17.1 Introduction

Many physical electronics devices have two ports (or terminal pairs) for input and output, respectively. Note voltage polarities and current orientations.

Examples of these devices include filters, transistors, coaxial cable (transmission line), power supplies, etc.

Likewise, the circuit models we use to represent these devices are two-port models.
17.2 T-to-Π Transformation and Two-Port Three-Terminal Networks

Two basic circuit configurations that occur frequently are the T network and the Π network.

![T-network](image)

(a) T-network (Y)

![Π-network](image)

(b) Π-network (Δ)

Sometimes it is more convenient to convert from one to the other. To develop the conversion, it is best done by representing the T- and Π-networks as three terminal networks.

![T-to-Π conversion](image)

(a) T-network

(b) Π-network

(a) T-network (Y)

(b) Π-network (Δ)
For equivalence, the two networks must have the same impedance when measured between the same pair of terminals.

Between terminal pair a-c:

\[ Z_1 + Z_3 = \frac{Z_A(Z_B + Z_C)}{Z_A + Z_B + Z_C} \]

Between terminal pair a-b:

\[ Z_1 + Z_2 = \frac{Z_C(Z_A + Z_B)}{Z_A + Z_B + Z_C} \]

Between terminal pair b-c:

\[ Z_2 + Z_3 = \frac{Z_B(Z_A + Z_C)}{Z_A + Z_B + Z_C} \]

After some effort the conversion from \( \Pi \) to T becomes...

And T to \( \Pi \) becomes...

\[ Z_A = Z_1Z_2 + Z_2Z_3 + Z_3Z_1 \]

\[ Z_B = Z_1Z_2 + Z_2Z_3 + Z_3Z_1 \]

\[ Z_C = Z_1Z_2 + Z_2Z_3 + Z_3Z_1 \]

(pg 802 and 803)

If all impedances are equal in these networks, then

\[ Z_T = \frac{Z_\Pi}{3} \quad \text{or} \quad Z_\Pi = 3Z_T \]
Exercises in Section 17-2

Exercise 17.2.1 (pg 804)

Find the T circuit equivalent to the Π circuit shown in Figure E 17.2.1.

\[ Z_1 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} = \frac{(100)(25)}{100 + 125 + 25} = \frac{2500}{250} = 10 \Omega \]

\[ Z_2 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} = \frac{(125)(25)}{250} = \frac{3125}{250} = 10 \Omega \]

\[ Z_A = 100 \Omega \]
\[ Z_B = 125 \Omega \]
\[ Z_C = 25 \Omega \]
Exercise 17.2.1 (cont’d)

\[ Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} = \frac{(100)(125)}{100 + 125 + 25} = \Omega \]

17.3 Equations of Two-port Networks

There are four variables associated with a two-port network: \( V_1 \), \( I_1 \), \( V_2 \), and \( I_2 \).

This means there are six combinations of two variables as input (independent variable) and the other two variables as output (dependent variables).

<table>
<thead>
<tr>
<th>Inputs ( (I_1, I_2) )</th>
<th>Outputs ( (V_1, V_2) )</th>
<th>Circuit parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1, I_2 )</td>
<td>( V_1, V_2 )</td>
<td>Impedance ( Z )</td>
</tr>
<tr>
<td>( V_1, V_2 )</td>
<td>( I_1, I_2 )</td>
<td>Admittance ( Y )</td>
</tr>
<tr>
<td>( V_1, I_2 )</td>
<td>( I_1, V_2 )</td>
<td>Hybrid ( g )</td>
</tr>
<tr>
<td>( I_1, V_2 )</td>
<td>( V_1, I_2 )</td>
<td>Hybrid ( h )</td>
</tr>
<tr>
<td>( V_2, I_2 )</td>
<td>( V_1, I_1 )</td>
<td>Transmission ( T )</td>
</tr>
<tr>
<td>( V_1, I_1 )</td>
<td>( V_2, I_2 )</td>
<td>Inverse Transmission ( T' )</td>
</tr>
</tbody>
</table>
### Table 17.3-1 Equations for the Six Sets of Two-Port Parameters (pg 805)

