OPTIMUM DIVERSITY COMBINING WITH FINITE-TAP DECISION FEEDBACK EQUALIZATION IN DIGITAL CELLULAR MOBILE RADIO

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ABSTRACT

This paper analyzes the performance of space diversity structures combined with finite-tap decision-feedback equalization technique in a mobile radio environment with QAM transmission and frequency-selective fading. Two optimum diversity structures, combining-diversity and selection-diversity, are considered in our analysis. By analytic techniques and simulation, we show that for the same number of taps the improvement is always greater with combining-diversity structure. The combining-diversity with a certain finite number of taps can outperform the selection-diversity with infinite taps. The numbers of taps to approach optimum performance (infinite-tap bound) in outage for different mobile environments are also determined. The decision-feedback equalizer not only provides better performance than the linear equalizer, but also needs a smaller number of taps to approach the optimum performance, especially in the combining diversity system or in the severe time-dispersion environment.

I. INTRODUCTION

The radio propagation environment places fundamental limitations on the performance of wireless communications systems. Multipath effects are one of the major sources of propagation degradation. Because of multipath effects, mobile radio channels in urban environment are rapidly time varying functions with fading rates dependent on Doppler effects caused by vehicle speed. For a narrow-band signal, multipath results in deep flat fading of the received signal. For a wide-band signal, the fading may be frequency-selective leading to intersymbol interference (ISI).

Diversity techniques are often used to combat the effect of multipath fading, since deep fades seldom occur simultaneously during the same time intervals over two or more independent paths. It has been known that fading effects can be ameliorated [4], if the receiver has multiple antennas, spaced sufficiently far apart such that the received signals are statistically independent. Diversity by itself may not effectively combat frequency-selective fading. Therefore, for digital transmission at high symbol rates, most space diversity receptions are used in combination with various equalization techniques to mitigate intersymbol interference caused by frequency-selective fading.

An optimum combining-diversity receiver using linear or decision-feedback equalization operating in the two-path mobile radio channel has been presented by Balaban and Salz [1]. The numerical results show that dual diversity combining and equalization can provide about two orders-of-magnitude improvement in the average error rate or in outage probability. In their analysis, infinite-tap equalizers have been considered to obtain the best performance benchmarks. However, only finite-tap equalizers can be used in practical receiver design and this case has not been considered by them. Other authors have reported on the performance of diversity receivers with the finite-tap zero-forcing equalizer for point-to-point radio [2], but the use of finite-tap decision-feedback equalizers, which are extensively employed in digital radio systems, has not been reported. Therefore, analyzing the combining-diversity with finite-tap decision-feedback equalization operating in mobile radio channel is of considerable interest.

The selection-equalization diversity is still used to reduce multipath fading effects. A performance analysis of selection-diversity in mobile radio has been presented [3], but no equalizer is used in the structure. The number of ISI terms may be increased using combining diversity such that more taps are required to suppress ISI. Thus, the performance of combining-diversity may be inferior to that of selection-diversity for some cases where only a certain number of taps are used [2]. Our intent is to compare the performance of selection-equalization and combining-diversity structures, when finite-tap equalization are employed. In addition, we also determine the number of taps which can approach optimum performance (infinite-tap bound) in outage probability for the mobile environments with different values of delay spread.

The covariance matrix inversion method for computing the optimum tap setting of finite-tap linear and decision-feedback equalizers are developed. In order to compare the simulation results with those of the Balaban & Salz, the same two-ray Rayleigh-fading channel model, QAM modulation, Monte-Carlo method and probability bound estimation are followed, but with some corrections and new approaches. We evaluate numerical results only for 2-path diversity with 4-QAM modulation, statistically independent channel parameters and identical SNR’s on diversity branches.

We only show the simulation results in outage probability, which is considered as the best performance criteria for digital radio. Simulation results show that combining-diversity always provides better performance than non-diversity and selection-diversity for the same number of taps. This conclusion is slightly different from that using finite-tap zero-forcing equalizer in point-to-point channels [2]. Combining-diversity with a certain number of taps can outperform the infinite-tap selection-diversity. The decision-feedback equalizer performs better than the linear equalizer for any diversity structure, as expected. In addition, a much fewer number of taps in decision-feedback equalization are required to approach the optimum performance, especially in the combining-diversity system or in the channel with severe time dispersion.

