1. (12 pts) A 16 kVA, 480 Vrms single-phase load, as shown in the figure below, has a power factor of 0.85 lagging. Find the value of the parallel impedance, $Z_p$ in ohms, that will correct the power factor to 0.95 leading.

![Diagram of the circuit](image)

Solution

\[
\begin{align*}
\theta_L &= \cos^{-1}(0.85) = 31.788^\circ, \quad \text{r.f.} = \sin \theta_L = 0.5268 \quad (\text{lag}) \quad (4 \text{ pts}) \\
S_L &= 16,000 (0.85+j0.5268) = 13,600+j8,428.5 \\
\text{After } Z_p \text{ placed in parallel:} \\
\theta_N &= \cos^{-1}(0.95) = -18.195^\circ \quad (\text{lead}) \quad (4 \text{ pts}) \\
Q_N &= P_L \tan \theta_N = 13,600 \tan(-18.195^\circ) = -4,470.1 \\
\text{Since } S_N = S_L + S_p, \text{ then } S_p = S_N - S_L \quad (4 \text{ pts}) \\
\text{or } S_p &= (13,600-j4,470.1) - (13,600+j8,428.5) \\
&= -j12,898.6 \\
Z_p &= \frac{|S_p|^2}{S_p^*} = \frac{(480)^2}{j12,898.6} = \frac{-j17.86}{2} \\
Z_p &= -j17.86 \, \Omega
\end{align*}
\]
2. (12 pts) Determine the current $I_0$ in phasor form for the circuit shown below. Express your answer in polar form.

Solution

\[ I_0 = I_1 - I_2 \] (4 pts)

1) \((4 + j24)I_1 - j16I_2 = 40\) (4 pts)

2) \(-j16I_1 + (2 + j2)I_2 = 0\) (4 pts)

\[ I_1 = \begin{bmatrix} 40 & -j16 \\ 0 & 2 + j2 \end{bmatrix} = \frac{80 + j80}{216 + j56} = 0.507/30.465^\circ \] (4 pts)

\[ I_2 = \begin{bmatrix} 4 + j24 & 40 \\ -j16 & 0 \end{bmatrix} = \frac{j640}{216 + j56} = 2.868/75.465^\circ \] (4 pts)

\[ I_0 = I_1 - I_2 = 2.535/-96.404^\circ \]

Alternate method

\[ (4 + j24)I_1 - j16I_2 + (2 + j2)I_2 - j16I_1 = 40 \]

\[ (4 + j8)I_1 + j(2 - j14)I_2 = 40 \]

\[ -j16I_1 + (2 + j2)I_2 = 0 \]

\[ I_0 = \frac{2.535}{-96.404^\circ} \text{ amps} \]
3. (12 pts) Find the voltage across the 4 \( \Omega \) resistor, \( v(t) \), for the circuit below for \( t \geq 0 \).

Solution

For \( t < 0 \), current source \( i(t) = -2 \text{ A} \) dc

\[
\begin{align*}
V_c(0^-) &= -8 \text{ V} \\
V_L(0^-) &= 0 \text{ V}
\end{align*}
\]

For \( t \geq 0 \)

\[
\begin{align*}
\text{KCL: } & \quad \frac{V}{4} + \frac{V + 8/5}{5S + 6 + 10/5} = \frac{6}{5} \\
& \quad \frac{V}{4} + \frac{V + 8}{5S^2 + 6S + 10} = \frac{6}{5}
\end{align*}
\]

\[
\begin{align*}
(5S^2 + 6S + 10 + 4S)V + 32 &= \frac{6}{5} \\
4(5S^2 + 6S + 10) &= \frac{6}{5}
\end{align*}
\]

\[
\begin{align*}
V &= \frac{120S^2 + 112S + 240}{(5)(5)(5S^2 + 2S + 2)} = \frac{K_1}{5} + \frac{K_2}{s + 1 - j1} + \frac{K_2^*}{s + 1 + j1} \\
K_1 &= 24, \quad K_2 = 12.8/90^\circ
\end{align*}
\]