<table>
<thead>
<tr>
<th>Circuit parameters</th>
<th>Equations</th>
<th>Matrix Expressions</th>
</tr>
</thead>
</table>
| Impedance Z | \[
\begin{align*}
V_1' &= Z_{11} I_1 + Z_{12} I_2 \\
V_2' &= Z_{21} I_1 + Z_{22} I_2
\end{align*}
\] | \[
V = Z I = Y^{-1} I
\] |
| Admittance Y | \[
\begin{align*}
I_1 &= Y_{11} V_1 + Y_{12} V_2 \\
I_2 &= Y_{21} V_1 + Y_{22} V_2
\end{align*}
\] | \[
I = Y V = Z^{-1} V
\] |
| Hybrid h | \[
\begin{align*}
I_1 &= h_{11} I_1 + h_{12} I_2 \\
I_2 &= h_{21} I_1 + h_{22} I_2
\end{align*}
\] | \[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix} G \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\
I_2
\end{bmatrix}
\] |
| Inverse Hybrid g | \[
\begin{align*}
I_1 &= g_{11} V_1 + g_{12} V_2 \\
I_2 &= g_{21} V_1 + g_{22} V_2
\end{align*}
\] | \[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix} G \end{bmatrix} \begin{bmatrix} V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix} H \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\
V_2
\end{bmatrix}
\] |
| Transmission T | \[
\begin{align*}
V_1 &= AV_2 - BI_2 \\
I_1 &= CV_2 - DI_2
\end{align*}
\] | \[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix} T \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\
V_2
\end{bmatrix}
\] |
| Inverse Transmission T' | \[
\begin{align*}
V_1' &= A' V_2' - B' I_1' \\
I_1' &= C' V_2' - D' I_1'
\end{align*}
\] | \[
\begin{bmatrix}
V_1' \\
V_2'
\end{bmatrix} = \begin{bmatrix} T' \end{bmatrix} \begin{bmatrix} I_1' \\
I_2'
\end{bmatrix} = \begin{bmatrix} T' \end{bmatrix}^{-1} \begin{bmatrix} V_1' \\
V_2'
\end{bmatrix}
\] |

### Obtaining the Z Parameters

Z are called the “Open-Circuit: parameters
Y are called the “Short-Circuit: parameters

\[
\begin{align*}
V_1 &= Z_{11} I_1 + Z_{12} I_2 \\
V_2 &= Z_{21} I_1 + Z_{22} I_2
\end{align*}
\]

The impedance parameters also called “Driving Point and Transfer Impedances”

\[
\begin{align*}
Z_{11} &= \left. \frac{V_1}{I_1} \right|_{I_2 = 0} \\
Z_{21} &= \left. \frac{V_2}{I_1} \right|_{I_2 = 0} \quad \text{Terminal 2 is open}
\end{align*}
\]

\[
\begin{align*}
Z_{12} &= \left. \frac{V_1}{I_2} \right|_{I_1 = 0} \\
Z_{22} &= \left. \frac{V_2}{I_2} \right|_{I_1 = 0} \quad \text{Terminal 1 is open}
\end{align*}
\]
Obtaining the Y Parameters …

\[ I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{Admittance Parameters} \]
\[ I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{The admittance parameters also called “Driving Point and Transfer Admittances”} \]

\[
\begin{align*}
Y_{11} &= \left. \frac{I_1}{V_1} \right|_{V_2=0} \\
Y_{21} &= \left. \frac{I_2}{V_1} \right|_{V_2=0} \\
Y_{12} &= \left. \frac{I_1}{V_2} \right|_{V_1=0} \\
Y_{22} &= \left. \frac{I_2}{V_2} \right|_{V_1=0}
\end{align*}
\]

\[ \begin{cases} 
\text{Terminal 2 is short circuited} \\
\text{Terminal 1 is short circuited}
\end{cases} \]

For linear bi-lateral circuit with no dependent sources

\[
\begin{align*}
Z_{12} &= Z_{21} \\
Y_{12} &= Y_{21}
\end{align*}
\]

Exercises in Section 17-3
Exercise 17.3-1 (pg 808)

Find the Z and Y parameters of the circuit of Fig E 17.3-1

![Circuit Diagram]

Exercise 17.3-1 (Cont’d)

I will introduce an alternative method to that described in Table 17.3-3 (pg. 807) for obtaining the Z parameters.

Step 1. To determine $Z_{11}$ and $Z_{21}$, connect a current source equal to 1 A to the input terminals. Injection of 1 A is consistent with assumed direction of $I_1$.

Step 2. Determine $V_1$ and $V_2$, thus $Z_{11} = V_1 / I_1$ and $Z_{21} = V_2 / I_1 = V_2$.

Step 3. To determine $Z_{12}$ and $Z_{22}$, connect a current source equal to 1 A to the output terminals. Injection of 1 A is consistent with assumed direction of $I_2$.

Step 4. Determine $V_1$ and $V_2$, thus $Z_{22} = V_2 / I_2 = V_2$ and $Z_{12} = V_1 / I_2 = V_1$. 
Exercise 17.3-1 (Cont’d)

Back to our Exercise:

\[ I_1 = 1 \text{ A} \]

Current divider to determine current through 42 ohm times 42 ohm resistor yields \( V_1 = Z_{11} \).

\[
V_1 = (1) \left[ \frac{1}{42} + \frac{1}{42 + \frac{1}{10.5 + 21}} \right] (42) = 18
\]

\[
\therefore Z_{11} = \frac{10.5}{21 + 10.5} \Omega
\]

Current divider to determine current through 10.5 ohm times 10.5 ohm resistor yields \( V_2 = Z_{21} \).