* This work was done while the author was at the University of Texas at Arlington.
II. SYSTEM MODELING

In this section we define the mobile radio system with QAM data transmission, including transmitter, diversity channel model, and diversity receiver.

Transmitted Signals

We denote the transmitted signal by \( s(t) = g(t) \). Where \( s(t) \) is the complex baseband signal and \( g(t) \) is the carrier frequency. The equivalent baseband signal \( s(t) \) is given by

\[
 s(t) = \sum_n c_n g_n(t-nT)
\]

with \( g_n(t) \), the impulse response of the transmitter shaping filter as a real pulse having square-root raised cosine spectra, and \( T \), the signaling interval. Also, \( c_n = a_n + j b_n \) are the complex data sequences, where \( a_n, b_n \in \{ \pm 1, \pm 3, ..., \pm L \} \) define the signal constellation with \( L^2 \) points in QAM system.

Diversity Reception

Diversity reception is one of the useful methods to combat multipath fading effects. In this approach, several antennas separated sufficiently in space are used to process the independent received fading signals. After ideal coherent demodulation, the baseband signal received on the \( k \)th diversity branch, \( k = 1, ..., N \), is expressed as

\[
 v_k(t) = s(t) \otimes h_k(t) + n_k(t)
\]

where \( h_k(t) \) and \( n_k(t) \) represent the impulse response and additive white Gaussian processes with common spectral densities \( N_0 \) on the \( k \)th diversity branch.

Diversity Channel Model

The frequency-selective channel can be characterized by a small number of short-term fading beams with a delay spectrum [1]. The multipath channel model with Rayleigh-fading beams and double spike delay spectrum are used for our simulation. Thus, the impulse response of the \( k \)th diversity path can be characterized as

\[
 h_k(t) = \rho_k^{(k)} \left( \alpha_k^{(k)} + j \beta_k^{(k)} \right) \delta(t) + \rho_k^{(k)} \left( \alpha_k^{(k)} + j \beta_k^{(k)} \right) \delta(t-\tau)
\]

where \( \alpha_k = \rho_k^{(k)} \left( \alpha_k^{(k)} + j \beta_k^{(k)} \right) \) and \( \beta_k = \rho_k^{(k)} \left( \alpha_k^{(k)} + j \beta_k^{(k)} \right) \) represent the two Rayleigh-fading beams. \( \alpha_k^{(k)}, \beta_k^{(k)} \) are zero-mean, i.i.d. Gaussian random variables. Parameter \( \rho_k^{(k)} \) is the amplitude of the \( k \)th scatter and \( \tau \) is the relative delay of the two paths. For a channel with a double-spike delay spectrum, \( \rho_k^{(k)} = \rho_k^{(k)} \). On the principle of microscopic diversity, it is reasonable to assume the mean power and relative delay are identical for all diversity paths [4]. The double spike spectrum tends to yield worst-case result for a given root mean square (RMS) delay spread. For a radio system, the error rate due to intersymbol interference may become intolerable for a certain value of delay. If the delay spread exceeds this value, it is necessary to use equalization to reduce the error rate.

III. DIVERSITY RECEIVER AND FINITE-TAP EQUALIZATION

We now consider the design of two optimum diversity schemes in combination with equalization techniques. The optimum selection-diversity structure consists of an optimum receiver on each diversity path [2]. Each optimum receiver is the cascade of a matched filter and an infinite-tap transversal filter, as shown in Fig. 1. The basic algorithm for the selection-diversity is based on the principle of selecting the best signal with least mean-square-error among all branches. The optimum combining-diversity reception revealed by the theory is composed of a set of matched filters whose outputs are first summed and then sampled. These samples are then passed through an infinite tapped-delay transversal filter [1], as described in Fig. 2.
Traditional adaptive algorithms for equalization are based on the criterion of minimizing the mean-square-error (MSE) between the desired output \( c_i \) and the actual output \( \hat{c}_i \)

\[
MSE = E\left[ (c_i - \hat{c}_i)^2 \right]
\]

where the equalizer output \( \hat{c}_i \), the estimate of the information sequence \( c_i \), depends on what kind of equalizer is used. The covariance matrix inversion approach for computing the optimum equalizer tap setting is employed in our simulation for the finite-tap equalization [6]. No whitening filter is cascaded at the receiver front end [5][7] in our analysis, since this approach is not realistic in actual implementation of receivers.

