Performance of 16-QAM Demodulation in the Rapid Multipath Environment

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Abstract

Land mobile communication systems must overcome the rapid changes in received signal amplitude and phase that result from multipath effects. The effects of multipath are most significant in applications where coherent demodulation is required, namely high-order QAM. In this paper, the effects of noise and multipath distortion on direct 16-QAM at microwave frequencies is described with the techniques being applicable to 64-QAM, 256-QAM and other high-order modulation types. Multipath performance is characterized herein using dynamic characteristics, for example, the change rates of power, amplitude, and phase. The distributions of these characteristics are predicted using analytical methods and then verified by direct measurement of shadowed multipath channels on a ground mobile collection platform. It is shown that the multipath effects may be overcome with the use of adaptive demodulation techniques to compensate for amplitude and phase distortion, and with the use of diversity.

I. Introduction

In applications that require RF transmission to a land-based moving platform, communication is likely to be along signal paths that are not direct line-of-sight, but blocked with the receive location in a shadow. When in a shadow, the signal will arrive at the receiver along multiple paths. Propagation in shadows will change characteristics as vehicle or nearby objects move, changing the signal path geometry. This multipath propagation will result in rapid changes in the received signal amplitude and phase which must be either tolerated or compensated in the receiver.

The phase and amplitude change rates have been modeled and shown to have a distribution proposed by Rice, and a Gaussian distribution respectively [1-2]. Since the long term channel effects on amplitude are due to path geometry and change slowly compared to the multipath fades, the fractional rate of change for received amplitude or power is of interest and presented as a distribution similar to that for phase change rate. These rates constrain the symbol rates that may be supported by the channel and define the compensation required in a receiver to track carrier.

When signals with higher-ordered modulation types, namely m-ary QAM, are used, multipath changes affect not only the recovered signal, but also the estimate of carrier reference, required in the demodulator. An instantaneous estimate of carrier created using the receive signal will be noisy. Averaging the carrier estimate over time will reduce the effect of noise received noise, but will worsen the error due to multipath.

Using optimal carrier recovery, the resulting performance is evaluated for a proposed signal, 16-QAM, at a nominal symbol rate of 4800 baud. The evaluation is based on a receiver which provides amplitude and phase tracking at the demodulator using decision directed feedback. The error performance relative to average received SNR and improvement resulting from addition of diversity are presented. Error results from channel noise and the uncompensated multipath effects. Time domain diversity is proposed to provide Error Detection And Correction (EDAC) using Bose-Chaudhuri-Hocquenghem (BCH) codes.

II. Background

The basic model for determining rates of change for amplitude and phase have been developed by Rice for application of Gaussian noise to a sine wave [1-3]. These same models are applicable to describing the multipath waveform. In other work, the model was expanded to provide fractional rates of change for the received power [4]. The rapid effects of multipath may be treated separate from the effects due to terrain blocking and diffraction as these latter effects occur over a nominal period of about ten wavelengths of travel by the vehicle [5]. Differential demodulation of phase modulated signals has been used to overcome multipath, using the signal at one symbol period previous in time for carrier reference [6].

Demodulator output is thus the difference in phase between the previous symbol and the symbol being detected. Decision directed feedback has been used to provide carrier tracking over symbol-length intervals when higher-ordered modulation types are used [7]. Performance of direct digital modulations has been derived for noise environments assuming no carrier error [8] and extended to show effects of carrier and gain error for terrestrial applications [9]. This work provided verification of the curves derived herein. Automatic gain control [10] and diversity [11] are additional receiver design elements proposed for use in the land mobile channel.
III. Dynamic Multipath Characteristics

The amplitude of a multipath signal follows a Rayleigh distribution [12] while the phase changes shown in (1) as defined by Rice [13]. The narrowband nature of multipath limits the change rates of both amplitude and phase. The probability distributions for the change rates of amplitude (or power) and phase are discussed in the following paragraphs.