\[
V_2 = (1) \left[ \frac{1}{21 + 10.5} + \frac{1}{21 + 10.5 + 21} \right] (10.5) = 6
\]

\[
\therefore Z_{21} = \frac{1}{21 + 10.5} \Omega
\]
Exercise 17.3-1 (Cont’d)

Current divider to determine current through 10.5 ohm times 10.5 ohm resistor yields \( V_2 = Z_{22} \).

\[
V_2 = (1) \left[ \frac{1}{10.5} \left( \frac{1}{10.5} + \frac{1}{42 + 21} \right) \right] (10.5) = 9
\]

\[ \therefore Z_{22} = \text{[value]} \ \Omega \]

Exercise 17.3-1 (Cont’d)

Current divider to determine current through 42 ohm times 42 ohm resistor yields \( V_1 = Z_{12} \).

\[
V_1 = (1) \left[ \frac{1}{42 + 21} \left( \frac{1}{10.5} + \frac{1}{42 + 21} \right) \right] (42) = 6
\]

\[ \therefore Z_{12} = \text{[value]} \ \Omega \]
Exercise 17.3-1 (Cont’d)

Now let’s determine the Y parameters using the 1 A current injection method.

\[ I_1 = 1 \text{ A} \]

Current divider to determine current through 42 ohm times 42 ohms yields \( V_1 \). Thus \( Y_{11} = I_T / V_1 = 1 / V_1 \)

\[
V_1 = (1) \left[ \frac{1}{42} \right] \left( \frac{1}{42} \right) = 14, \quad \therefore Y_{11} = \frac{1}{V_1} = \frac{1}{14}
\]

Exercise 17.3-1 (Cont’d)

Current divider to determine \( I_2 \). Then \( Y_{21} = I_2 / V_1 \)

\[
I_2 = (-1) \left[ \frac{1}{21} \right] = -\frac{42}{63}
\]

\[
Y_{21} = \frac{-42}{14} = \frac{-42}{(14)(63)} = \frac{1}{14}
\]
Exercise 17.3-1 (Cont’d)

Current divider to determine current through 10.5 ohm times 10.5 ohm resistor yields $V_2$. Thus $Y_{22} = I_T / V_2 = 1 / V_2$

$$V_2 = (1) \left[ \frac{1}{10.5} \right] \left( \frac{1}{10.5 + \frac{1}{21}} \right) (10.5) = 7, \quad \therefore Y_{22} = \frac{1}{7}$$

Current divider to determine current through $I_1$. Then $Y_{12} = I_1 / V_2$

$$I_1 = (-1) \left[ \frac{1}{10.5 + \frac{1}{21}} \right] = -\frac{1}{3}$$

$$\therefore Y_{12} = \left( -\frac{1}{3} \right) \left( \frac{1}{7} \right) = \frac{1}{21}$$
17.4 Z and Y Parameters for Circuits with Dependent Sources

Again, I will use the current injection method to illustrate an alternative. Please refer to the author’s example for their methods described in Tables 17.3-3 and 17.3-4 (pp 807 & 808).

We will do example 17.4-1 (pg 808) but determine both Z (Example 17.4-1) and Y (Exercise 17.4-1, pg 810).

Where \( m = \frac{2}{3} \)

---

Inject 1 A at input with output open.

KCL involving node associated with \( V_x \)

\[
\frac{2}{3} V_2 + I = 1 \quad \text{also} \quad I = \frac{V_2}{3}
\]
\[ \frac{2}{3} V_2 + \frac{V_2}{3} = 1 \quad \text{or} \quad V_2 = Z_{21} = \frac{I}{3} = \frac{V_2}{3} = \frac{1}{3} \]

Now, \( V_x = I(2 + 3) = \left( \frac{1}{3} \right) (5) = \frac{5}{3} \)

\( \therefore V_2 = (1)(4) + \frac{5}{3} = Z_{11} = \frac{\Omega}{3} \)

Inject 1 A at output terminal with input open.

---

KCL at “+” terminal of \( V_2 \).

\[ \frac{V_2}{3} + \frac{2}{3} V_2 = I_T = 1 \quad \text{or} \quad V_2 = Z_{22} = \frac{\Omega}{3} \]

Then current

\( 2/3 V_2 = 2/3 A, \text{ and } V_x = V_2 = (2/3) = 1 - 4/3 = -1/3 \)

\( \therefore Z_{12} = \frac{\Omega}{3} \)

\[ Z = \begin{bmatrix} \end{bmatrix} \]
Exercises in Section 17-4

Exercise 17.4-1 (pg 810)

Find Y parameters

\[ I_1 = I_T \]

\[ I_1 = \frac{V_V}{R} \]

\[ V_2 = 0 \]

Since \( V_2 = 0 \) the Dependent Current source is 0 A

\[ I_1 = I_T \]

\[ I_2 = \frac{V_V}{R} \]

\[ I_2 = 0 \]

\[ V_2 = 0 \]
Exercise 17.4-1 (cont’d)

\[ V_1 = (4 + 2)(1) = 6 \]

\[ \therefore Y_{11} = \frac{I_1}{V_1} = \quad \]