A. Linear Equalization

As shown in Fig. 3, the output of the linear equalizer in the signaling interval is expressed as

\[
\hat{c}_i = \sum_{j=-K}^{K} u_j V_{i-j}
\]

(8)

Based on the MSE criterion [6], the orthogonality condition provides us with a set of \( 2K+1 \) linear equations that can be solved for the optimum tap coefficients of a post-combining linear finite-tap equalizer

\[
\sum_{j=-K}^{K} u_j E[V_{i-j} V_{i-j}^*] = E[c_i c_i^*]
\]

(9)

The paths of the space diversity reception are assumed relatively independent and the sequence of data \( c_i \) is assumed to be an uncorrelated process. Substituting (4) into the both sides of (9), we obtain a set of linear equations for the tap coefficients

\[
\sum_{j=-K}^{K} u_j \left[ \sum_{k=1}^{N} r_{(j)}^{(k)} \right] = \sum_{k=1}^{N} r_{(k)}^{(j)}
\]

(10)

Substituting (10) to (7), the minimum mean-square-error for combining-diversity with linear equalization is derived as

\[
J_{min} = \sigma_c^2 \left[ \frac{1}{N} \sum_{k=1}^{N} \left( \sum_{j=-K}^{K} u_j r_{(j)}^{(k)} \right)^2 \right] = \sigma_c^2 \left( 1 - g_{\alpha}^{(k)} \right)
\]

(11)

where \( g_{\alpha}^{(k)} \) is the overall single pulse response taken at time instant \( t=0 \) at the post-combining equalizer output. We observe that the Eq. (10) can be obtained by replacing the single pulse response in the non-diversity case with the summation of the single pulse response of all paths.

B. Decision-feedback Equalization

The decision-feedback equalizer which consists of two filters, a feedforward filter (FF) and a feedback filter (FB) as described in Fig. 4, has been shown to be more effective than linear equalization [5]. The equalizer output sampled at \( t=T \) can be expressed as

\[
\hat{c}_i = \sum_{q=-K}^{0} u_q V_{i-q} + \sum_{p=-K}^{K} u_p \hat{c}_{i-p}
\]

(12)

where \( V_r \) represents the input signal to the FF equalizer. \( u_q \) and \( u_p \) are the weight coefficients of the FF and FB equalizers separately. The equalizer is assumed to have \( K_1+1 \) taps in its feedforward section and \( K_2 \) taps in its feedback section.

Substituting (12) into (7) and taking the derivative of the MSE with respect to the coefficient, \( u_n \), in the feedback section and setting the result to zero

\[
\frac{\partial J}{\partial u_n^*} = E[(c_i - \hat{c}_i) (\hat{c}_{i-n} - \hat{c}_i)] = 0 \Rightarrow E[(c_i - \hat{c}_i) (\hat{c}_{i-n} - \hat{c}_i)] = 0
\]

(13)

Since \( n \neq 0 \) and \( (i-n) \neq 1 \), \( E[c_i \hat{c}_{i-n}] = E[c_i c_{i-n}] = 0 \). Then Eq. (13) is rewritten as

\[
E\left[ \sum_{q=-K}^{0} u_q V_{i-q} \hat{c}_{i-n}^* \right] = -E\left[ \sum_{p=-K}^{K} u_p \hat{c}_{i-p} \hat{c}_{i-n}^* \right]
\]

(14)

where \( E[\hat{c}_{i-n} \hat{c}_{i-n}] = \delta_{i-n} \). Substitution of (4) into (14), yields

\[
u_n = -\sum_{k=1}^{N} \left( \sum_{q=-K}^{0} u_q r_{(k)}^{(q)} \right)
\]

