Solve ALL FIVE problems. Time: 1 hr. 30 min.
Maximum score: 36 points.

Problem 1 Transfer Function Evaluation
Evaluate the transfer function \( G(s) \) from \( r \) to \( y \) using any method. (8 points)

Solution:

\[
\begin{align*}
L_1 &= -\frac{2}{5}, \quad L_2 = -\frac{18}{5^2}, \quad L_3 = \frac{4}{5} \\
\Delta &= 1 - (L_1 + L_2 + L_3) + L_1 L_3 = 1 + \frac{2}{5} + \frac{18}{5^2} - \frac{4}{5} - \frac{36}{5^2} = \frac{36}{5^2} \quad (1 \text{ pt.}) \\

P_1 &= \frac{1}{5^2}, \quad A_1 = 1 \\
P_2 &= -\frac{1}{5}, \quad A_2 = 1 - A_1 = 1 + \frac{2}{5} \quad (3 \text{ pts.}) \\

\therefore \quad G(s) &= \frac{P_1 A_1 + P_2 A_2}{\Delta} = \frac{1 - (5+2)}{5^2 + (2-1)5 - 2} = \frac{-5+1}{5^2 + (2-1)5 - 2} = -5(5) \quad (1 \text{ pt.}) \\
\text{TOTAL: 8 pts.}
\end{align*}
\]
Problem 2 (Solution continued):

**ALTERNATIVE METHOD (BLOCK DIAGR. MANIPULATION):**

\[ g(s) = \frac{1}{s} \cdot \frac{-s+1}{s+2} \]

\[ \frac{1}{s+2} - 1 = \frac{-s+1}{s+2} \]

\[ g(s) = \frac{-s+1}{s+2} \]

\[ g(s) = \frac{s+1}{s(s+2)-\kappa(s+1)} \]

\[ \text{Total: 8 pts.} \]
Problem 2  Signal-Flow Graph

Consider the following set of equations describing a certain network with only one independent input $i$:

$$C_1 \dot{v}_1 + G_1 v_1 - C_1 \dot{v}_2 = i$$
$$- C_1 \dot{v}_1 + (C_1 + C_2) \dot{v}_2 + G_2 v_2 = 0$$

Draw a signal-flow graph showing all five variables $v_1, \dot{v}_1, v_2, \dot{v}_2, i$.  

(6 points)

Solution:

$$\dot{v}_1 = - \frac{G_1}{C_1} v_1 + \frac{1}{C_1} i$$
$$\dot{v}_2 = - \frac{G_2}{C_1 + C_2} v_2 + \frac{C_1}{C_1 + C_2} \dot{v}_1$$

$\leftarrow$ EACH ERROR
ONE PT. DEDUCTION

Total: 6 pts.
Problem 3. Linearization about an Operating Point

Consider the following multiplier with two inputs $u$ and $v$, and one output $y$. The relation between the inputs and the output is given by $y = uv$.

(a) Let $u = u^0 = 2$ and $v = v^0 = 3$ be constant inputs to the multiplier. Calculate the corresponding constant output $y = y^0$. (2 points)

(b) Linearize the above system about the operating point in Part (a) and thus evaluate the gains $k_1$ and $k_2$ in the following model. (4 points)

Solution:

1. $y^0 = u^0 v^0 = 2 \times 3 = 6 = y^0$ (2 pts.)

2. $(y^0 + Ay) = (u^0 + Au)(v^0 + Av)$

   $y^0 + Ay = u^0 v^0 + u^0 Av + u^0 Au + Au Av$

   $Ay = u^0 Av + u^0 AU + Au AV$

   $\frac{Ay}{k_1} \approx \frac{Au AV}{k_2}$

   Neglected compared with $(u^0 Au + u^0 AV)$

   $\Rightarrow \frac{P_1}{P_2} = 3$, $\frac{P_2}{P_3} = 2$ (4 pts.)

   Total: 6 pts.
Problem 4  Steady-State Error

Consider the following feedback loop. The design parameter $k$ is a real constant which may take on negative or positive values, or zero.

Let error be defined as $e = r - y$. Let the reference input $r$ be zero, and let the disturbance $w$ be a unit ramp. Find the range of values for $k$ such that the absolute value of the steady-state error is less than 0.1. (8 points)

Solution:

$$E(s) = \frac{R(s)}{0} - \frac{Y(s)}{W(s)}$$

$$= - G_{w \to y}(s) W(s)$$

$$= - \frac{k}{s^2 + s + 1} \frac{1}{1 + \frac{s}{b} \cdot \frac{1}{s^2 + s + 1} \cdot \frac{10}{s + 20}}$$

$$= - \frac{k(s + 20)}{5(s^2 + s + 1)(s + 20) + 10k}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{0}{5}$$  

$$\frac{0}{5} < 0.1 \Rightarrow |12| > 20 \Rightarrow \begin{cases} k > 20 \\ \text{or} \\ k < 20 \end{cases}$$  

TOTAL = 8 pts.
Problem 5  Stability

Consider the same system as in Problem 3. The parameter $k$ is real and can be negative.
Find the range of values of $k$ such that the closed loop is stable.  

Solution:

\[ \Delta = 1 + \frac{k}{3} \cdot \frac{1}{s^2 + 3s + 2} \cdot \frac{2}{s^2 + 3s + 2} = 0 \]

\[ s(s^2 + 5s + 20) + 10k = 0 \]

\[ \frac{s^3 + 215 + 215 + 20}{s^3 + 215 + 215 + 20} \]

\[ s^4 + 215 + 215 + 205 + 10k = 0 \quad (3 \text{ pts}) \]

\[ s^4 \quad 1 \quad 21 \quad 10k \quad \frac{21}{421} \]

\[ s^3 \quad 21 \quad 20 \quad \frac{21}{421} \]

\[ s^2 \quad \frac{421}{21} \quad 10k \quad \frac{21}{421} \]

\[ s^1 \quad \left( \frac{421}{21} - 210k \right) \cdot \frac{21}{421} > 0 \quad (2 \text{ pts}) \]

\[ s^0 \quad 10k > 0 \quad (1 \text{ pt}) \]

\[ \Rightarrow \left\{ \begin{array}{l}
\frac{421}{21} - 210k > 0 \Rightarrow k < \frac{421}{21} \cdot \frac{20}{21} = 1.909 \\
\end{array} \right. \]

\[ k > 0 \]

\[ \Rightarrow 0 < k < 1.909 \quad (2 \text{ pts}) \]

TOTAL = 8 pts.