Several different sets of parameters can be used to describe a transistor. We may use impedance, admittance, hybrid or s-parameters, although others are possible as well. In this class we will use primarily hybrid parameters.

Hybrid Parameters

\[ V_{be} = h_{ie} I_b + h_{re} V_{ce} \]
\[ I_c = h_{fe} I_b + h_{oe} V_{ce} \]

is the set that is appropriate to the common-emitter configuration.

Definitions:

\[ h_{ie} = \left. \frac{dV_{BE}}{di} \right|_{i_b=0} = \frac{V_{be}}{i_b} \bigg|_{V_{ce}=0} = \frac{kT(\beta+1)}{q|Ie|} + r_b \]

From p-n junction equation

\[ = \frac{kT}{q|Ib|} + r_b \]

Physical resistance of base region.
Notice that $h_{re}$ is the CE input resistance with collector voltage held constant at the Q point.

$$h_{re} = \frac{\Delta V_{BE}}{\Delta V_{CE}} = \frac{V_{BE}}{V_{CE}} \bigg|_{i_b = 0}$$  
Result of base-narrowing effect.

This parameter is the CE voltage feedback ratio.

$$h_{fe} = \frac{\Delta I_C}{\Delta I_b} = \frac{i_c}{i_b} \bigg|_{v_a = 0}$$

This is the CE small-signal current gain.

$$h_{oe} = \frac{\Delta I_C}{\Delta V_{CE}} = \frac{i_c}{V_{CE}} \bigg|_{i_b = 0}$$  
Result of base-narrowing effect.

This is the CE output impedance.

Show examples
Hybrid-$\pi$ Model

This model is somewhat superior wrt high-frequency effects. However, it has slightly greater complexity.

Let's find the hybrid parameters in terms of the circuit components.

\[
V_{be} = R_b i_b + V_{\pi}
\]

\[
I_C = V_{be} \left( \frac{1}{g_0 + g_m} \right) + V_{\pi} \left( g_m - g_0 \right)
\]

Also,

\[
V_{\pi} = \frac{1}{g_m} \left[ i_b + \left( V_{ce} - V_{\pi} \right) g_m \right]
\]

We can combine these 3 expressions to obtain

\[
V_{be} = i_b \left( R_b + \frac{1}{1 + r_m g_m} \right) + V_{ce} \left( \frac{r_m g_m}{1 + r_m g_m} \right)
\]

\[
I_C = i_b \left( \frac{g_m - g_0}{1 + r_m g_m} \right) + V_{ce} \left( \frac{g_0 + g_m + \frac{r_m g_m^2}{1 + r_m g_m}}{1 + r_m g_m} \right)
\]
Comparing these last 2 equations with the form shown at the onset we find

$$h_{ie} = r_b + \frac{r_{\pi}}{1+r_{\pi}g_m} \approx r_b + r_{\pi}$$

$$h_{fe} = \frac{r_{\pi}(g_m - g_u)}{1+r_{\pi}g_m} \approx r_{\pi}g_m$$

$$h_{re} = \frac{r_{\pi}g_m}{1+r_{\pi}g_m} \approx r_{\pi}g_m \approx 0$$

$$h_{oe} = g_o + g_u + \frac{r_{\pi}g_m(g_m - g_u)}{1+r_{\pi}g_m} \approx g_o + r_{\pi}g_u g_m \approx g_o$$

**Mutual conductance or transconductance**

$$g_m = \frac{\partial I_e}{\partial V_{BE}} \bigg|_{V_{CE}=0} = \frac{g_o |I_e|}{kT}$$

Using the approximations shown above

$$r_{\pi} = \frac{h_{fe}}{g_m}$$

$$r_b = h_{ie} - r_{\pi}$$

$$g_o = h_{oe} - g_m h_{re}$$

$$r_m = \frac{r_{\pi}}{h_{re}}$$