(15)

where \( n \neq 0 \). Functionally the feedback filter is used to remove that part of the ISI from the present estimate caused by previously detected symbols \( \hat{c}_j \), where it is assumed that \( \hat{c}_i = \hat{c}_j \) after a suitable training period. Similarly, for the coefficient in the feedforward section, \( u_i \)

\[
\frac{\partial J}{\partial u_i^*} = E[(c_i - \hat{c}_i) (V_{i-n}^* - \hat{c}_i)] = 0 \Rightarrow E[(c_i - \hat{c}_i) (V_{i-n}^*)] = 0
\]

(16)

or, equivalently, the orthogonality condition for feedback equalizer is expressed in the form

\[
E\left[ \sum_{q=-K}^{0} u_q V_{i-q} + \sum_{p=-K}^{K} u_p \hat{c}_{i-p} \right] V_{i-n}^* = E[c_i V_{i-n}^*]
\]

(17)

where \( i \neq q \leq 0 \). Similarly, substituting (4) and (15) into (17), we obtain the set of linear equations for the tap coefficients of feedforward filter.
Substituting (12) to (7), the mean-square-error for decision-feedback equalization is derived as

$$J_{\text{min}} = \sigma_e^2 \left[ 1 - \sum_{k=1}^{N} \left( \sum_{q=-K}^{0} \sum_{m=-1}^{N} \frac{r_k^{(q)}}{\sigma_e^2} \right) \right] = \sigma_e^2 (1 - \tilde{g}_0^{D2})$$

(19)

where $\tilde{g}_0^{D2}$ represents the sampled single pulse response taken at $T=0$ in the feedforward equalizer. Provided that previous decisions are correct and that $K_s \geq L$ in the feedback filter, Eq. (18) can be rewritten as

$$\sum_{q=-K}^{0} \sum_{m=-L}^{L} \frac{r_k^{(q)} \cdot m}{\sigma_e^2} = \sum_{k=1}^{N} r_k^{(q)}$$

(20)

As $K_s \to \infty$ in the feedforward filter, the minimum mean-square-error has been expressed as [1]

$$J_{\text{min}} = \sigma_e^2 \exp \left\{ -\frac{T}{2\pi} \sum_{n=-\infty}^{\infty} \ln \left[ 1 + \frac{\sigma_e^2}{N} \sum_{k=1}^{N} R_k(w) \right] \right\}$$

(21)

where $R_k(w)$ is the periodic frequency functions and defined as

$$R_k(w) = \mathcal{F}(r_k^{(q)}) = \frac{1}{T} \sum_{n=-\infty}^{\infty} |F_k(w + \frac{2\pi n}{T})|^2$$

(22)

Further progress can be made by calculating the minimum MSE using equation (24).

$$MSE = (g_0 - 1)^2 \sigma_e^2 + \sum_{k=0}^{N} |g_k|^2 + \sigma_e^2$$

(28)

of the equalizer. $\hat{c}_t$ represents the equalizer output and can be written as

$$\hat{c}_t = a_t + j b_t \quad \text{where}$$

$$a_t = a g_0 + \sum_{j \neq 1} b_j g_{k-1} + v_t$$

$$b_t = b g_0 + \sum_{j \neq 1} a_j g_{k-1} + v_t$$

(25)

A good estimate of the error rate for a QAM system is $P_e = P_e^{(e)} + P_e^{(c)}$, where $P_e^{(e)}$ and $P_e^{(c)}$ represent the error probabilities on the real and imaginary axis, respectively.

$$P_e^{(e)} = P_r \left[ v_{r_t} \geq g_0 - \sum_{j \neq 1} a_j g_{k-1} - b_j g_{k-1} \right]$$

$$P_e^{(c)} = P_r \left[ v_{i_t} \geq g_0 - \sum_{j \neq 1} b_j g_{k-1} - a_j g_{k-1} \right]$$

