1. (25 points) Calculate the effective value of the voltage across the resistance R shown in the circuit below when $\omega = 200 \text{ rad/s}$.

![Circuit Diagram]

**Solution**

The expression for effective value of a periodic function is the following:

$$V_{\text{eff}} = \sqrt{\frac{1}{T} \int_{t=0}^{T} (iR)^2 \, dt} \quad \text{but } i = i_{dc} + i_{ac1} + i_{ac2} \ , \text{ then}$$

$$i^2 = (i_{dc} + i_{ac1} + i_{ac2})^2 = i_{dc}^2 + i_{ac1}^2 + i_{ac2}^2 + 2i_{dc}i_{ac1} + 2i_{dc}i_{ac2} + 2i_{ac1}i_{ac2}$$

Note that integration of the double product terms over the period T yield 0.0

Note that $i_{ac1}^2 = (10 \cos \omega t)^2 = 10^2 \left(1 + \cos 2\omega t\right) \frac{1}{2}$

and $i_{ac2}^2 = (8 \sin \omega t)^2 = 8^2 \left(1 - \cos 2\omega t\right) \frac{1}{2}$

But integration of $\cos 2\omega t$ over the period T is zero, thus

$$V_{\text{eff}} = \sqrt{\frac{R^2}{T} \left(5^2 + \frac{10^2}{2} + \frac{8^2}{2}\right) T} = \sqrt{5^2 + \left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{8}{\sqrt{2}}\right)^2} = 10.34$$

The above result indicates a simple approach for a problem such as this where you have sources with different frequencies (yes, dc is a frequency – just zero). The “rule” from the above derivation is that the overall effective value is the square-root of the sum of squares of the individual rms values. Note that the rms value of a dc quantity is the quantity itself.

$$V_{\text{eff}} = \sqrt{(5)^2 + \left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{8}{\sqrt{2}}\right)^2} = \sqrt{107} = 10.34$$

$$V_{\text{eff}} = 10.34 \text{ volts}$$
2. (25 points) Determine the current, \( i_o(t) \), for the circuit shown below.

![Circuit Diagram]

Solution

The phasor form of this circuit is the following:

\[
\begin{align*}
\text{KCL: } I_1 &= I_o + I_2 \\
\text{(1) KVL including short and } j40 \text{ } \Omega \text{ coil} & \quad 0 = -j30I_1 + j40I_2 \\
\text{(2) KVL including source, short, and } j60 \text{ } \Omega \text{ coil} & \quad 20 = j60I_1 - j30I_2
\end{align*}
\]

Substitute \( I_1 = I_o + I_2 \) for each occurrence of \( I_1 \) in above two KVL equations.

\[
\begin{align*}
0 &= -j30(I_o + I_2) + j40I_2 = -j30I_o + j10I_2 \\
20 &= j60(I_o + I_2) - j30I_2 = j60I_o + j30I_2
\end{align*}
\]

Combine the two equations with two unknowns and solve for \( I_o \):

\[
I_o = \begin{vmatrix}
0 & j10 \\
20 & j30 \\
-j30 & j10 \\
j60 & j30
\end{vmatrix} = \frac{-j200}{1500} = 0.133 - 90^\circ
\]

Transforming back to the time-domain: \( i_o(t) = 0.133\cos(5t-90^\circ) \) Amps

\[
i_o(t) = 0.133\cos(5t-90^\circ) \text{ Amps}
\]
3. (25 points) Determine $V_2$ and $I_2$ for the circuit shown below. Express answers in polar form.

**Solution**

For this ideal transformer, $3 = V_2 / V_1$ (note that positive polarities for $V_1$ and $V_2$ are “consistent”) and $I_1 = 3I_2$ (note that currents are “inconsistent” with respect to how they enter dots).

$$Z_{equiv} = \frac{V_1}{I_1} = \frac{V_2}{3I_2} = \frac{1}{3} \frac{V_2}{I_2} = \frac{1}{9} (180 - j180) = 20 - j20$$

KVL around first loop:

$$20 = (20 + j20)I_1 + Z_{equiv} I_1 = (20 + j20)I_1 + (20 - j20)I_1 = 40I_1$$

Thus, $I_1 = 0.5$ and $V_1 = (20 - j20)(0.5) = 10 - j10$

Therefore:

$I_2 = I_1 / 3 = 0.5 / 3 = 0.167[0^\circ]$ 
$V_2 = 3V_1 = 3(10 - j10) = 42.43[45^\circ]$ 

$$V_2 = 42.43[45^\circ] \text{ Volts}$$

$$I_2 = 0.167[0^\circ] \text{ Amps}$$
(25 points) The figure below shows a balance $Y - \Delta$ three-phase circuit. The phase voltages of the $Y$-connected source are $V_a = 208/0^\circ$, $V_b = 208/-120^\circ$, and $V_c = 208/120^\circ$ volts rms. The line impedance are each $Z_L = 5+j10 \ \Omega$. The impedances of the $\Delta$-connected load are each $Z_\Delta = 90 + j120 \ \Omega$. Determine the average power delivered to the $\Delta$-connected load.

Solution

Convert $Z_\Delta$ to equivalent $Z_Y$: $Z_Y = (90 + j120)/3 = 30 + j40 \ \Omega$

Since three-phase system is balanced, a single-phase equivalent circuit is used:

$$V_a = (5 + j10 + 30 + j40) I_{ad} \ \text{or,}$$

$$I_{ad} = \frac{V_a}{35 + j50} = \frac{208}{35 + j50} = 3.408 -55.01^\circ$$

$$P_\phi = |I_{ad}|^2 R_r = (3.408)^2 (30) = 348.43$$

$$P_{\text{load}} = 3P_\phi = 3(348.43) = 1045.3$$

$$P_{\text{load}} = 1045.3 \ \text{watts}$$