And \(-I_2 = 1 \text{ A or } I_2 = -1 \text{ A}\)

\[ \therefore Y_{21} = \frac{I_2}{V_1} = \quad \]

Exercise 17.4-1 (cont’d)

Inject 1 A at output terminal with input shorted. (i.e., \( V_1 = 0 \))

\[ I_T = I_2 = 1 \text{ A} \]
Exercise 17.4-1 (cont’d)

KCL about nodes associated with \( V_x \) and \( V_2 \)

Node "x": \( \frac{V_x}{4} + \frac{2}{3} V_2 + \frac{V_x - V_2}{2} = 0 \)

Node "2": \( \frac{V_2}{3} + \frac{V_2 - V_x}{2} = 1 \)

Rearranging these two equations yields the following:

\[
\frac{3}{4} V_x + \frac{1}{6} V_2 = 0 \\
-\frac{1}{2} V_x + \frac{5}{6} V_2 = 1
\]

Solving for \( V_x \) and \( V_2 \):

\( V_2 = \frac{18}{17} \) but \( Y_{22} = \frac{I_2}{V_2} = \frac{17}{18} \)

\( V_x = -\frac{4}{17} \)

\( -I_1 = \frac{V_x}{4} = \left( -\frac{4}{17} \right) \left( \frac{1}{4} \right) = -\frac{1}{17} \)

\( \therefore I_1 = \frac{1}{17} \) and \( Y_{12} = \frac{I_1}{V_2} = \frac{1/17}{18/17} = \frac{1}{18} \)

\[
Y = \begin{bmatrix}
\end{bmatrix}
\]

Work Exercise 17.5-1
For Z & Y parameters
(pg 812)
17.5 Hybrid and Transmission Parameters

Hybrids are used principally in transistor circuits

**Hybrid h**

\[ V_1^h = h_{11} I_1 + h_{12} V_2 \]
\[ I_2^h = h_{21} I_1 + h_{22} V_2 \]

**Hybrid g**

\[ I_1^g = g_{11} V_1 + g_{12} I_2 \]
\[ V_2^g = g_{21} V_1 + g_{22} I_2 \]

Hybrid **h**

\[ h_{11} = \frac{V_1}{I_1} \bigg|_{V_2=0} \]
\[ h_{21} = \frac{I_2}{I_1} \bigg|_{V_2=0} \]
\[ h_{12} = \frac{V_1}{V_2} \bigg|_{I_1=0} \]
\[ h_{22} = \frac{I_2}{V_2} \bigg|_{I_1=0} \]

Hybrid **g**

\[ g_{11} = \frac{I_1}{V_1} \bigg|_{I_2=0} \]
\[ g_{21} = \frac{V_2}{V_1} \bigg|_{I_2=0} \]
\[ g_{12} = \frac{I_1}{I_2} \bigg|_{V_1=0} \]
\[ g_{22} = \frac{V_2}{I_2} \bigg|_{V_1=0} \]
Transmission Parameters are used to describe cable, fiber, and transmission lines

\[ V_1 = AV_2 - BI_2 \]
\[ I_1 = CV_2 - DI_2 \]
or in matrix form, as

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix} = T
\begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix}
\]

\[
A = \frac{V_1}{V_2} \bigg|_{I_2=0} \\
C = \frac{I_1}{V_2} \bigg|_{I_2=0}
\]
\[
B = \frac{V_1}{-I_2} \bigg|_{I_1=0} \\
D = \frac{I_1}{-I_2} \bigg|_{I_1=0}
\]

Note: The reason \( I_2 \) is negative is because the Transmission model Requires that \( I_2 \) be in the opposite direction of the two-port model.

Inverse Transmission Parameters

\[ V_2 = AV_1 - B'I_1 \]
\[ I_2 = CV_1 - D'I_1 \]
or in matrix form, as

\[
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
A & B' \\
C & D'
\end{bmatrix}
\begin{bmatrix}
V_1 \\
-I_1
\end{bmatrix} = T'
\begin{bmatrix}
V_1 \\
-I_1
\end{bmatrix}
\]

\[
A' = \frac{V_2}{V_1} \bigg|_{I_1=0} \\
C' = \frac{I_2}{V_1} \bigg|_{I_1=0}
\]
\[
B' = \frac{V_2}{-I_1} \bigg|_{I_1=0} \\
D' = \frac{I_2}{-I_1} \bigg|_{I_1=0}
\]

Note: The reason \( I_1 \) is negative is because the Transmission model Requires that \( I_1 \) be in the opposite direction of the two-port model.
Exercises in Section 17-5

Exercise 17.5-1 (pg 812)

Find the hybrid h parameter model of the circuit shown.

