Diffraction from a straight edge

As a basis for this discussion we consider the case of a uniform plane wave incident on an aperture.

If the point of observation is sufficiently far from the aperture, the pattern of radiation remains constant and the power density at point P is dependent only on $\frac{1}{R^2}$. This region, the Fraunhofer region, is described by $R > \frac{2D^2}{\lambda}$.

On the other hand if $\frac{2D^2}{\lambda} < R < \frac{z_{min}}{\omega}$, the radiation pattern is a function of $R$, because the beam has not yet fully formed. This region is referred to as the Fresnel region.
Diffraction (cont.)

For the problem of interest (diffraction from a straight edge), the obstacle above the aperture goes away, and so \( D \to \infty \). In this case, though, for practical purposes we replace the uniform plane wave to the left hand side with a spherically incident wave. We treat this wavefront in terms of Huygens principle.

**Huygens’ principle** - This principle relates fields at an observation point (in this case receiver, \( R \)) to isotropic sources along the wavefront. It can be rigorously related to Maxwell’s equations, although there is an approximation that has negligible effect for most practical apertures.
Diffraction (cont)

Steps in the analytical development.

According to Huygen's principle, at point R due to the angular strip, du

\[ dE = k_1 du e^{-j\beta r} \]

Since radiation is into unbounded freespace, \( \beta = \frac{2\pi}{\lambda} \), \( f(r) \) is some function of \( r \), and \( k_1 \) is a constant.

Law of cosines, \((QR)^2 = r^2 = (d_x + dx)^2 + d_z^2 - 2d_x(dx)(d_y + dy)\cos(\theta)\)

If the largest contributions (to \( E \)) are for small values of \( u \), then we can use

\[ \cos(\theta) \approx 1 - \frac{u^2}{2d_y^2} \]
Diffraction (cont)

So,

\[ r^2 = (d_1 + d_2)^2 + d_1^2 - 2d_1(d_1 + d_2)(1 - \frac{u^2}{2d_1^2}) \]

\[ = d_1^2 + d_2^2 + 2d_1d_2 + d_1^2 - 2d_1^2 - 2d_1d_2 + u^2 + \frac{d_2}{d_1}u^2 = d_2^2 + u^2\left(\frac{d_1 + d_2}{d_1}\right) \]

If we assume that \( \delta \ll d_2 \) then we can write \( d_2\left(1 + \frac{d_1}{d_2} + \frac{\delta}{d_2}\right) \approx d_2\left(1 + \frac{\delta}{d_2}\right) \)

\[ r^2 = (d_2 + \delta)^2 = d_2^2\left(1 + \frac{\delta}{d_2}\right)^2 = d_2\left(1 + \frac{u^2(d_1 + d_2)}{2d_1d_2}\right) \]

\[ \Rightarrow \delta = \frac{u^2(d_1 + d_2)}{2d_1d_2} \approx \frac{u^2(d_1 + d_0)}{2d_1d_0} \]

Since for practical cases \( d_2 \) is not too different than \( d_0 \).

\[ \text{So, } E = \frac{k_1}{f(r)} \int_{u_0}^{u_1} e^{-j\beta r} du = \frac{k_1}{f(d_2)} \int_{u_0}^{u_1} e^{j\beta d_2} du \]

Assumed \( \propto \) constant for amplitude

\[ = \frac{k_1 e^{-j\beta d_2}}{f(d_2)} \left( \int_{u_0}^{u_1} u_1 \cos \beta d_2 du - j \int_{u_0}^{u_1} u_1 \sin \beta d_2 du \right) \]
Diffraction (cont)

Notice that \[ \beta \tau = \frac{2\pi}{\lambda} \left( \frac{d_1 + d_2}{d_1 d_2} \right) u^2 \]

The magnitude of the field strength at \( R \) is thus,

\[ |E|^2 = \frac{k_0^2}{f^2(dz)} \left[ \left( \int_{u_0}^{u_1} \cos \beta \tau du \right)^2 + \left( \int_{u_0}^{u_1} \sin \beta \tau du \right)^2 \right] \]

We will make use of the cosine and sine integrals, where

\[ C = \int_{0}^{\nu} \cos \frac{\pi u^2}{2} dv \quad S = \int_{0}^{\nu} \sin \frac{\pi u^2}{2} dv \]

and \( C(\nu) - jS(\nu) = \int_{0}^{\nu} e^{-j\frac{\pi u^2}{2}} dv \) is a standard Fresnel integral.

We make the appropriate transformation from \( u \) to \( \nu \) by noticing that

\[ \frac{\pi}{2} \nu^2 = \beta \tau = \frac{2\pi}{\lambda} \left( \frac{d_1 + d_2}{d_1 d_2} \right) u^2 \]

\[ \Rightarrow \nu = \sqrt{\frac{2}{\lambda}} \left( \frac{d_1 + d_2}{d_1 d_2} \right) u = k_{\nu} u \]

\[ d\nu = \frac{2}{\sqrt{\lambda \pi}} \left( \frac{d_1 + d_2}{d_1 d_2} \right) d\nu = k_{\nu} d\nu \]
Diffraction (cont.)

So, \[ E = \frac{k_1 e^{-j \beta d_2}}{k_2 f(d_2)} \int_{v_0}^{\infty} \frac{e^{-j \frac{\pi}{2} v^2}}{v_{\infty}} dv \]

We generally assume that \( v_{\infty} \rightarrow \infty \), since wavelength is assumed to be small.

Hence, \[ E_0 = \frac{k_1 e^{-j \beta d_2}}{k_2 f(d_2)} \int_{v_0}^{\infty} \frac{e^{-j \frac{\pi}{2} v^2}}{v_{\infty}} dv \]

Received \( E_0 \) 

Note that \[ E_0 = \frac{k_1 e^{-j \beta d_2}}{k_2 f(d_2)} \int_{v_0}^{\infty} e^{-j \frac{\pi}{2} v^2} dv \]

So, \[ E_0 = K \left( \frac{1}{2} - \frac{i}{2} \right) - j \frac{1}{2} \left( \frac{1}{2} - \frac{i}{2} \right)^2 \]

\[ = K \left( 1 - j \right) \text{ or } K = \frac{E_0}{2} \left( 1 + j \right) \]

So,

\[ \frac{E}{E_0} = \frac{1}{2} (1 + j) \int_{v_0}^{\infty} e^{-j \frac{\pi}{2} v^2} dv \]

\[ = \frac{1}{2} (1 + j) \left[ \frac{e^{\frac{1}{2} - c(v_0)^2} - j \left\{ \frac{1}{2} - s(v_0)^2 \right\}}{c_{\infty} - c(-\infty)} \right] \]

\[ V_0 = \sqrt{2d_1 + d_2} \]

[Limitations due to approximations: \( d_1, d_2 \gg \lambda_0 \) \( d_1, d_2 \gg \lambda \)]