1. (25 points) Consider the circuit shown below. The source voltage, \( v_s(t) \), is sinusoidal with a frequency \( \omega = 40 \text{ rad/s} \). The response is the voltage, \( v_o(t) \), across the capacitor. For \( C = 2 \mu F \), determine the value of \( R \) that will cause a phase shift equal to \(-30^\circ\).

\[
\begin{align*}
\text{Solution} \\
V_0 &= \left(\frac{1}{j\omega C}\right)V_s \quad \text{or} \quad H(\omega) = \frac{V_0}{V_s} = \frac{1}{1+j\omega RC} \\
H(\omega) &= \frac{1}{\sqrt{1+(\omega RC)^2}} \left(-\tan^{-1}(\omega RC)\right) \\
\therefore -\tan^{-1}(\omega RC) &= -30^\circ \\
R &= \frac{\tan(30^\circ)}{\omega C} = \frac{0.57735}{(40)(2 \times 10^6)} \\
R &= 7.217 \Omega \\
\therefore R &= 7,217 \Omega
\end{align*}
\]
2. For the circuit shown below:
(a) [15 points] Derive the ratio $V_t/V$, in symbolic form and express in polar form.
(b) [10 points] Sketch the magnitude Bode plot when $RC = 0.01$ and $R_1/R_2 = 5$.

![Circuit Diagram]

**Solution**

$$V_t = \left(\frac{j \omega C}{R + \frac{1}{j \omega C}}\right)V_s = \left(\frac{R_2}{R_1 + R_2}\right)V_0$$

$$\therefore \frac{V_0}{V_s} = \left(\frac{R_1 + R_2}{R_2}\right)\left(\frac{1}{1 + j \omega RC}\right)$$

$$= \frac{1 + R_1/R_2}{1 + j \omega RC} = \left(1 + \frac{R_1}{R_2}\right)\frac{1}{\sqrt{1 + (\omega RC)^2}} \frac{-\tan^{-1}(\omega RC)}{1}$$

(a) $V_t/V_s = \frac{1 + R_1/R_2}{\sqrt{1 + (\omega RC)^2}} \frac{-\tan^{-1}(\omega RC)}{1}$

(problem 2 continued)
For $RC = 0.01$ and $R_1/R_2 = 5$

$$\left| \frac{V_o}{V_s} \right| = \frac{1 + 5}{\sqrt{1 + (\omega \times 0.01)^2}} = \frac{6}{\sqrt{1 + (\frac{\omega}{100})^2}}$$

$$20\log \left| \frac{V_o}{V_s} \right| = 20\log 6 + 20\log \left[1 + (\frac{\omega}{100})^2\right]^{1/2}$$

$$= 15.56 - 20\log \left[1 + (\frac{\omega}{100})^2\right]^{1/2}$$

\[\text{(b)}\]

(0 $\rho \omega$)

\[\text{0 db}
\]

\[\text{20 db}
\]

\[\text{40 db}
\]

\[\text{0 db}
\]

\[\text{10 db}
\]

\[\text{20 db}
\]

\[\text{40 db}
\]

\[\text{0 db}
\]

\[\text{10 db}
\]

\[\text{20 db}
\]

\[\text{40 db}
\]

(1) (2) (0)

3
3. (25 points) For the circuit shown below, determine the voltage, $v_c(t)$, for $t \geq 0$.

Solution

Initial condition $(t < 0)$

$V_c(0^-) = \left[ \frac{5}{3+6} \right] 2A = 4 \text{ Volts}$

For $t > 0$

$(2+\frac{1}{3})I_c = \frac{4}{5} + \frac{5I_c}{3}$

$(6s+1)I_c - 5sI_c = 12$

$(s+1)I_c = 12$

$I_c = \frac{12}{5+1}$, then $V_0 = 5I_c = \frac{60}{5+1}$

$V_0(t) = 60 - t$

$v_c(t) = \frac{60}{5} e^{-t}$

Volts
4. (25 points) Use the convolution integral (time domain) to determine the output, \( y(t) \), for the following. Clearly indicate the range of times for each expression for \( y(t) \).

\[ h(t) = 4[u(t) - u(t-4)] \] and
\[ f(t) = 2[u(t+1) - u(t+5)]\]

**Solution**

\[ h(t) \]

\[ f(t) \]

\[ y(t) = \int_{-1}^{3} 8 \, dt = 8(t+1) \]

\[ y(t) = \int_{-5}^{5} 4 \, dt = 32 \]

\[ y(t) = \int_{-5}^{9} 4 \, dt = -8(t-9) \]

(Problem 4 continued on next page)
5. (Extra credit) The transfer function of a circuit is given as follows:

\[ H(s) = \frac{V_o(s)}{V_i(s)} = \frac{kz}{s^2 + (a-k)s + b^2} \]

(a) [8 points] State the necessary relationship between the parameters "a" and "k" that would cause the output voltage, \(v_o(t)\), to be unstable (i.e., \(a < k\), or \(a = k\), or \(a > k\)).

(b) [7 points] State the necessary relationship between parameters "a" and "k" that would cause the output voltage, \(v_o(t)\), to be purely sinusoidal (i.e., \(a < k\), or \(a = k\), or \(a > k\)).

Solution

(a) Roots of denominator

\[ s = -\frac{(a-k)}{2} \pm \sqrt{\frac{(a-k)^2 - 4b^2}{4}} \]

\(v_o(t)\) will be unstable if real part of root is on the right half of the complex plane

\[ \frac{(a-k)}{2} > 0 \]

(i.e., positive value)

\[ a + k > 0 \]

or \(a > k\)

(b) \(v_o(t)\) will be sinusoidal when real part of pole is zero causing roots to be imaginary.

\[ a - k = 0 \]

\[ a = k \]

(a) \(a < k\)

(b) \(a = k\)

7