The phase change rate is independent of the receiver tuning error which results in a constant phase change rate added to the multipath phase change rate. The pdf of the rate of phase change \( \phi \) may be expressed as

\[
pdf(\phi) = \frac{1}{2} \sqrt{\frac{b_0}{b_2}} (1 + \frac{b_0}{b_2} \phi^2)^{-3/2}
\]

where \( b_0 \) and \( b_2 \) are the variances of amplitude and amplitude change rate respectively. The amplitude variation, follows a Rayleigh distribution:

\[
P(r) = \frac{r}{b_2} e^{-r^2/2b_2}.
\]

In systems that employ automatic gain control (AGC), small amplitude variations in the receive signal result in small variations at the detector when the incoming signal is large. However, when the incoming signal is small, very small variations in the receive signal result in large variations at the detector. The fractional power change rate characterizes the relative rate at which the signal is changing. Letting \( s \) = the fractional power change rate \( \text{pwr}^\gamma / \text{pwr} \), the pdf may be expressed as shown below [4]:

\[
pdf(s) = pdf \left( \frac{I dh/dt + Q dQ/dt}{(I^2 + Q^2)} \right) = \frac{\sigma_a}{2 \sigma_b (1 + s^2 \sigma_a^2 / \sigma_b^2)^{3/2}}
\]

where the variances \( \sigma_a^2 \) and \( \sigma_b^2 \) are the same as \( b_0 \) and \( b_2 \) in (1).

IV. Carrier Recovery Window Selection

The period for which a carrier may be considered stable is very short, usually just a few times the length of a symbol. An accurate carrier estimate requires consideration of the signal near the point of interest in time, however, longer periods of time should be considered to average out received noise. The carrier may be estimated at a point in time using only previous signal or using both previous and future signal.

If signal phase is averaged across the period of one symbol to provide a basic phase estimate \( \phi_i \), then carrier phase may be estimated for symbol \( i \) from the \( N \) preceding symbols using a single sided estimate:

\[
est(\phi) = \frac{1}{N} \sum_{k=1}^{N} (\phi_{i-k} - \Delta \phi_{\text{avg}})
\]

where \( \Delta \phi_{\text{avg}} \) is the average phase change per symbol due to carrier frequency offset error. Amplitude is estimated as

\[
est(A) = \frac{1}{N} \sum_{k=1}^{N} (A_{i-k}).
\]

using \( A_{i-k} \) to represent the amplitude of one symbol. Phase and amplitude may be estimated for symbol \( i \) using \( N \) preceding and \( N \) following measurements in a double sided estimate as follows:

\[
est(\phi) = \frac{1}{2N} \sum_{k=-N}^{N} (\phi_{i+k} + \phi_{i+k}), \quad k \neq N
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Recorded multipath channel measurements were processed to determine the effect of time averaging for both single-sided and double-sided methods. The measurements included actual quantization noise, receiver noise, and passband phase error. The average error for carrier phase and amplitude are shown in Figures 1 and 2 respectively for both single sided and double sided estimates. The results shown are from measured data using the processing methods of (4) through (7).

![Figure 1. Carrier phase error in a measured channel vs. period considered.]

![Figure 2. Fractional amplitude error in a measured channel vs. period considered.]

As the consideration period extends through four symbol periods, the performance improves slightly due to the averaging effect on the random noise. For greater periods of consideration, phase drift will increase the error. The double-sided estimate less sensitive to drift than the single-sided estimate.
Effect of Carrier Recovery Errors on Noise Performance

A rectangular constellation 16-QAM signal may be demodulated using coherent maximum likelihood detection on each channel, I and Q. As shown by the dashed lines in Figure 3, the I and Q channels are each divided into four regions by threshold levels, with each region centered to surround a constellation point.

![Figure 3. Constellation of 16-QAM signal with noise](image)

Noise in the channel will move the received sample away from the constellation point. If the added noise exceeds a certain value, the sample cross the threshold line, resulting in error.

Carrier phase error will skew the sample points from the center of a partition. As a result, a smaller amount of noise is required to move the sample across threshold boundaries. Large phase errors move the sample points across threshold lines without the noise contribution.

