1. (11 pts) Determine the current, \( i_0(t) \), for the circuit shown below when:
\[ v_s(t) = 20 \cos(5t + 45^\circ) \text{ volts} \]

![Circuit Diagram]

Solution

Note that \( I_2 = I_1 - I_o \)

Loop A: \( 10I_0 + j30I_1 - j40I_2 = 0 \)

Loop B: \( j40I_2 - j30I_1 + j60I_1 - j30I_2 = 20 \angle 45^\circ \)

Replace \( I_2 \) with \( I_1 - I_0 \) in above equations and combine terms

Loop A: \( (10 + j40)I_0 - j10I_1 = 0 \)

Loop B: \( -j10I_0 + j40I_1 = 20 \angle 45^\circ \)

Solve these two equations for \( I_0 \):

\[
I_0 = \begin{bmatrix} 0 & -j10 \\ 20 \angle 45^\circ & j40 \\ 10 + j40 & -j10 \\ -j10 & j40 \end{bmatrix}^{-1} = \begin{bmatrix} 200 \angle 135^\circ \\ 1552.42 \angle 165.07^\circ \end{bmatrix} = 0.129 \angle -30.07^\circ
\]

\[ \therefore i_0(t) = 0.129 \cos(5t - 30.07^\circ) \text{ amps} \]
2. (11 pts) Determine \( V_2 \) and \( I_2 \) for the circuit shown below. Express answers in polar form.

Solution

Equal volts per turn: \( \frac{V_1}{1} = -\frac{V_2}{3} \) (negative since polarities are inconsistent with dots)

Equal ampere turns: \( 1 \times I_1 = -3I_2 \) (negative since orientations are consistent with dots)

\[
Z_{\text{eq}} = \frac{V_1}{I_1} = -\frac{V_2}{3I_2} = \frac{1}{9} \frac{V_2}{I_2} = \frac{1}{9} (270 - j90) = 30 - j10
\]

\[
I_1 = \frac{40 \angle 0^\circ}{20 + j20 + 30 - j10} = \frac{40 \angle 0^\circ}{50 + j10} = 0.784 \angle -11.31^\circ
\]

\[
V_1 = Z_{\text{eq}} I_1 = (30 - j10)(0.784 \angle -11.31^\circ) = 24.81 \angle -29.74^\circ
\]

\[
V_2 = -3V_1 = 74.42 \angle 150.25^\circ \text{ volts}
\]

\[
I_2 = -\frac{1}{3} I_1 = 0.261 \angle 168.69^\circ \text{ amps}
\]

\[ V_2 = 74.42 \angle 150.25^\circ \text{ volts} \]

\[ I_2 = 0.261 \angle 168.69^\circ \text{ amps} \]
3. (11 pts) For the circuit shown below, $R/L = 8$ and $R_1/R_2 = 4$:
   a) Determine the transfer function $H(s) = V_o(s)/V_s(s)$
   
   b) For $v_s(t) = 20 \cos(30t + 45^\circ)$, determine $v_o(t)$.

Solution

$$H(s) = \frac{V_o}{V_s} = \left(\frac{R_1 + R_2}{R_1 + R_2}\right) \left(\frac{R}{R + sL}\right) = \left(1 + \frac{R_1}{R_2}\right) \left(\frac{R}{s + R/L}\right) = \left(1 + 4\right) \left(\frac{8}{s + 8}\right) = \frac{40}{s + 8}$$

$$H(30) = \frac{40}{j30 + 40} = 1.288 \angle -75.07^\circ$$

$$V_o = H(30)V_s = (1.288 \angle -75.07^\circ)(20 \angle 45^\circ) = 25.77 \angle -30.07^\circ$$

$$\therefore v_o(t) = 25.77 \cos(30t - 30.07^\circ) \text{ volts}$$

a) $H(s) = \frac{40}{s + 8}$

b) $v_o(t) = 25.77 \cos(30t - 30.07^\circ) \text{ volts}$
4. (11 pts.) Determine the voltage $v_o(t)$ for $t \geq 0$ for the circuit shown below.

Solution

For $t = 0^-$

Note that since $v_L(0^-) = 0$, $v_0(0^-) = 0$ and $i_L(0^-) = 4$

For $t \geq 0$

Write KCL equation

$$\frac{V_L}{2s} + \frac{V_L}{2} + \frac{V_L}{8} - \frac{3V_L}{8} = 0$$

$$V_L \left( \frac{1}{2s} + \frac{1}{2} - \frac{1}{4} \right) = \frac{-8}{2s} = \frac{-4}{s}$$

$$V_L \left( \frac{1}{2s} + \frac{1}{4} \right) = \frac{-4}{s}$$

$$V_L \left( \frac{s + 2}{4s} \right) = \frac{-4}{s}$$

$$V_L = \frac{-16}{s + 2}$$

$$V_0 = 3V_L = \frac{-3 \times 16}{s + 2} = \frac{-48}{s + 2}$$

$\therefore v_o(t) = -48e^{-2t}u(t)$ volts

$v_o(t) = -48e^{-2t}u(t)$ volts
5. (11 pts) A band-pass transfer function can be obtained as the product of low-pass and high-pass transfer functions, \( H_{BP}(s) = H_L(s) \times H_H(s) \), provided that the lower cutoff frequency, \( \omega_a \), is much smaller than the upper cutoff frequency, \( \omega_b \). Both the low-pass and high-pass filters in cascade are passive second order Butterworth (i.e., max gain in pass-band = 1). Obtain the transfer function of a fourth order band-pass filter having cutoff frequencies equal to 200 rad/s and 3000 rad/s.