(26)

since the $a_j$ and $b_j$ are identically independent, the intersymbol and cross-symbol interference terms have identical probability distribution, and then $P_e^{(e)} = P_e^{(c)} \Rightarrow P_e < 2P_e^{(e)}$. $v_r$ and $v_i$ are ideally distributed Gaussian random variables. Using the Chernoff bound [9] to estimate (26), yields the upper bound for M-ary QAM system as

$$P_e^{(c)} \leq 2 \left( 1 - \frac{1}{\sqrt{M}} \right) \exp \left\{ -\frac{\sigma_e^2}{2} \frac{g_0^2}{g_0^2 - 1} \right\}$$

(27)

Further progress can be made by calculating the minimum MSE using equation (24).

$$MSE = (g_0 - 1)^2 \sigma_e^2 + \sum_{k=0}^{N} |g_k|^2 + \sigma_e^2$$

(28)

where $|g_k|^2 = g_k^2$, $\sigma_e^2 = \sigma_e^2$, $\sigma_i^2 = \sigma_i^2$, $\sigma_e^2 = \sigma_e^2$ and by hypothesis, $\sigma_e^2 = \sigma_i^2$ and $\sigma_e^2 = \sigma_i^2$. Substituting (28) to (27), yields

$$P_e \leq 4 \left( 1 - \frac{1}{\sqrt{M}} \right) \exp \left\{ -\frac{\sigma_e^2}{2M - \sigma_e^2} (g_0 - 1)^2 \right\}$$

(29)

From (11) and (19), we know $MSE = \frac{1}{M} (1 - g_0)$ and substituting (29), a useful exponential relationship between the probability of error and MSE in 4-QAM is expressed as

$$P_e \leq 2 \exp \left\{ -\frac{1 - (MSE)/(M)}{\sqrt{M}} \right\}$$

(30)

In the Balaban and Salz’s paper, the factor 2 was omitted in the formula. This formula was extensively used to estimate the error rate bound when the system is degraded by intersymbol interference.

### Outage Probability

The best measure of the performance of mobile radio system is the outage probability $P_o$, defined as the probability that error rate exceeds a specified threshold value $P_{th}$, given that multipath fading is occurring [10]. In formulas, the outage domain $P_o = P_r(P > P_{th})$ can be expressed for the mobile channel with relative delay $\tau$ as

$$P_o(\tau) = \int_D f_{\eta, \theta, \beta_1, \beta_2}(u, v, x, y) \, du \, dv \, dx \, dy$$

where $D = \alpha_1, \beta_1, \beta_2 : P_e > P_{th}$ . The threshold is typically $10^{-2}$ for voice circuit applications and $10^{-6}$ for data [2]. Since outage probability is the best performance
criteria, not average error probability, we only show the
simulation results in outage probability for various diversity
receiver designs. The threshold is defined as $P_{th} = 10^{-3}$ in our
simulation results.

V. SIMULATION RESULTS

We evaluate the performance of the various receiver structures
by using the Monte-Carlo simulation method. Generally, the total
number of iterations $N$ should be on the order of $10^6/p$, where $p$

Table 1: Infinite-tap Linear Equalization.

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Delay = 0T</th>
<th>Delay = 0.6T</th>
<th>Delay = 1.6T</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2.62</td>
<td>2.18</td>
<td>1.88</td>
</tr>
<tr>
<td>20</td>
<td>7.10</td>
<td>6.19</td>
<td>6.86</td>
</tr>
<tr>
<td>25</td>
<td>21.25</td>
<td>25.17</td>
<td>44.59</td>
</tr>
</tbody>
</table>

P_{th} = 10^{-3}$ as described in
Figs. 5-6 [6]. We also can estimate the outage probability at the
same time.