To compute $h_{11}$ and $h_{21}$, we inject a 1 A current at the input and short the output ($V_2 = 0$)
Exercise 17.5-1 (cont’d)

I₁ = Iᵣ

I₁ = 1 A

V₁ + \frac{V₁}{9} = 1 \text{ or } V₁ = \frac{9}{10} = 0.9

h₁₁ = \left. \frac{V₁}{I₁} \right|_{V₂=0} =

Exercise 17.5-1 (cont’d)

The current I = \frac{V₁}{I₁} = 0.9

\therefore 1.0 - I = 5I - I₂

I₂ = 6I - 1.0 = (6)(0.9) - 1.0 = 4.4

\therefore h₂₁ = \left. \frac{I₂}{I₁} \right|_{V₂=0} =
Exercise 17.5-1 (cont’d)

Now, we inject 1 A at output and open the input.

\[ I_2 = I_T = 1 \text{A} \]

\[ V_1 = (1 + 9)I = \frac{10}{6} \text{V} \]

\[ V_2 = (1)I = \frac{1}{6} \text{V} \]

\[ \begin{bmatrix} h_{12} = \frac{V_1}{V_2} \bigg|_{I_i=0} & = \frac{1}{6} \bigg|_{10/6} \\ h_{22} = \frac{I_2}{V_2} \bigg|_{I_i=0} & = \frac{1}{10/6} \end{bmatrix} \]

\[ H = \begin{bmatrix} h_{12} & \end{bmatrix} \]
Exercise 17.5-1 (cont’d)

Let’s continue the Exercise by computing the “g” parameters.
We find $g_{11}$ and $g_{21}$ by opening $I_2 = 0$

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2 = 0} \quad g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2 = 0}$$

We inject 1 A current for $I_1$ and determine $V_1$ and determine $V_2$.

\[ I_1 = 1 \text{ A} \]

\[ I_1 = I_T \]

\[ I_2 = 0 \]

\[ I_1 = 1 \text{ A} \]

\[ \begin{align*}
  KCL: \quad & I + 5I = I_1 = 1 \\
  & I = 1/6 \text{ A} \\
  \therefore V_1 = (1)I = 1/6 \\
  g_{11} = \frac{1}{V_1} = \\
  KVL: \quad & (9)(5I) + V_2 = V_1 \\
  & V_2 = V_1 - 45I = 1/6 - 45/6 = -44/6 \\
  g_{21} = \frac{V_2}{V_1} = \frac{-44/6}{1/6} = \\
\end{align*} \]
Exercise 17.5-1 (cont’d)

We now inject 1 A current at the output terminal and short circuit the input terminals (i.e., $V_1 = 0$)

$I = 0$ due to short circuit, thus “5I” dependent current source is zero.

\[
\begin{align*}
I_1 = -I_2 = -1A \\
V_2 = (9)(I_2) = 9 \\
g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1 = 0} = \boxed{1} \\
g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1 = 0} = \boxed{9} \\
\therefore g = \begin{bmatrix} 1 \\ 9 \end{bmatrix}
\end{align*}
\]
Exercise 17.5-1 (cont’d)

Let’s continue our Exercise by computing the T and T’ parameters

\[
A = \frac{V_1}{V_2} \bigg|_{I_1=0} \quad B = -\frac{V_1}{I_2} \bigg|_{V_1=0}
\]

\[
C = \frac{I_1}{V_2} \bigg|_{I_1=0} \quad D = -\frac{I_2}{I_2} \bigg|_{V_2=0}
\]

We inject 1.0 A current at the input terminal open circuit the output

From previous slide where we calculated “g” parameters:

\[
V_1 = \frac{1}{6}, \quad V_2 = -\frac{44}{6}
\]

\[
\therefore A = \frac{V_1}{V_2} = \frac{\frac{1}{6}}{-\frac{44}{6}} = -\frac{1}{44}
\]

Open Circuit Voltage Ratio

\[
C = \frac{I_1}{V_2} = -\frac{6}{44} = -\frac{3}{22}
\]

Open Circuit transfer admittance
Exercise 17.5-1 (cont’d)

The B and D parameters are determined by leaving the 1.0 A current injected at the input terminals and shorting the output terminals.

From previous slides:

\[ V_1 = 0.9, \quad I_2 = 4.4 \]

\[
B = -\frac{V_1}{I_2} = \frac{-0.9}{4.4} = \text{Short circuit terms for impedance}
\]

\[
D = -\frac{I_1}{I_2} = \frac{1}{4.4} = \text{Short circuit current ratio}
\]

---

Exercise 17.5-1 (cont’d)

For the inverse transmission, T' parameters, inject 1 A current at the output terminal

\[
A' = \frac{V_2}{V_1} \bigg|_{I_i=0}, \quad B' = -\frac{V_2}{I_1} \bigg|_{V_i=0}
\]

\[
C' = \frac{I_2}{V_1} \bigg|_{I_i=0}, \quad D' = -\frac{I_2}{I_1} \bigg|_{V_i=0}
\]

see previous slide

\[ V_1 = \frac{1}{6}, \quad V_2 = \frac{10}{6} \]
Exercise 17.5-1 (cont’d)

\[ A' = \frac{V_2}{V_1} = \frac{10}{6} = \frac{5}{3} \]  open circuit inverse voltage ratio

\[ C' = \frac{I_2}{I_1} = \frac{1}{1 / 6} = 6 \]  open circuit admittance transfer

