Let's go back to the Chebyshev multiple $\frac{A}{4}$-section example from the previous class period.

**Example.**

Design a 3-section transformer having $Z_L = 100\Omega$ and $Z_{in} = Z_0 = 50\Omega$ using Chebyshev weights where $F_m = 0.2$.

**First find $\Theta_m$ from**

$$\sec \Theta_m = \cosh \left( \frac{1}{N} \cosh^{-1} \left( \frac{\frac{2}{N}}{2F_m} \right) \right)$$

$= 1.074$

$$\Theta_m = \cos^{-1} \left( \frac{1}{1.074} \right)$$

$= .373$
Recall that bandwidth can be obtained from
\[
\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \theta_m \quad \text{[pg. 12]}
\]
\[
= 2 - \frac{4}{\pi} (0.373) = 1.525
\]
Of course this is a fairly wide bandwidth.

Let's next determine \( A \).
\[
A = \frac{\ln \left( \frac{\theta_m}{\theta_{m0}} \right)}{2 T_3 (\sec \theta_m)}
\]
\[
= \frac{\ln (2)}{2 T_3 (1.074)}
\]
\[
= \frac{\ln (2)}{2 (1.733)} = .2
\]
Notice that $T_3(\sec \Theta_m)$ is calculated from

$$T_3(\sec \Theta_m) = \cosh(3 \cosh^{-1}(\sec \Theta_m))$$

Hence, we have

$$2 \left[ \rho_0 \cos(3\Theta) + \rho_1 \cos(\Theta) \right]$$

$$= \frac{2}{3} T_3(\sec \Theta_m \cos \Theta)$$

$$= \frac{2}{3} \left[ (\sec^3 \Theta_m \cos^3 \Theta) + 3 \sec \Theta_m \cos \Theta \right]$$

But $\cos^3 \Theta = 2^{-3} e^{-j3\Theta} (1 + e^{j2\Theta})^3$

$$= \frac{1}{4} \left( \frac{e^{j3\Theta} + e^{-j3\Theta}}{2} + 3 \frac{e^{j\Theta} + e^{-j\Theta}}{2} \right)$$

$$= \frac{1}{4} \cos 3\Theta + \frac{3}{4} \cos \Theta$$
So, \[ 2[p_0 \cos 3\theta + p_1 \cos \theta] \]

\[ = .2 \left[ \sec^3 \Theta_m (\cos 3\theta + 3\cos \theta) 
- 3\sec \Theta_m \cos \theta \right] \]

Hence,

\[ 2p_0 = .2 \sec^3 \Theta_m \]

or \[ p_0 = .1 (1.074)^3 = p_3 \]

\[ p_0, p_3 = .124 \]

\[ 2p_1 = .2 \left( 3 \sec^3 \Theta_m - 3 \sec \Theta_m \right) \]

or \[ p_1 = .2 (3(1.074)^3 - 3(1.074)) = p_2 \]

\[ p_1, p_2 = .099 \]
The characteristic impedances can be determined from the approximation shown earlier, i.e.

\[ 2 \rho_0 \approx \ln \frac{Z_1}{Z_0} \quad \text{or} \quad Z_1 = Z_0 e^{2 \rho_0} \]

\[ Z_1 = 50 \, \text{e}^{2 \rho_1} = 64 \, \Omega \]

\[ Z_2 = Z_1 \, e^{2 \rho_1} = 64 \, e^{2 \rho_1} = 78 \, \Omega \]

\[ Z_3 = Z_2 \, e^{2 \rho_2} = 78 \, e^{2 \rho_2} = 95 \, \Omega \]

These transformers may be implemented in either a balanced or an unbalanced configuration.
Transmission-line Transformers

Previously we indicated that conventional transformers are not suited for high frequency applications for several reasons:

- parasitic capacitances and inductances cause the transformer to behave unpredictably with frequency. These effects generally reduce efficiency.

- losses in core increase as magnetic flux increases due to inefficiencies in the core material.
Transmission-line transformers incorporate parasitic components into a transmission-line structure so frequency behavior is much better. Also, since the individual conductors of a transmission line carry equal and opposite currents, core fluxes can be kept small.

Consider the following arrangement:

[Diagram of a toroidal core phase reversing circuit]
Notice the bifilar (twisted pair) winding. This can be redrawn as an ideal conventional transformer (neglecting parasitic effects and core losses):

Notice equal and opposite currents at the terminals; the same is true of the unmarked terminals.
Schematic for the phase reversing circuit:

\[
\begin{align*}
V_g &= R_g I + Z_L I \\
0 &= -Z_L I + R_L I
\end{align*}
\]

⇒ \( I = \frac{V_g}{R_g + R_L} \)

\[ V_L = -R_L I = \frac{-V_g R_L}{R_g + R_L} \]

\( Z_L \) accounts for self- and mutual-inductance effects of the transformer winding.
Consider now a circuit that performs an impedance transformation.