Consider the receive signal at a point in time with additive zero-mean Gaussian noise of variance $\sigma^2$. If $x$ is the level of signal received (over one symbol period) then the probability that $x$ will exceed a given threshold due to noise, i.e. be less than a given value $-a$ is given by

$$Pr \{ x < -a \} = \frac{1}{2} \text{erfc} \left( \frac{a}{\sqrt{2} \sigma} \right) \quad (8)$$

Now, let the energy required to over a given symbol period to produce error be represented as $a^2$, and the average received power in the signal be $e$. When Gaussian noise of in-band power $N_0/2$, is added to the signal, the variance of the resulting signal is

$$s^2 = N_0/2, \quad s = \frac{\sqrt{N_0}}{\sqrt{2}} \quad (9)$$

[16], and the error rate may be related to SNR by

$$P_e = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{f(e)}{N_0}} \right). \quad (10)$$

For analysis of 16-QAM, the method presented by Benedetto et. al. [14] was modified to using constellation points that have been skewed in both amplitude and phase directions. The resulting probability of error dependent upon fractional amplitude error $e_a$ and phase error $e_p$ is given by

$$P_e = \frac{1}{8} \text{erfc} \left( \frac{d_{i1}}{\sqrt{N_0}} \right) + \frac{1}{8} \text{erfc} \left( \frac{d - d_{i1}}{\sqrt{N_0}} \right)$$

$$+ \frac{1}{8} \text{erfc} \left( \frac{d_{q1}}{\sqrt{N_0}} \right) \left\{ 1 - \frac{1}{2} \text{erfc} \left( \frac{d_{i1}}{\sqrt{N_0}} \right) - \frac{1}{2} \text{erfc} \left( \frac{d - d_{i1}}{\sqrt{N_0}} \right) \right\}$$

$$+ \frac{1}{4} \text{erfc} \left( \frac{d_{i2}}{\sqrt{N_0}} \right) + \frac{1}{4} \text{erfc} \left( \frac{d_{q2}}{\sqrt{N_0}} \right) \left\{ 1 - \frac{1}{2} \text{erfc} \left( \frac{d_{i2}}{\sqrt{N_0}} \right) \right\}$$

$$+ \frac{1}{8} \text{erfc} \left( \frac{d_{i3}}{\sqrt{N_0}} \right)$$

$$+ \frac{1}{8} \text{erfc} \left( \frac{d_{q3}}{\sqrt{N_0}} \right) \left\{ 1 - \frac{1}{2} \text{erfc} \left( \frac{d_{i3}}{\sqrt{N_0}} \right) \right\} \quad (11)$$

where the average energy per symbol in terms of distance $d$ is given by

$$E_{avg} = \frac{10}{4} d^2, \quad \text{thus } d = \sqrt{\frac{4}{10} E_{avg}}, \quad (12)$$

and the distances from each constellation point center to the cell edges is expressed in terms of average signal as

$$d_{i3} = e_a \frac{3}{\sqrt{2}} \sqrt{\frac{4}{10} E_{avg}} \cos (45^\circ + e_p) - \sqrt{\frac{4}{10} E_{avg}}, \quad (13)$$

$$d_{q3} = e_a \frac{3}{\sqrt{2}} \sqrt{\frac{4}{10} E_{avg}} \sin (45^\circ + e_p) - \sqrt{\frac{4}{10} E_{avg}},$$

$$d_{q2} = e_a \frac{3}{\sqrt{2}} \sqrt{\frac{4}{10} E_{avg}} \sin (18.43^\circ + e_p),$$

$$d_{i2} = d_{q2} - \sqrt{\frac{4}{10} E_{avg}},$$

$$d_{i1} = e_a \frac{1}{\sqrt{2}} \sqrt{\frac{4}{10} E_{avg}} \cos (45^\circ + e_p), \quad \text{and}$$

$$d_{q1} = d_{i1}.$$
average received power level [15]. The bit error rate performance for 16-QAM will vary with the SNR. The performance may be determined for a local mean or average SNR level by

\[ P_S = \int_{r=0}^{\infty} P_e(r) P_r(r) \, dr \]  \hspace{1cm} (14)

where \( r \) is the signal amplitude, \( P_e(r) \) is the probability of symbol error for level \( r \), and \( P_r(r) \) is the pdf of \( r \).