We now short circuit the input terminal \((V_1 = 0)\) see previous slide

\[ I_i = -1 \, A, \quad V_2 = 9 \]

Exercise 17.5-1 (cont’d)

\[ B' = \frac{V_2}{I_i} = \frac{-9}{-1} = 9 \]  Short circuit impedance transfer

\[ D' = \frac{I_2}{I_i} = \frac{-1}{-1} = 1 \]  Short circuit inverse current ratio
17.6 Relationships Between Two-Port Networks

If all the two-port parameters for a circuit exist, it is possible to relate one set of parameters to another, since the variables $V_1$, $I_1$, $V_2$ and $I_2$ are interrelated by the parameters.

### Table 17.6-1 Parameter Relationships between Two-Port Networks

<table>
<thead>
<tr>
<th></th>
<th>$Z$</th>
<th>$Y$</th>
<th>$H$</th>
<th>$G$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
<td>$Z_{11}$</td>
<td>$Z_{21}$</td>
<td>$Y_{11}$</td>
<td>$Y_{12}$</td>
<td>$Y_{21}$</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>$Z_{22}$</td>
<td>$Y_{22}$</td>
<td>$Y_{12}$</td>
<td>$Y_{11}$</td>
<td>$Y_{21}$</td>
</tr>
<tr>
<td>$H_{11}$</td>
<td>$h_{11}$</td>
<td>$h_{12}$</td>
<td>$h_{11}$</td>
<td>$h_{12}$</td>
<td>$h_{11}$</td>
</tr>
<tr>
<td>$H_{12}$</td>
<td>$h_{12}$</td>
<td>$h_{11}$</td>
<td>$h_{12}$</td>
<td>$h_{11}$</td>
<td>$h_{12}$</td>
</tr>
<tr>
<td>$G_{11}$</td>
<td>$g_{11}$</td>
<td>$g_{12}$</td>
<td>$g_{11}$</td>
<td>$g_{12}$</td>
<td>$g_{11}$</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>$g_{12}$</td>
<td>$g_{11}$</td>
<td>$g_{12}$</td>
<td>$g_{11}$</td>
<td>$g_{12}$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>$t_{11}$</td>
<td>$t_{12}$</td>
<td>$t_{11}$</td>
<td>$t_{12}$</td>
<td>$t_{11}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$t_{22}$</td>
<td>$t_{22}$</td>
<td>$t_{21}$</td>
<td>$t_{21}$</td>
<td>$t_{22}$</td>
</tr>
</tbody>
</table>

$\Delta Z = Z_1 Z_2 - Z_2 Z_1$, $\Delta Y = Y_{11} Y_{22} - Y_{21} Y_{12}$, $\Delta H = h_{11} h_{22} - h_{21} h_{12}$, $\Delta G = g_{11} g_{22} - g_{21} g_{12}$, $\Delta T = AD - BC$
Relationships between Two-Port Parameters

Provided parameters exist for $Z$, $Y$, $h$, $g$, $T$ and $T'$ they can be related to one another (see table 17.6-1, pg 813)

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = Z \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}	ext{ and }
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = Z^{-1} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

\[Y = Z^{-1} \quad \text{and} \quad Z = Y^{-1}\]

\[
\begin{bmatrix}
Y_{11} \\
Y_{12} \\
Y_{21} \\
Y_{22}
\end{bmatrix} = \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
h_{11} = \frac{V_1}{I_1} \\
h_{12} = \frac{V_1}{I_2}
\end{bmatrix}_{I_2=0} \quad \begin{bmatrix}
h_{21} = \frac{I_2}{I_1} \\
h_{22} = \frac{I_2}{V_2}
\end{bmatrix}_{I_2=0}
\]
Relationships between Two-Port Parameters

\[ Y \text{ & } H \text{ cont'd} \]

\[ h_{11} = \left. \frac{I_1}{V_1} \right|_{V_2' = 0} = \frac{1}{Y_{11}} \]

\[ h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2' = 0} = \frac{I_2}{V_1} \left| \frac{I_1}{V_1} \right|_{V_2' = 0} = \frac{Y_{21}}{Y_{11}} \]

\[ Y \text{ & } H \text{ cont'd} \]

\[ h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1' = 0} = \frac{1}{V_2} \left| \frac{I_2}{I_1} \right|_{I_1' = 0} = \frac{1}{Z_{22}} \]

\[ But \quad Z_{22} = \frac{Y_{11}}{\Delta Y} \quad \text{where} \quad \Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} \]

\[ h_{22} = \frac{\Delta Y}{Y_{11}} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}} \]
Relationships between Two-Port Parameters