![Fig. 5. Monte-Carlo simulation procedure for QAM mobile radio.](image)

![Fig. 6. Confidence bands for the Monte Carlo technique.](image)

A. Infinite-tap Equalization

An optimum combining-diversity receiver using linear or
decision-feedback equalization has been presented by Balaban
and Salz. Here, we include the selection-diversity for comparing
these two different diversity systems. The simulation shows that
the outage probability of error decreases to asymptotic value at
about 0.6T due to a greater effect of intrinsic frequency diversity
[1], which provides about an order-of-magnitude improvement in
outage probability for the non-diversity and selection-diversity
and about two orders-of-magnitude for dual combining-diversity.
The diversity receiver structure can be compared by calculating
the improvement factor.

$$I = \frac{\text{Outage probability of non-diversity}}{\text{Outage probability of diversity}} P_{th} \quad (32)$$

The improvements for each structure with the linear equalizer
and the decision-feedback equalizer are given in Table 1 and
Table 2, respectively. The flat fading channel and frequency-
selective channels with relative delay, 0.6T and 1.6T, are
evaluated. The results show the combining-diversity structure
provides greater improvement than selection-diversity structure.
Also, for decision feedback equalization, the outage probability versus SNR for different number of taps in feedforward and feedback sections at the same relative delay is described in Figs. 9-10. If the number of taps in a feedback equalizer is over two or three, almost all post-cursors of ISI can be eliminated and then there is significant improvement in error probability. However, the improvement is often limited by the number of taps of the feedback filter. For example, at the relative delay \( = 0.6T \), the performance with (FF, FB) = (3,2) is better than (FF, FB) = (4,1), given total number of taps = 5. Therefore, it is important for finite-tap decision-feedback equalization to determine the terms of post-cursors of ISI.

Fig. 9. Outage versus SNR in finite-tap decision-feedback equalization.

Fig. 10. Outage versus SNR in finite-tap decision-feedback equalization.

In finite-tap equalization, the combining-diversity is still superior to the selection-diversity and non-diversity in performance for both equalization algorithms, but the advantage is reduced when the number of taps is not sufficient to eliminate ISI terms. This result is a little different from that of finite-tap zero-forcing equalizer for point-to-point radio, where the five-tap combining-diversity does not outperform the selection-diversity [2]. Additionally, the decision feedback equalizer performs much better than the linear equalizer even using fewer taps in any diversity structure.

Figs. 11-12 and Figs. 13-14 show the outage probability decreases to an asymptotic value (infinite-tap) with the increasing of taps in linear and decision-feedback equalization at \( SNR = 20 \) dB for delay = 0.6T and 1.6T. We notice that the taps capable of eliminating most post-cursors in the feedback section, the performance improvement due to the increase in DFE taps converges faster to an asymptotic value than in the linear equalizer, especially in cases of the combining-diversity structure.

Fig. 11. Outage versus number of taps in linear equalization (delay = 0.6T).

Fig. 12. Outage versus number of taps in decision-feedback equalization.

Fig. 13. Outage versus number of taps in linear equalization (delay = 1.6T).

Fig. 14. Outage versus number of taps in decision-feedback equalization.
Number of taps to approach optimum performance

We have also developed a new method for comparing the performance of these two diversity configurations with various number of taps in equalization by defining the factor

\[ I = \frac{(\text{Outage of finite-tap equalizer with } K \text{ taps})}{(\text{Outage of infinite-tap equalizer })} \]

Given the factor \( I \) below a certain value = 1.2, Table 3 and Table 4 show the minimum number of taps required for 3 different values of SNR at relative delay = 0.6T and 1.6T.

**TABLE 3** Finite-tap Linear Equalization. \( I \leq 1.2 \)

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>( \text{No diversity} )</th>
<th>( \text{Selection} )</th>
<th>( \text{Combining} )</th>
<th>( \text{No diversity} )</th>
<th>( \text{Selection} )</th>
<th>( \text{Combining} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>5</td>
<td>7*2</td>
<td>7</td>
<td>11</td>
<td>17*2</td>
<td>17</td>
</tr>
<tr>
<td>20</td>
<td>9</td>
<td>13*2</td>
<td>15</td>
<td>23</td>
<td>29*2</td>
<td>29</td>
</tr>
<tr>
<td>25</td>
<td>17</td>
<td>25*2</td>
<td>25</td>
<td>39</td>
<td>49*2</td>
<td>49</td>
</tr>
</tbody>
</table>