Solution

\[
H_{LP} = \frac{1}{s^2 + 1.414s + 1} = \frac{1}{\left(\frac{s}{3000}\right)^2 + 1.414 \left(\frac{s}{3000}\right) + 1} = \frac{(3000)^2}{s^2 + 4242.6s + (3000)^2}
\]

\[
H_{HP} = \frac{s^2}{s^2 + 1.414s + 1} = \frac{s^2}{\left(\frac{s}{200}\right)^2 + 1.414 \left(\frac{s}{200}\right) + 1} = \frac{s^2}{s^2 + 282.8s + (200)^2}
\]

\[
H = H_{LP}H_{HP} = \left(\frac{9 \times 10^6}{s^2 + 4242.6s + 9 \times 10^6}\right) \left(\frac{s^2}{s^2 + 282.8s + 4 \times 10^4}\right)
\]

\[
H = \frac{9 \times 10^6 s^2}{(s^2 + 4242.6s + 9 \times 10^6)(s^2 + 282.8s + 4 \times 10^4)}
\]
6. (12 pts) A band-pass filter can be achieved by using two operational amplifiers as illustrated in the circuit shown below. Assume the Op-Amps are ideal.

a) Derive the transfer function \( H(s) = \frac{V_o}{V_s} \)

b) Find \( \omega_o, Q, \text{Bandwidth (BW)}, \text{and K (pass-band gain)} \) for this filter when \( R_1 = 2000 \, \Omega, \, R_2 = 200 \, \Omega, \, C_1 = 1 \, \mu F, \text{and} \, C_2 = 0.1 \, \mu F \)

Solution

Write two KCL equations, noting that input voltage at negative terminal (\( V_a \)) of first stage is equal to output voltage of first stage, and voltage at negative terminal of second stage (\( V_b \)) is zero:

1) \( sC_1(V_a - V_s) + \frac{1}{R_1}(V_a - V_0) = 0 \) 

2) \( \frac{1}{R_2}(V_b - V_a) + sC_2(V_b - V_0) = 0 \)

From equation 2:

\( -\frac{V_a}{R_2} - sC_2V_0 = 0 \) or, \( V_a = -sR_2C_2V_0 \)

Back to equation 1:

\( \left( sC_1 + \frac{1}{R_1} \right) V_a - sC_1V_s - \frac{V_0}{R_1} = 0 \)

Substitute for \( V_a \):

\( \left( \frac{sR_1C_1 + 1}{R_1} \right) \left( sR_2C_2 \right) V_0 + \frac{V_0}{R_1} = -sC_1V_s \)

\( V_0 \left[ \left( \frac{sR_1C_1 + 1}{R_1} \right) \left( sR_2C_2 \right) + 1 \right] = -sC_1V_s \)

\( H(s) = \frac{V_o}{V_s} = \frac{-sR_1C_1}{s^2R_1C_1R_2C_2 + R_2C_2s + 1} = \frac{-sR_2C_2}{s^2 + \left( \frac{1}{R_1C_1} \right)s + \frac{1}{R_1C_1R_2C_2}} \)

(problem 6 continued on next page)
The “standard” expression for a second order band-pass filter is compared to $H(s)$:

$$\frac{K \left( \frac{\omega_0}{Q} \right) s}{s^2 + \left( \frac{\omega_0}{Q} \right) s + \omega_0^2} = \frac{s}{R_2 C_2}$$

$$\frac{1}{R_1 C_1} = \frac{1}{(2 \times 10^3)(10^{-6})} = 500$$

$$\frac{1}{R_2 C_2} = \frac{1}{(200)(10^{-7})} = 50,000$$

$$\omega_0 = \sqrt{\frac{1}{R_1 C_1 R_2 C_2}} = \sqrt{(500)(50,000)} = 5,000$$

$$Q = \frac{\omega_0}{BW} = \frac{5,000}{500} = 10$$

$$K \times BW = \frac{1}{R_2 C_2} = 50,000 \text{ or}$$

$$K = \frac{50,000}{500} = 100$$

\[ a) \quad H(s) = \frac{-s}{R_2 C_2} \]  
\[ b) \quad \omega_0 = 5,000 \text{ rad/s} \]

$$Q = 10$$

$$BW = 500 \text{ rad/s}$$

$$K = 100$$
7. (11 pts) Determine the “Y” parameters of the circuit shown below.

\[ I_1 = Y_{11}V_1 + Y_{12}V_2 \\
I_2 = Y_{21}V_1 + Y_{22}V_2 \]