\[
v(t) = \begin{cases} 
24 + 25.6e^{-t} \cos(t + 90^\circ) & \text{if } t < 0 \\
24 - 25.6e^{t} \sin t & \text{if } t \geq 0
\end{cases} u(t)
\]
4. (12 pts) Use the convolution integral (time domain) to find \( y(t) \) when \( h(t) = 3[u(t-1) - u(t-3)] \) and \( x(t) = 4[u(t-1) - u(t-5)] \). Clearly indicate the range of times for each expression for \( y(t) \).

Solution

\[ y(t) = \int_{t-1}^{t} 12 \, dt = 12(t-1) = 12(t-2) \]

Range: \( t-1 \leq 0 \) or \( t = 2 \)
\( t-1 = 3 \) or \( t = 4 \)
\( 2 \leq t \leq 4 \)

\[ y(t) = \int_{t-1}^{3} 12 \, dt = 12(3-1) = 24 \]

Range: \( t-5 = 1 \) or \( t = 6 \)
\( 4 \leq t \leq 6 \)

\[ y(t) = \int_{t-5}^{3} 12 \, dt = 12(3-t+5) \]
\[ = -12(t-8) \]

Range: \( t-5 = 3 \) or \( t = 8 \)
\( 6 \leq t \leq 8 \)

(continued on next page)
5. (13 pts) A band-pass filter can be achieved by using two operational amplifiers in the circuit shown below with parameter values of \( R_1 = 10 \text{k}\Omega, R_2 = 1 \text{k}\Omega, C_1 = 0.1 \mu\text{F}, C_2 = 1 \mu\text{F} \). Assume the operational amplifiers are ideal.
   a. Derive the transfer function \( H(s) = \frac{V_o}{V_i} \).
   b. Find \( \omega_0 \), Bandwidth \( BW \), and \( Q \).

\[ \text{Solution} \]

\[ \text{KCL at node 1} \]
\[ (V_i - V_i)C_1s + \frac{(V_i - V_o)}{R_1} = 0 \]
\[ V_i(C_1s + \frac{1}{R_1}) - V_iC_1s - \frac{V_o}{R_1} = 0 \]
\[ -V_o = R_2C_2(\frac{SR_1C_1 + 1}{R_1}) - \frac{V_o}{R_1} - V_iSC_i = 0 \]
\[ V_o \left[ s^2 + \left( \frac{1}{R_1C_1} \right)s + \frac{1}{R_1C_1R_2C_2} \right] = -sV_i \left( \frac{1}{R_2C_2} \right) \]
\[ H(s) = \frac{V_o}{V_i} = -\left( \frac{1}{R_2C_2} \right)s \]
\[ s^2 + \left( \frac{1}{R_1C_1} \right)s + \frac{1}{R_1C_1R_2C_2} \]
\[ \omega_0 = \sqrt{\frac{1}{R_1C_1R_2C_2}} = \sqrt{10^4 \times 10^7 \times 10^9 \times 10^6} = 1,000 \text{ rad/s} \]
\[ BW = \frac{1}{R_1C_1} = \frac{1}{10^4 \times 10^7} = 1,000 \text{ rad/s} \]
\[ Q = \frac{\omega_0}{BW} = 1 \]

(continued on next page)
6. (13 pts) Design a low-pass filter circuit that has the following transfer function. Note that the denominator for a normalized 3rd order low-pass Butterworth filter is \((s+1)(s^2+s+1)\). Assume a value for capacitance of 0.1 \(\mu F\) throughout.

\[
H(s) = -\frac{100^3}{(s+100)(s^2+100s+100^2)}
\]