**Y & H cont’d**

\[ h_{12} = \frac{V_1}{V_2} \bigg|_{I_1=0} = \frac{V_1}{I_2} \bigg|_{I_1=0} = \frac{Z_{12}}{Z_{22}} = \frac{-Y_{12}}{Y_{11}/\Delta Y} \]

\[ \therefore h_{12} = \frac{-Y_{12}}{Y_{11}} \]

<table>
<thead>
<tr>
<th>( Z )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{11} = \frac{V_1}{I_1} \bigg</td>
<td><em>{I_2=0} ) &amp; ( Y</em>{11} = \frac{I_1}{V_1} \bigg</td>
</tr>
<tr>
<td>( Z_{21} = \frac{V_2}{I_1} \bigg</td>
<td><em>{I_2=0} ) &amp; ( Y</em>{21} = \frac{I_2}{V_1} \bigg</td>
</tr>
<tr>
<td>( Z_{12} = \frac{V_1}{I_2} \bigg</td>
<td><em>{I_1=0} ) &amp; ( Y</em>{12} = \frac{I_1}{V_2} \bigg</td>
</tr>
<tr>
<td>( Z_{22} = \frac{V_2}{I_2} \bigg</td>
<td><em>{I_1=0} ) &amp; ( Y</em>{22} = \frac{I_2}{V_2} \bigg</td>
</tr>
</tbody>
</table>

\[ Z = Y^{-1} \text{ or } Y = Z^{-1} \]
\[ h_{11} = \frac{V_1}{I_1} | V_2 = 0 \]
\[ h_{21} = \frac{I_2}{I_1} | V_2 = 0 \]
\[ h_{12} = \frac{V_1}{V_2} | I_1 = 0 \]
\[ h_{22} = \frac{I_2}{V_2} | I_1 = 0 \]

\[ g_{11} = \frac{I_1}{V_1} | I_2 = 0 \]
\[ g_{21} = \frac{V_2}{V_1} | I_2 = 0 \]
\[ g_{12} = \frac{I_1}{I_2} | V_1 = 0 \]
\[ g_{22} = \frac{V_2}{I_2} | V_1 = 0 \]

\[ h = g^{-1} \text{ or } g = h^{-1} \]

\[ T = \begin{vmatrix} \frac{V_1}{V_2} & \frac{I_1}{I_2} \\ \frac{V_2}{I_2} & \frac{I_2}{V_2} \end{vmatrix} \]
\[ T' = \begin{vmatrix} \frac{I_1}{I_2} & \frac{V_2}{V_1} \\ \frac{I_2}{V_1} & \frac{V_2}{I_1} \end{vmatrix} \]

\[ T = T^{-1} \text{ or } T' = T'^{-1} \]
Exercises in Section 17-6

Exercise 17.6-1 (pg 814)

Determine $Z$ when $Y = \begin{bmatrix} 2/15 & -1/5 \\ -1/10 & 2/5 \end{bmatrix}$

$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{2/5}{(2/15)(2/5)-(1/10)(1/5)} = \frac{2/5}{1/30} = \Omega$

$Z_{12} = -\frac{Y_{12}}{\Delta Y} = -\frac{(-1/5)}{(1/30)} = \Omega$

$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{(-1/10)}{(1/30)} = \Omega$

$Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{(2/15)}{(1/30)} = \Omega$
Exercise 17.6-2 (pg 814)

Determine the T-parameters from the Y-parameters of exercise 17.6-1.

\[
A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \frac{Z_{11}}{Z_{21}}
\]

But \( Z_{11} = \frac{Y_{22}}{\Delta Y} \) and \( Z_{21} = -\frac{Y_{21}}{\Delta Y} \)

\( \therefore A = \frac{Y_{22}}{\Delta Y} = -\frac{Y_{21}}{\Delta Y} = -\frac{2/5}{-1/10} = \)

Exercise 17.6-2 (cont’d)

\[
C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = \left. \frac{1}{V_2/I_1} \right|_{I_2=0} = \frac{1}{Z_{21}}
\]

But \( Z_{21} = -\frac{Y_{21}}{\Delta Y} \)

\( \therefore C = -\frac{\Delta Y}{Y_{21}} = -\frac{1/30}{-1/10} = \)

\[
B = \left. \frac{V_1}{-I_2} \right|_{I_2=0} = \left. \frac{1}{-I_2/V_1} \right|_{I_2=0} = -\frac{1}{Y_{21}} = -\frac{1}{-1/10} = \)

\[
D = \left. \frac{I_1}{-I_2} \right|_{I_2=0} = \left. \frac{I_1/V_1} {-I_2/V_1} \right|_{I_2=0} = \frac{Y_{11}}{-Y_{21}} = \frac{2/15}{-1/10} = \)
17.7 Interconnection of two-port Networks

A.1: Parallel Connection (use Y-parameters)
A.2: Series Connection (use Z-parameters)
B: Cascade Connection (use T-parameters)

Parallel Case (Y).

