \]

\[ I_i = \frac{V_i}{R_e} + \frac{V_i}{r_{in} + r_b} + \frac{V_i}{r_o} - \frac{V_o}{r_o} \approx gm V_i \]

Substituting in forms for \( A_{vi} \) and \( V_{in} \) we obtain

\[ R_i = \frac{V_i}{I_i} = \frac{r_{in} + r_b}{G_c (r_{in} + r_b) + 1 + gm r_{in}} \approx \frac{1}{G_c + gm} \]

Current Amplification

As before

\[ A_i = \frac{I_o}{I_i} = \frac{V_o}{V_i} \frac{R_i}{R_L} = \frac{gm}{(G_c + G_L) + \frac{G_c (r_{in} + r_b) + gm r_{in}}{G_c + gm}} \]

\[ \approx \frac{gm G_L}{(G_c + G_L)(G_c + gm)} \]