Solution

Note that since \( V_2 \) is zero, dependent current source is zero
\( V_1 = (4 + 2)I_1 = 6 \) and \( I_2 = -I_1 = -1 \)

\[ Y_{11} = \frac{I_1}{V_1} = \frac{1}{6} \quad \text{and} \quad Y_{21} = \frac{I_2}{V_1} = -\frac{1}{6} \]

Write two KCL equations, one for \( V_x \), one for \( V_2 \)

KCL for \( V_x \):
\[ \frac{V_x}{4} + 2V_x + \frac{V_x - V_2}{2} = 0 \]
\[ V_x \left( \frac{3}{4} \right) + V_2 \left( \frac{3}{2} \right) = 0 \]

KCL for \( V_2 \):
\[ \frac{V_2 - V_x}{2} + \frac{V_2}{3} = 1.0 \]
\[ -V_x \left( \frac{1}{2} \right) + V_2 \left( \frac{5}{6} \right) = 1.0 \]

Solve these two equations for \( V_x \) and \( V_2 \)
\( V_x = -\frac{12}{11} \) and \( V_2 = \frac{6}{11} \)
\( I_1 = -\frac{V_x}{4} = \frac{3}{11} \)
\( I_{12} = \frac{I_1}{V_2} = \frac{3/11}{6/11} = \frac{1}{2} \)
\( I_{22} = \frac{I_2}{V_2} = \frac{1/6}{1/11} = \frac{11}{6} \)

\[ Y_{11} = 1/6 \text{ S} \]
\[ Y_{21} = -1/6 \text{ S} \]
\[ Y_{12} = 1/2 \text{ S} \]
\[ Y_{22} = 11/6 \text{ S} \]
8. (11 pts) Determine the hybrid $g$ parameters for the circuit shown below (same circuit as in problem 7). Do not use the table of Parameter Relationships – compute the hybrid $g$ parameters from the circuit model below.

![Circuit Diagram](image)

$I_1 = g_{11}V_1 + g_{12}I_2$

$V_2 = g_{21}V_1 + g_{22}I_2$

**Solution**

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} \quad \text{and} \quad g_{21} = \left. \frac{V_2}{V_1} \right|_{I_1=0}$$

**KCL at "x":**

$$2V_2 + \frac{V_2}{3} = 1.0 \text{ or } V_2 = \frac{3}{7}$$

Current in $3\Omega$ resistor: 

$$I = \frac{V_2}{3} = \frac{1}{7}$$

$$V_1 = 4 \times I_1 + (2 + 3)I = 4 + 5\left(\frac{1}{7}\right) = \frac{33}{7}$$

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} = \frac{\frac{3}{7}}{\frac{33}{7}} = \frac{7}{33} \quad \text{and} \quad g_{21} = \left. \frac{V_2}{V_1} \right|_{I_1=0} = \frac{\frac{3}{7}}{\frac{33}{7}} = \frac{1}{11}$$

Since the second set of “$g$” parameters requires a short-circuit test on port 1, results from Problem 7 are used: $V_2 = \frac{6}{11}$, $I_1 = \frac{3}{11}$, and $I_2 = 1.0$

$$g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0} = \frac{\frac{3}{11}}{1} = \frac{3}{11}$$

$$g_{12} = \left. \frac{V_2}{I_2} \right|_{V_1=0} = \frac{\frac{6}{11}}{1} = \frac{6}{11}$$

$$g_{11} = \frac{7}{33}$$

$$g_{21} = \frac{1}{11}$$

$$g_{12} = \frac{3}{11}$$

$$g_{22} = \frac{6}{11}$$
9. (11 pts) Determine the inverse transmission $T'$ parameters (i.e., $A'$, $B'$, $C'$, $D'$) for the circuit shown below (same circuit as in problems 7 and 8).

Solution

Write KCL at $V_2$: \[ \frac{V_2}{2} + 2V_2 = 1.0 \text{ or } V_2 = \frac{3}{7} \]

Write KCL at $V_x$: \[ 2V_2 + \frac{V_x - V_2}{2} = \frac{4V_2 - V_2 + V_x}{2} = \frac{3V_2 + V_x}{2} = 0 \]

\[ \therefore V_x = -3V_2 = -3\left(\frac{3}{7}\right) = -\frac{9}{7} = V_1 \]

\[ A' = \left. \frac{V_2}{V_1} \right|_{I_1=0} = \frac{3/7}{-9/7} = -\frac{1}{3} \text{ and } C' = \left. \frac{I_2}{V_1} \right|_{V_1=0} = \frac{1}{-9/7} = -\frac{7}{9} \]

Since the second set of $T'$ parameters requires a short-circuit test on port 1, results from Problem 7 are used: $V_2 = 6/11$, $I_1 = 3/11$, and $I_2 = 1$.

\[ B' = \left. \frac{V_2}{-I_1} \right|_{V_1=0} = \frac{6}{-3/11} = -2 \text{ and } D' = \left. \frac{I_2}{-I_1} \right|_{V_1=0} = \frac{1}{-3/11} = -\frac{11}{3} \]

\[ A' = -\frac{1}{3} \]
\[ C' = -\frac{7}{9} \]
\[ B' = -2 \]
\[ D' = -\frac{11}{3} \]