Gain - closely related to directive gain and directivity but, in addition, it accounts for efficiency of the antenna.

Definition: Power gain is defined as "$4\pi$ times the ratio of the radiation intensity in that direction to the net power accepted by the antenna from a connected transmitter."

In other words $\text{gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input power}}$

or $G_g = \frac{4\pi U(\theta, \phi)}{P_{in}}$

Note that $P_{in}$ is related to $P_{rad}$ by $P_{rad} = \epsilon_t P_{in}$, where $\epsilon_t$ is the total antenna efficiency.

Hence, $G_g = \frac{4\pi U(\theta, \phi)}{P_{rad}/\epsilon_t} = \epsilon_t D_g$

When we refer to the term "gain", we are usually referring to its maximum value; i.e. $G_g = G_g(\theta, \phi)_{\text{max}} = \epsilon_t D_g(\theta, \phi)_{\text{max}} = \epsilon_t D_o$
A useful approximation for gain is

\[ G_0 \approx \frac{30000}{\theta_1 \theta_2} \quad (\theta_1, \theta_2 \text{ in degrees}) \]

Most commonly, \( G_0 \) is expressed in dB such that

\[ G_0(\text{dB}) = 10 \log_{10} [G_0] \]

**Antenna Efficiency**

Antennas do not radiate 100% of the energy input to them. Major losses are due to:

1. Impedance mismatch at the antenna terminals; and
2. Losses in the antenna's conductors and dielectrics.

We will use the following terms:

1. \( et \) = total overall efficiency (\( \text{of input} \))
2. \( er \) = reflection efficiency = \( (1 - \text{reflection})^2 \)
3. \( ec \) = conduction efficiency
4. \( ed \) = dielectric efficiency

\( ec \) and \( ed \) cannot usually be determined separately; together \( ec \cdot ed \) can be referred to as the antenna radiation efficiency.
Beam Efficiency

This parameter indicates the total power in the major lobe (or designated portion of the major lobe) relative to the total power transmitted (i.e., including major and minor lobes).

\[ BE = \frac{\text{Power transmitted (revd) within angle}}{\text{Power transmitted (revd) by the antenna}} \]

where \( \theta_1 \) is selected so as to include only the half power beamwidth or the entire major lobe between first nulls, depending on the application.

\[ BE = \frac{\int_0^{2\pi} \int_0^{\theta_1} U(\theta, \phi) \sin \theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta d\theta d\phi} \]

(This definition assumes the major lobe lies along the \( z \) axis)
Bandwidth

Bandwidth is a general term which can be used to refer to the behavior of input impedance, beamwidth, polarization, side lobe level, gain, beam direction, or efficiency, over a band of frequencies. Bandwidth also refers to the parameter that is most important in a particular application, although it frequently refers to the parameter that is most important. Distinction is generally made between pattern bandwidth and impedance bandwidth. The former is associated with gain, side lobe level, beamwidth, polarization, and beam direction. The latter is associated with input impedance and radiation efficiency.

The bandwidth of broadband antennas is specified in the form 10:1, indicating the upper freq. is 10x the lower freq. Broadband antennas are specified as a %, where 
\[ \% = \frac{\text{Upper freq} - \text{Lower freq}}{\text{Center freq}} \times 100 \]
Input impedance - often implies the ratio of voltage to current at the input terminals of the antenna. (Note that the def. may be different if the antenna is fed using waveguide)

\[ z_0 = R_g + jX_g \]

has as an equiv. circuit

\[ V_g \]

\[ R_g + jX_g \]

\[ I_g = \frac{V_g}{(R_L + R_r + R_g) + j(X_A + X_g)} \]

\[ R_L = \text{loss resistance} \]
\[ R_r = \text{radiation resistance} \]
\[ X_A = \text{antenna reactance} \]

\[ \text{Peak value} \]

\[ P_{\text{rad}} = \frac{1}{2} |I_g|^2 R_r \]

\[ P_{\text{radiated power}} = \frac{1}{2} |V_g|^2 \left[ \frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2} \right] \]

\[ P_L = \frac{1}{2} |I_g|^2 R_L = \frac{1}{2} |V_g|^2 \left[ \frac{R_L}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2} \right] \]

\[ \text{Power lost as heat in antenna} \]
\[ P_g = \frac{1}{2} |I_g|^2 R_g = \frac{\frac{1}{2} |V_g|^2 R_g}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2} \]

Max. power transfer to antenna occurs for the case:
\[ R_r + R_L = R_g \]
\[ X_A = -X_g \]

In this case,
\[ I_g = \frac{V_g}{2 (R_r + R_L)} \]

Hence,
\[ P_{rad} = \frac{1}{2} |V_g|^2 \left[ \frac{R_r}{4 (R_r + R_L)^2} \right] \]
\[ P_L = \frac{1}{2} |V_g|^2 \left[ \frac{R_L}{4 (R_r + R_L)^2} \right] \]
\[ P_g = \frac{1}{2} |V_g|^2 \left[ \frac{R_g}{4 (R_r + R_L)^2} \right] \]

Receive mode
\[ = R_T + j X_1 \]
Antenna radiation efficiency

\[
e_{cd} = \frac{P_{rad}}{P_L + P_{rad}} = \frac{P_{rad}}{P_{in}} = \frac{\frac{1}{2} \text{Volts}^2 [\frac{R_r}{\pi (R_r + R_t)^2}]}{\frac{1}{2} \text{Volts}^2 \left[\frac{R_r}{\pi (R_r + R_t)^2}\right]} = \frac{R_r}{R_r + R_L}
\]

The text works through a very simple example for a half-wavelength dipole.

Effective Aperture

Effective aperture = \( A_e = \frac{P_L}{\text{Power delivered to load}} = \frac{|I_I|^2 R_r/2}{\text{effective incident power density}} \)

\[ I_T = \frac{V_T}{R + R_L + R_T + j(X_A + X_T)} \]

Hence,
\[ A_e = \frac{|V_T|^2 R_T}{2 \omega_I \left[(R + R_L + R_T)^2 + (X_A + X_T)^2\right]} \]

For \( R_L + R_L = R_T \), \( X_A = -X_T \).
\[ A_{e_m} = \frac{|V_T|^2 R_T}{8 \omega_I (R_T + R_L)} \]

Max. effective aperture

In general,
\[ A_{e_m} = \frac{2}{\pi} G_0 \text{ (pol. eff.)} \]