Figure 17.1-1 (p. 764)
A transistor amplifier.
DESIGN PROBLEM – Transistor Amplifier

Figure 17.1-1 shows the small signal equivalent circuit of a transistor amplifier. The data sheet for the transistor describes the transistor by specifying its $h$ parameters to be

\[ h_{ie} = 1250 \, \Omega, \quad h_{oe} = 0, \quad h_{ie} = 100, \quad h_{re} = 0 \]

The value of the resistance $R_c$ must be between $300 \, \Omega$ and $5000 \, \Omega$ to ensure that the transistor will be biased correctly. The small signal gain is defined to be

\[ A_v = v_o/v_{in} \]

The challenge is to design the amplifier so that

\[ A_v = -20 \]

(There is no guarantee that these specifications can be satisfied. Part of the problem is determining whether it is possible to design this amplifier so that $A_v = -20$.)

To solve this problem we need to define the $h$ parameters that describe the transistor. We will return to this problem at the end of the chapter.

17.2 Amplifiers and Filters

Continued improvements in vacuum tubes and amplifier circuits made long-distance telephone lines possible, and in 1915 Alexander Graham Bell placed the first transcontinental telephone call to his famous assistant Thomas Watson. In 1921 Bell Telephone was experimenting with a 1000-mile telephone line with three voice channels, but the nonlinearity of even the best tubes introduced an intolerable amount of distortion.
Amplifiers and Filters …

Transmission of messages over long lines required the insertion of amplifiers at points along the line. These devices have two terminals for input and two terminals for output to the line. Harold S. Black, a 23-year-old engineer at Bell Laboratories, concluded that, in a rapidly growing country 4000 miles wide, a new approach would be required. First he tried to improve the amplifier tubes, but he decided that the necessary 1000-fold reduction in distortion could never be achieved that way.

Amplifiers and Filters …

After hearing an inspiring talk by Charles Steinmetz, he clearly stated his problem: how to remove all the distortion from an imperfect amplifier. His first scheme was to compare the output (suitably reduced) to the input, amplify the difference (distortion) separately, and use it to cancel the distortion in the actual output. Within one day he had built a working feed-forward amplifier.

But this 1923 invention required very precise balances and subtractions. For example, every hour on the hour somebody had to adjust the filament current to the tubes. For four years, Black struggled, and failed, to turn his idea into a practical amplifier.
Amplifiers and Filters …

As he related 50 years later in the December 1977 IEEE spectrum:

“Then came the morning of Tuesday, August 2, 1927, when the concept of negative feedback amplifier came to me in a flash while I was crossing the Hudson river on the Lackawanna Ferry, on my way to work. I suddenly realized that if I fed the amplifier output back to the input, in reverse phase, I would have exactly what I wanted… On a page of the New York Times, I sketched a simple diagram… and the equations for amplification with feedback.”

Amplifiers and Filters …

Four months later, his goal was surpassed when a 100,000-to-1 reduction of distortion was realized in a practical one-stage amplifier. Now Black’s negative feedback principle is applied practically in all amplifiers.
17.3 Two ports Networks

Many physical electronics devices have two ports (or terminal pairs) for input and output, respectively.

Examples of these devices include filters, transistors, coaxial cable (transmission line), power supplies, etc. Likewise, the circuit models we use to represent these devices are two-port models.

17.4 T-to-Π Transformation and Two-Port Three-Terminal Networks

Two basic circuit configurations that occur frequently are the T network and the Π network

\[(a) \text{T-network (Y)} \quad (b) \text{Π-network (Δ)}\]
Sometimes it is more convenient to convert from one to the other. To develop the conversion, it is best done by representing the T- and \( \Pi \)-networks as three terminal networks.

![T-network](a) ![\Pi-network](b)

For equivalence, the two networks must have the same impedance when measured between the same pair of terminals.

