1. (20 pts) Determine the rms value, \( V_{\text{rms}} \), for the waveform shown below.

![Waveform Diagram]

\[ v(t) = \left( \frac{9 - 3}{4 - 1} \right) t + b = \frac{6}{3} t + b = 2t + b \]

where \( b \) is the intercept. At \( t = 1 \) sec, \( v(1) = 3 \).

\[ 3 = 2(1) + b \] or \( b = 3 - 2 = 1 \), thus \( v(t) = 2t + 1 \)

\[ V_{\text{rms}} = \sqrt{\frac{1}{T} \int_{t=a}^{t=b} (2t + 1)^2 dt} = \sqrt{\frac{1}{3} \int_{t=1}^{4} (4t^2 + 4t + 1) dt} = \sqrt{\frac{1}{3} \left[ \frac{4}{3} t^3 + \frac{4}{2} t^2 + t \right]_{t=1}} \]

\[ = \sqrt{\frac{1}{3} \left[ (85.33 + 32 + 4) - (1.33 + 2 + 1) \right]} = \sqrt{\frac{117}{3}} = 6.245 \]

\( V_{\text{rms}} = 6.245 \) volts
2. (20 pts) An inductor has been added to the load in the circuit shown below in order to maximize the power absorbed by the resistor $R$. What value of resistance $R$ and what value of inductance, $L$, should be used to accomplish that objective?

![Circuit Diagram]

Solution

$$X_C = \frac{1}{\omega C} = \frac{10^6}{4 \times 10^3} = 250$$

$$Z_{load} = Z_{thev}$$ then

$$\frac{1}{Z_{load}} = \frac{1}{Z_{thev}}$$ therefore,

$$Y_{load} = Y_{thev}$$

$$Y_{thev} = \left(\frac{1}{500 - j250}\right)^* = 0.0016 - j0.0008$$

$$Y_{load} = \frac{1}{R} - j\frac{1}{\omega L} = 0.0016 - j0.0008$$

$$\therefore R = \frac{1}{0.0016} = 625 \, \Omega$$

$$\therefore \omega L = \frac{1}{0.0008} = 1,250 \, \Omega$$

$$\therefore L = \frac{1.250}{4,000} = 0.3125 \, H$$

$R = 625 \, \Omega$

$L = 0.3125 \, H$
3. (20 pts) Find the input impedance $Z$ for the circuit shown below.

![Circuit Diagram]

**Solution**

Express $I_3$ in terms of $V_1$ and $V_2$. Then express $V_2$ in terms of $V_1$

$$I_3 = \frac{V_1 - V_2}{6} = \frac{V_1 + 2V_1}{6} = \frac{3V_1}{6} = \frac{V_1}{2}$$

Current through 2 $\Omega$ resistor ($I_4$):

$$I_4 = I_2 + I_3 = \frac{V_2}{2}$$

but $I_3 = \frac{V_1}{2}$ therefore

$$I_2 = \frac{V_2}{2} - \frac{V_1}{2} = \frac{-2V_1}{2} - \frac{V_1}{2} = \frac{-3V_1}{2}$$

Now substitute for $I_2$

$$\frac{I_1}{2} = \frac{-3V_1}{2}$$

or $I_1 = 3V_1$ Now, back to $Z$

$$Z = \frac{V_1}{I_1 + I_3} = \frac{V_1}{3V_1 + \frac{V_1}{2}} = \frac{2}{6 + 1} = \frac{2}{7} \Omega = 0.2857 \Omega$$

$Z = 0.2857 \Omega$
4. A three-phase source with a line voltage of 35 kV rms is connected to two balanced loads. The Y-connected load has phase impedance of \( Z = 20 + j30 \, \Omega \), and the \( \Delta \) load has a phase impedance of \( 60 + j30 \, \Omega \). The connecting lines have an impedance of \( Z_{\text{line}} = 0.1 + j0.4 \, \Omega \). Determine (a) [10 pts] the three-phase power delivered to the loads, and (b) [10 pts] the three-phase power lost in the wires.

Solution

Convert the \( \Delta \)-load to Y-load equivalent

\[
Z_2 = \frac{60 + j30}{3} = 20 + j10
\]

The original Y-load impedance is \( Z_l = 20 + j30 \)

The equivalent source phase voltage is \( V_{an} = \frac{35,000[0^\circ]}{\sqrt{3}} = 20,207.26[0^\circ] \)

The equivalent impedance of both loads in parallel is

\[
Z_{\text{equ}} = \frac{(20 + j30)(20 + j10)}{20 + j30 + 20 + j10} = 11.25 + j8.75
\]

\[
I_{A1} = \frac{V_{an}}{Z_{\text{line}} + Z_{\text{equ}}} = \frac{20,207.26[0^\circ]}{0.1 + j0.4 + 11.25 + j8.75} = 1,386.06[-38.874^\circ]
\]

\[
P_{\text{load1}} = |I_{A1}|^2 R_{\text{equ}} = (1,386.06)^2 (11.25) = 21,613,076 \, \text{W}
\]

\[
P_{\text{load3}} = 3P_{\text{load1}} = 64,839,228 \, \text{W}
\]

\[
P_{\text{line1}} = |I_{A1}|^2 R_{\text{line}} = (1,386.06)^2 (0.1) = 192,116 \, \text{W}
\]

\[
P_{\text{line3}} = 3P_{\text{line1}} = 576,349 \, \text{W}
\]

(a) \( P_{\text{load}} = 64,839,228 \, \text{watts} \)

(b) \( P_{\text{line}} = 576,349 \, \text{watts} \)
5. The balanced three-phase load of a large commercial building requires 600 kW at a leading power factor of 0.707. The load is supplied by a connecting line with an impedance of $Z_{\text{line}} = 0.004 + j0.024 \ \Omega$ for each phase. The load has a line-to-line voltage of 480 V rms. Determine (a) [10 pts] the magnitude of the line current and the magnitude of the line voltage at the source, and (b) [10 pts] the power factor at the source. Use the line-to-neutral voltage at the load as the reference with an angle of zero degrees.

Solution

Power per phase in load: $P_{\phi} = \frac{600kW}{3} = 200kW$

Since power factor is 0.707 leading, the angle is $-45^{\circ}$, therefore $Q_{\phi} = -200kVAR$

Assume Y-connected load, then $V_{AN} = \frac{480}{\sqrt{3}} [0^{\circ}] = 277.13 [0^{\circ}]$

\[ I_{\phi} = \frac{S_{\phi}^{*}}{V_{AN}^{*}} = \frac{200,000 + j200,000}{277.13} = 1,020.61 [45^{\circ}] \]

\[ I_{\text{line}} = |I_{\phi}| = 1,020.61 \]

\[ V_{an} = Z_{\text{line}} I_{\phi} + V_{AN} = (0.004 + j0.024)1,020.61 [45^{\circ}] + 277.13 \]

\[ V_{an} = 263.47 [4.4^{\circ}] \]

\[ V_{line} = |V_{an}| = 263.47 \times \sqrt{3} = 456.35 \text{ volts} \]

$\theta_{pf} = \theta_{V_{an}} - \theta_{I_{\phi}} = 4.4^{\circ} - 45^{\circ} = -40.6^{\circ}$

\[ \text{power factor} = \cos(-40.6^{\circ}) = 0.759 \text{ (leading)} \]

(a) $I_{\text{line}} = 1.020.61 \text{ A}$

$V_{\text{line}} = 456.35 \text{ V}$

(b) power factor (source) 0.759 leading