\[
\begin{align*}
I_1 &= I_{1a} + I_{1b} & I_2 &= I_{2a} + I_{2b} \\
V_1 &= V_{1a} = V_{1b} & V_2 &= V_{2a} = V_{2b} \\
\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} &= Y_a \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = Y_a \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \\
\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} &= Y_b \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = Y_b \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \\
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} &= \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = (Y_a + Y_b) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \\
\therefore Y &= Y_a + Y_b
\end{align*}
\]
Series Case (Z)

\[ I_1 = I_{1a} = I_{1b} \quad I_2 = I_{2a} = I_{2b} \]

\[ V_1 = V_{1a} + V_{1b} \quad V_2 = V_{2a} + V_{2b} \]

\[
\begin{bmatrix}
V_{1a} \\
V_{2a}
\end{bmatrix} = Z_a 
\begin{bmatrix}
I_{1a} \\
I_{2a}
\end{bmatrix} = Z_a 
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_{1b} \\
V_{2b}
\end{bmatrix} = Z_b 
\begin{bmatrix}
I_{1b} \\
I_{2b}
\end{bmatrix} = Z_b 
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
V_{1a} \\
V_{2a}
\end{bmatrix} + \begin{bmatrix}
V_{1b} \\
V_{2b}
\end{bmatrix} = (Z_a + Z_b) \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

\[ \therefore Z = Z_a + Z_b \]
Cascade Case (T)

Note that:

\[
\begin{bmatrix}
V_{1a} \\
I_{1a}
\end{bmatrix} = T_a \begin{bmatrix}
V_{2a} \\
-I_{2a}
\end{bmatrix},
\begin{bmatrix}
V_{1b} \\
I_{1b}
\end{bmatrix} = T_b \begin{bmatrix}
V_{2b} \\
-I_{2b}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = T_a \begin{bmatrix}
V_{1a} \\
-I_{2a}
\end{bmatrix} = T_a \begin{bmatrix}
V_{2a} \\
-I_{2a}
\end{bmatrix} = T_a T_b \begin{bmatrix}
V_{1b} \\
-I_{1b}
\end{bmatrix} = T_a T_b \begin{bmatrix}
V_{2b} \\
-I_{2b}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = T_a T_b \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix} = T \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix}
\]

Where \( T = T_a T_b \)
Exercises in Section 17-7

Exercise 17.7-1 (pg 817)

Determine the total transmission parameters of the cascade connection of the three-port networks showing in Figure E17.7-1.
Exercise 17.7-1 (cont’d)

\[
\begin{align*}
A_a &= \left. \frac{V_{1a}}{V_{2a}} \right|_{I_{2a} = 0} = \quad \text{(note that } I_{1a} \text{ and } I_{2a} \text{ are zero)} \\
C_a &= \left. \frac{I_{1a}}{V_{2a}} \right|_{I_{2a} = 0} = \\
B_a &= \left. \frac{V_{1a}}{-I_{2a}} \right|_{V_{1a} = 0} = \quad \Omega \\
D_a &= \left. \frac{I_{1a}}{-I_{2a}} \right|_{V_{1a} = 0} = \\
\therefore T_a &= \left[ \begin{array}{c} \end{array} \right] 
\end{align*}
\]
Exercise 17.7-1 (cont’d)

\[ A_b = \frac{V_{1b}}{V_{2b}} \bigg|_{I_{2b}=0} = \]

\[ C_b = \frac{I_{1b}}{V_{2b}} \bigg|_{I_{2b}=0} = \]

Exercise 17.7-1 (cont’d)

\[ B_b = \frac{V_{1b}}{-I_{2b}} \bigg|_{V_{2b}=0} = \]

\[ D_{2b} = \frac{I_{1b}}{-I_{2b}} \bigg|_{V_{2b}=0} = \]

\[ T_b = \left[ \begin{array}{c} \vdots \end{array} \right] \]
Exercise 17.7-1 (cont’d)

\[
\begin{align*}
A_c &= \frac{V_{1c}}{V_{2c}} = \quad B_c &= \frac{V_{1c}}{-I_{2c}} = \Omega \\
C_a &= \frac{I_{1c}}{V_{2c}} = \quad D_a &= \frac{I_{1c}}{-I_{2c}} = \quad \therefore T_e = 
\end{align*}
\]

Exercise 17.7-1 (cont’d)

\[
T = T_a T_b T_c = \\
\begin{bmatrix}
1 & 12 \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
1/6 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 & 3 \\
0 & 1 \\
\end{bmatrix}
= \\
\begin{bmatrix}
3 & 12 \\
1/6 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 & 3 \\
0 & 1 \\
\end{bmatrix}
= \\
\begin{bmatrix}
3 & 9 + 12 \\
1/6 & 1/2 + 1 \\
\end{bmatrix}
= \\
\begin{bmatrix}
3 & 3 \\
1/6 & 3/2 \\
\end{bmatrix}

Or \quad A = 3, \quad B = 21, \quad C = 1/6, \quad D = 3/2
\]