For 16-QAM, \( P_e(r) \) is calculated by removing the phase and amplitude error from (11), resulting in

\[ P_e(r) = \frac{6}{4} \text{erfc} \left( \sqrt{\frac{r^2}{10 N_0}} \right) \left( 1 - \frac{3}{8} \text{erfc} \left( \sqrt{\frac{r^2}{10 N_0}} \right) \right) \]  \hspace{1cm} (15)

and \( P_r(r) \) is Rayleigh distributed as

\[ P_r = \frac{r}{a} e^{-r^2/2a^2}. \]  \hspace{1cm} (16)

Combining (14), (15), and (16) the symbol error for an average SNR may be shown to be

\[ P_S = \frac{3}{4\sqrt{\pi}} \sqrt{\frac{1}{1 + \frac{10\pi N_0}{4 R v g}}} \]  \hspace{1cm} (17)

The resulting symbol error vs. average SNR is plotted in Figure 6. These curves describe performance in a system having no carrier recovery error. The errors in recovery of signal amplitude and phase may be considered as the dominant level of background noise for signal performance modeling.

**Correlation of Phase and Amplitude errors**

The error due to noise only may be used as a limit on total system performance if optimal carrier recovery is used. As shown in Figure 5, a histogram of measured phase, the majority of time the signal exhibits significant (>8 degree) phase error is during fading conditions.

**Performance relative to Average SNR**

Since the mobile signal is constantly changing in amplitude while the noise remains relatively constant, it is often useful to express SNR using the local mean or average received power level [15]. The bit error rate performance for 16-QAM will vary with the SNR. The performance may be determined for a local mean or average SNR level by

\[ P_S = \int_{r=0}^{\infty} P_e(r) P_r(r) \, dr \]  \hspace{1cm} (14)

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**Diversity Improvement**

Since the bit error rate for 16-QAM in a multipath environment is limited by the low instantaneous SNR during fades, diversity may be used to improve signal quality. Time Diversity may be incorporated to use the signal transmitted during the periods of high instantaneous SNR and ignore the signal transmitted during the fading period. For this example, BCH codes were used to effect time diversity. The resulting improvement in symbol error rate is shown in Figure 6.

The error is corrected by a \((n,k,t)\) BCH code where \( n \) is the length of coded block in bits, \( k \) is the number of original bits encoded, and \( t \) is the number of errors corrected. BCH codes of \((n,k,t) = (7,4,1), (15,7,2), (31,16,3), \) were considered. The Probability of a word error resulting in \( k \) bits of data error is given by the probability that more than \( t \) errors will occur in the block of \( n \) transmitted bits [16]. This is given by

\[ P_w(e) = \sum_{m=t+1}^{n} \frac{n!}{m! (n-m)!} P_s^m (1 - P_s)^{n-m}. \]  \hspace{1cm} (18)
Multipath Noise Contribution

The portion of the amplitude and phase distortion resulting from multipath that may not be compensated may be represented as a noise level that is proportional to the transmitted power. Thus, for signals with a large received power with respect to non-multipath noise, the multipath noise will become the dominant noise source. In the example, for a channel measured in a suburban shadow environment with a vehicle speed of 15 mph, the standard deviation of phase change rate and fractional power change rates were measured as 9.6 Hz and 124%/second respectively. In a differential demodulator, using a technique that does not remove the effect of multipath, at a 4800 baud symbol rate, these values become for carrier phase error $\sigma_c$, and power error $\sigma_p$ are

$$\sigma_c = \frac{9.6 \times 360}{4800} = 0.72 \text{ deg/symbol, and}$$

$$\sigma_p = \frac{124}{4800} = 0.026 \%\text{/symbol.}$$

(19)

In this worse-case example, the carrier and power components together have a multipath noise power of 0.029 times the signal power, thus limiting the achievable SNR to 15.3 dB.

VI. Conclusion

The land mobile communication channel will support direct digital modulation of 16-QAM at symbol rates of 4,800 baud and higher, up to the coherence bandwidth of about 30,000 baud. At high baud rates, the rapid amplitude and phase changes due to multipath may be left uncompensated and considered as noise. In this case, the multipath noise becomes the dominant noise in the channel, and the signal must be corrected using diversity. Averaging techniques, employing a time constant of one to four symbol periods may be used to reduce the non-multipath noise in the signal. Techniques which employ longer time constants will be susceptible to multipath effects. Double-sided techniques which consider the signal previous and subsequent to a point of interest provide compensation for doppler and receiver tuning errors. In contrast to terrestrial applications, the performance of 16-QAM in land mobile applications improves as the symbol rate increases up to the channel coherence bandwidth. Diversity may be used to improve the signal error performance to levels acceptable for voice or data transmission.

References