**TABLE 4** Finite-tap Decision-Feedback Equalization. \( I \leq 1.2 \)

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>( \text{No diversity} )</th>
<th>( \text{Selection} )</th>
<th>( \text{Combining} )</th>
<th>( \text{No diversity} )</th>
<th>( \text{Selection} )</th>
<th>( \text{Combining} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>(3,2)</td>
<td>(4,3)*2</td>
<td>(4,2)</td>
<td>(6,3)</td>
<td>(7,3)*2</td>
<td>(6,3)</td>
</tr>
<tr>
<td>20</td>
<td>(4,2)</td>
<td>(4,3)*2</td>
<td>(4,2)</td>
<td>(7,3)</td>
<td>(7,4)*2</td>
<td>(7,3)</td>
</tr>
<tr>
<td>25</td>
<td>(4,2)</td>
<td>(4,3)*2</td>
<td>(4,2)</td>
<td>(7,3)</td>
<td>(7,4)*2</td>
<td>(7,3)</td>
</tr>
</tbody>
</table>

where the number multiplied by a factor of 2 is the total number of taps required for the two-path selection-diversity. (FF, FB) represents taps number in feed-forward and feedback filters.

We observe that the decision-feedback equalizer needs fewer taps to approach the optimum performance than the linear equalizer, especially in the combining diversity system or in severe time-dispersive environments. One reason is that the number of ISI terms may be increased using combining diversity such that more taps are required to suppress ISI in linear equalization, but most of ISI terms can be removed by the feedback filter in decision-feedback equalization.

VI. CONCLUSION

We have presented the analysis and simulation of different optimal diversity structures with finite-tap decision-feedback equalization techniques. In comparison to the linear equalizer, the decision-feedback equalizer can achieve better performance by using fewer taps. As expected, with optimum structures, the combining-diversity always outperforms the selection-diversity for both equalization algorithms when infinite taps or finite taps are used. In absence of diversity or with diversity, we do not discuss how many taps are required to generate the outage or average error probability while still maintaining acceptable speech quality. We have presented objective figures-of-merit which hopefully can be used by the system designers to resolve the subjective issues.

Our analysis can be extended to system with co-channel interference (CCI) or adjacent channel interference (ACI) effects, and also to the systems in point-to-point radio environments. Other more efficient and exact methods, like moment method \[6][8], can be applied to the evaluation of error probability in the Monte Carlo simulation.

REFERENCES


APPENDIX

Signal-to-Noise Ratio Calibration

With zero-mean information symbols, the average power spectrum of the signal received by the \( k \)th diversity antenna is

\[ V_k(w) = \frac{1}{2\pi} \left| H_k(w) \right|^2 \]

where \( G(w) \)=\( |G_k(w)|^2 \) has a raised cosine spectral characteristic, \( G_k(w) \) is the impulse response of transmitter filter. Recall that \( T \) is the signaling interval, and \( \sigma_f^2 = E[|c(t)|^2] \) is the data symbol variance. The frequency response of channel, \( H_k(w) \), is

\[ H_k(w) = S_0 e^{j\theta} \]

Since \( a_k \) and \( b_k \) are random variables, the time-average power is

\[ <P(k) \{ a_k, b_k \} > = \frac{\sigma_f^2}{2\pi T} \left| G(w) \right|^2 dw = \frac{\sigma_f^2}{T} (\sigma_a^2 + \sigma_b^2) \]

The noise power measured in the Nyquist band is \( N_0/T \).

Therefore, for Rayleigh-fading channels, the average value of the SNR on the \( k \)th diversity path is defined by

\[ P = \frac{\sigma_f^2 (\sigma_a^2 + \sigma_b^2)/T}{N_0} = \frac{\sigma_f^2}{N_0} (\sigma_a^2 + \sigma_b^2) \]

We can denote \( \rho^2 = \sigma_f^2/N_0 \), while the mean powers are assumed to be identical for all diversity paths and the channels are normalized with \( \sigma_a^2 + \sigma_b^2 = 1 \). The average power of the Rayleigh-fading beam depends on the long-term shadow fading.