**Solution**

The given circuit diagram shows a low-pass filter with components labeled as \(R\) and \(C\). The transfer functions are:

\[
H_1(s) = \frac{100^2}{s^2+100s+100^2} = \frac{k_1\omega_0^2}{s^2+(\omega_0^2)+\omega_0^2}
\]

\[
H_2(s) = \frac{-100}{s+100} = \frac{-k_2\omega_0}{s+\omega_0}
\]

\[
\omega_0 = \frac{1}{RC} = 100, \quad (\omega_0 = 100)
\]

\[
R = \frac{1}{\omega_0 C} = \frac{10^7}{10^2} = 10^5 \Omega
\]

\[
Q = 1 = \frac{1}{3-A} \quad A = 3 - \frac{1}{Q} = 3 - 1 = 2
\]

Thus, \(k_1 = A = 2\)

\[
\omega_c = \frac{1}{RC} = 100
\]

\[
K_2 = \frac{R_2}{R_1} = \frac{1}{2}
\]

\[
R_1 = 2R_2 = 2 \times 10^5 \Omega
\]
7. (13 pts) Find the hybrid “h” parameter model (i.e., $h_{11}$, $h_{21}$, $h_{12}$, $h_{22}$) of the circuit shown below.

Solution

\[ V_1 = h_{11} I_1 + h_{12} V_2 \]
\[ I_2 = h_{21} I_1 + h_{22} V_2 \]
\[ h_{11} = \frac{V_1}{I_1} | V_2 = 0 \]
\[ h_{12} = \frac{V_1}{V_2} | I_1 = 0 \]
\[ h_{21} = \frac{I_2}{I_1} | V_2 = 0 \]
\[ h_{22} = \frac{I_2}{V_2} | I_1 = 0 \]

Due to short circuit, $I = 0$

\[ \therefore V_1 = 9 \Omega \times I_1 = 9 \]
\[ I_2 = -I_1 = -1 \]

\[ h_{11} = \frac{V_1}{I_1} = 9 \]
\[ h_{21} = \frac{I_2}{I_1} = -1 \]

\[ I + 5I = I_2 = 1.0 \]
\[ \therefore I = \frac{1}{6} \]
\[ V_2 = I \times 1\Omega = \frac{1}{6} \]

\[ V_1 = V_2 - 5I \times 9\Omega = \frac{1}{6} - \frac{45}{6} = -\frac{44}{6} \]

\[ \therefore h_{12} = \frac{V_1}{V_2} = \frac{-\frac{44}{6}}{\frac{1}{6}} = -44 \]
\[ h_{22} = \frac{I_2}{V_2} = \frac{1}{\frac{1}{6}} = 6 \]
8. (pts) Find the transmission “T” parameter model (i.e., \( A, B, C, D \)) of the circuit shown below (same as in problem 7).

\[
\begin{align*}
V_1 &= AV_2 - BI_2 \\
I_1 &= CV_2 - DI_2
\end{align*}
\]

Solution

\[
\begin{align*}
A &= \frac{V_1}{V_2} \bigg| _{I_2=0} \\
B &= -\frac{V_1}{I_2} \bigg| _{V_2=0} \\
C &= \frac{I_1}{V_2} \bigg| _{I_2=0} \\
D &= \frac{I_1}{I_2} \bigg| _{V_2=0}
\end{align*}
\]

\[
\begin{align*}
I_1 &= 1.0 \\
\frac{5I + I}{9} &= I_1 = 1.0 \\
\therefore I &= \frac{1}{6} \\
V_2 &= I \times 1.0 \cdot 2 = \frac{1}{6} \\
V_1 &= I \times (9.0 + 1.2) = 10/6
\end{align*}
\]

\[
A = \frac{10}{6} = 6 \\
B = -\frac{1}{6} = 9 \\
D = -\frac{1}{1.2} = 1
\]

Same test as first part of problem 7

\[
V_1 = 9, \ I_2 = -1
\]

\[
\begin{align*}
A &= 10 \\
C &= 6 \\
B &= 9 \\
D &= 1
\end{align*}
\]