Redrawing in the form of an ideal conventional transformer:
The schematic becomes:

2 equations:
\[ V_i = Z_L I + R_L I \]
\[ V_i = -Z_L I \]

Combining leads to
\[ 2V_i = R_L I \]
\[ V_o = 2V_i \]

Hence, \[ Z_{in} = \frac{V_i}{2I} = \frac{V_o}{4I} = \frac{R_L}{4} \]

For maximum power transfer we would like to have
\[ R_L = 4R_g \]
We can combine 2 stages to obtain a 1:16 impedance transformation.

Here \( Z_{in} = \frac{R_L}{16} \)

There are additional options by using trifilar, quadfilar, etc. windings.
1:9
impedance
transformer
with trifilar
winding

Looking at this arrangement
from a conventional transformer
perspective:
So, \( Z_{in} = \frac{V_i}{3I} = \frac{V_o}{9I} = \frac{R_L}{9} \)

Other impedance ratios can be obtained by tapping windings. We won't consider that here—it is available in the book.

Transmission Line Transformers
(Jerry Sevick)
(American Radio Relay League)
Consider converting from unbalanced to balanced.
(single ended)

If the load can be center tapped an option is:

This isn't generally practical as many loads can't be easily center tapped. A popular balun (75Ω to 300Ω) uses the following arrangement:

\[
\begin{align*}
2I \rightarrow I & \quad \rightarrow +V_1 \rightarrow \frac{1}{4} \\
\frac{75\Omega}{I} & \quad \rightarrow \frac{300\Omega}{I} \\
\text{unbalanced} & \rightarrow \frac{1}{4}
\end{align*}
\]
Practical aspects

Guidelines for designing:

(1) Select the self-inductance to be sufficiently large that the device operates in the transmission-line mode over the frequency range of interest. (This establishes a low frequency limit.)

(2) Select the transmission line to be less than about $\lambda/4$ at the highest frequency of interest. This ensures performance even with some mismatch. (This establishes a high frequency limit.)
Let's consider first low frequency behavior.

For low frequencies the transmission line concept is no longer appropriate as the parasitic capacitance between the windings becomes a very high impedance, i.e., ineffective coupling between the windings results.

The analysis will focus on the 1:4 balun just shown.

Consider the 2 loop equations (as shown).
\( \text{1. } V_g = (R_g + z)I_1 - (z + k^2)I_2 \)

\( \text{2. } V_g = (R_g - k^2)I_1 + (R_c + z + k^2)I_2 \)

where \( z \) is the series impedance of each half of the bifilar winding and \( k \) is the coefficient of coupling.

Subtracting to obtain \( \text{1} - \text{2} \) we have

\[
0 = (R_g + z - R_g + k^2)I_1 - (2z + 2k^2 + R_c)I_2
\]

and so

\[
\frac{I_1}{I_2} = \frac{R_c + 2z(1 + k)}{z(1 + k)}
\]

If \( 2z(1 + k) \gg R_c \) then

\[
\frac{I_1}{I_2} \to 2 \quad \text{as expected for the ideal case.}
\]
As long as the current through the 2 windings is balanced, we have

\[ V_i = (2 - k^2)I_2 \]

and

\[ V_L = 2(2 - k^2)I_2 = 2V_i = R_L I_2 \]

In this case,

\[ V_i = V_g - R_g I_1 = V_g - 2R_g I_2 \]

\[ = V_g - 2R_g \left[ \frac{V_L}{2(2 - k^2)} \right] \]

\[ = V_g - 2R_g \left[ \frac{V_i}{(2 - k^2)} \right] \]

\[ = V_g - 2R_g \left[ \frac{V_i}{R_L/2} \right] \]

\[ \Rightarrow V_i = \frac{V_g \frac{R_L}{4}}{\frac{R_L}{4} + R_g} \]
Note that the transformer for the low frequency case is basically an autotransformer:

Taking into account the magnetizing inductance, $L_m$, and assuming that $R_L = 4R_g$ the circuit becomes:

For a torroidal core
LM can be expressed as:

$$L_M = 0.4\pi N_P^2 \mu_c \left[ \frac{A_e (cm^2)}{I_e (cm^2)} \right] \times 10^{-8} H$$

So we examine the power transfer from input to output:

Consider the ratio $\frac{P_{out}}{P_{avail}}$.

where $P_{avail} = \frac{V_g^2}{4R_g}$

$$P_{out} = \text{Re} \left[ V_L I_L^* \right]$$

$$V_L = \frac{jwLMR_g}{jwLM + R_g} \frac{V_g}{R_g + \frac{jwLMR_g}{jwLM + R_g}}$$

$$= \frac{jwLMV_g}{R_g + j2\omega LM}$$
\[ I_L = \frac{V_g}{j\omega L_m R_g + R_g} + R_g = \frac{(j\omega L_m + R_g) V_g}{R_g R_g + j 2 \omega L_m} \]

\[ \text{Re} [V_L I_L^*] = \frac{V_g^2 (\omega L_m) \frac{1}{R_g}}{R_g^2 + 4 \omega^2 L_m^2} \]

So,
\[ \frac{P_{\text{avail}}}{P_{\text{out}}} = \frac{R_g^2 + 4 \omega^2 L_m^2}{4 \omega^2 L_m^2} \]

It is best to design so that only about 10% of power is lost at the low frequency end.

**High Frequency analysis**

Start by looking at the V-I characteristics for a section of transmission line.