Between terminal pair a-c:

\[
Z_1 + Z_3 = \frac{Z_A(Z_B + Z_C)}{Z_A + Z_B + Z_C}
\]

Between terminal pair a-b:

\[
Z_1 + Z_2 = \frac{Z_C(Z_A + Z_B)}{Z_A + Z_B + Z_C}
\]

Between terminal pair b-c:

\[
Z_2 + Z_3 = \frac{Z_B(Z_A + Z_C)}{Z_A + Z_B + Z_C}
\]
After some effort the conversion from Π to T becomes.

\[ Z_1 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} \]

\[ Z_2 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} \]

\[ Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} \]

And T to Π becomes

\[ Z_A = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} \]

\[ Z_B = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1} \]

\[ Z_C = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3} \]

(pg 766 and 767)

If all impedances are equal in these networks, then

\[ Z_{T} = \frac{Z_{\Pi}}{3} \]

or

\[ Z_{\Pi} = 3Z_{T} \]

Exercises in Section 17-4
Exercise 17.4.1 (pg 768)

Find the T circuit equivalent to the Π circuit shown in Figure E 17.4.1.

\[ Z_A = 100 \, \Omega \]
\[ Z_B = 125 \, \Omega \]
\[ Z_C = 25 \, \Omega \]

\[ Z_1 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} = \frac{(100)(25)}{100 + 125 + 25} = \frac{2500}{250} = 10 \, \Omega \]

\[ Z_2 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} = \frac{(125)(25)}{250} = \frac{3125}{250} = 12.5 \, \Omega \]

Exercise 17.4.1 (cont’d…)

\[ Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} = \frac{(100)(125)}{100 + 125 + 25} = \frac{12500}{250} = 50 \, \Omega \]
17.5 Equations of Two-port Networks

There are four variables associated with a two-port network: $V_1$, $I_1$, $V_2$, and $I_2$. This means there are six combinations of two variables as input (independent variable) and the other two variables as output (dependent variables).

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
<th>Circuit parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$, $I_2$</td>
<td>$V_1$, $V_2$</td>
<td>Impedance $Z$</td>
</tr>
<tr>
<td>$V_1$, $V_2$</td>
<td>$I_1$, $I_2$</td>
<td>Admittance $Y$</td>
</tr>
<tr>
<td>$V_1$, $I_2$</td>
<td>$I_1$, $V_2$</td>
<td>Hybrid $g$</td>
</tr>
<tr>
<td>$I_1$, $V_2$</td>
<td>$V_1$, $I_2$</td>
<td>Hybrid $h$</td>
</tr>
<tr>
<td>$V_2$, $I_2$</td>
<td>$V_1$, $I_1$</td>
<td>Transmission $T$</td>
</tr>
<tr>
<td>$V_1$, $I_1$</td>
<td>$V_2$, $I_2$</td>
<td>Inverse Transmission $T'$</td>
</tr>
</tbody>
</table>

Table 17.5-1 (pg 769)
### Obtaining the Z and Y Parameters

Z are called the “Open-Circuit: parameters
Y are called the “Short-Circuit: parameters

\[
V_1 = Z_{11} I_1 + Z_{12} I_2 \\
V_2 = Z_{21} I_1 + Z_{22} I_2
\]

The impedance parameters also called “Driving Point and Transfer Impedances”

\[
Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad \text{Terminal 2 is open}
\]

\[
Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} \quad \text{Terminal 1 is open}
\]
Obtaining the Z and Y Parameters …

\[ I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{Admittance Parameters} \]
\[ I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{The admittance parameters also called “Driving Point and Transfer Admittances”} \]

\[
\begin{align*}
Y_{11} &= \left. \frac{I_1}{V_1} \right|_{V_2 = 0} \quad \text{Terminal 2 is short circuited} \\
Y_{21} &= \left. \frac{I_2}{V_1} \right|_{V_2 = 0} \\
Y_{12} &= \left. \frac{I_1}{V_2} \right|_{V_1 = 0} \quad \text{Terminal 1 is short circuited} \\
Y_{22} &= \left. \frac{I_2}{V_2} \right|_{V_1 = 0}
\end{align*}
\]

For linear bi-lateral circuit with no dependent sources
\[ Z_{12} = Z_{21} \]
\[ Y_{12} = Y_{21} \]

Exercises in Section 17-5
Exercise 17.5-1 (pg 772)

Find the Z and Y parameters of the circuit of Fig E 17.5-1

Exercise 17.5-1 (Cont’d)

I will introduce an alternative method to that described in Table 17.5-3 (pg. 771) for obtaining the Z parameters

Step 1. To determine $Z_{11}$ and $Z_{22}$, connect a current source equal to 1 A to the input terminals. Injection of 1 A is consistent with assumed direction of $I_1$.

Step 2. Determine $V_1$ and $V_2$, thus $Z_{11} = \frac{V_1}{I_1}$ and $Z_{21} = \frac{V_2}{I_1} = V_2$.

Step 3. To determine $Z_{12}$ and $Z_{22}$, connect a current source equal to 1 A to the output terminals. Injection of 1 A is consistent with assumed direction of $I_2$.

Step 4. Determine $V_1$ and $V_2$, thus $Z_{22} = \frac{V_2}{I_2} = V_2$ and $Z_{12} = \frac{V_1}{I_2} = V_1$. 
Back to our Exercise:

\[ I_1 = 1 \text{ A} \]

Current divider to determine current through 42 ohm times 42 ohm resistor yields \( V_1 = Z_{11} \).

\[
V_1 = (1) \left( \frac{1}{\frac{42}{42} + \frac{1}{10.5 + 21}} \right) = 18
\]

\[
\therefore Z_{11} = \Omega
\]

Current divider to determine current through 10.5 ohm times 10.5 ohm resistor yields \( V_2 = Z_{21} \).

\[
V_2 = (1) \left( \frac{1}{\frac{21 + 10.5}{42} + \frac{1}{10.5 + 21}} \right) = 6
\]

\[
\therefore Z_{21} = \Omega
\]
Exercise 17.5-1 (Cont’d)

Current divider to determine current through 10.5 ohm times 10.5 ohm resistor yields \( V_2 = Z_{22} \).

\[
V_2 = (1) \left[ \frac{1}{\frac{10.5}{10.5} + \frac{1}{1}} \right] (10.5) = 9
\]

\[
\therefore Z_{22} = \Omega
\]

---

Exercise 17.5-1 (Cont’d)

Current divider to determine current through 42 ohm times 42 ohm resistor yields \( V_1 = Z_{12} \).

\[
V_1 = (1) \left[ \frac{1}{\frac{42}{10.5} + \frac{1}{1}} \right] (42) = 6
\]

\[
\therefore Z_{12} = \Omega
\]
Exercise 17.5-1 (Cont’d)

Now let’s determine the Y parameters using the 1 A current injection method.

\[ I_1 = 1 \text{ A} \]

Current divider to determine current through 42 ohm times 42 ohms yields \( V_1 \). Thus \( Y_{11} = \frac{I_T}{V_1} = \frac{1}{V_1} \)

\[
\begin{align*}
V_1 &= (1) \left[ \frac{1}{\frac{42}{42}} - \frac{1}{\frac{42}{21}} \right] (42) = 14, \quad \therefore \quad Y_{11} = \frac{1}{14} = \frac{1}{14}
\end{align*}
\]

Exercise 17.5-1 (Cont’d)

Current divider to determine \( I_2 \). Then \( Y_{21} = \frac{I_2}{V_1} \)

\[
I_2 = (-1) \left[ \frac{1}{\frac{21}{42}} + \frac{1}{\frac{1}{21}} \right] = \frac{-42}{63}
\]

\[
Y_{21} = \frac{-42}{63} \cdot \frac{14}{(14)(63)} = \frac{-42}{(14)(63)} = \frac{-1}{14(63)}
\]
Exercise 17.5-1 (Cont’d)

Current divider to determine current through 10.5 ohm times
10.5 ohm resistor yields $V_2$. Thus $Y_{22} = I_T / V_2 = 1 / V_2$

\[
V_2 = (10.5) = 7, \quad \therefore Y_{22} = \frac{1}{V_2} = \frac{1}{7}
\]

Current divider to determine current through $I_2$. Then $Y_{12} = I_1 / V_2$

\[
I_2 = (-1) \left[ \frac{1}{10.5} + \frac{1}{21} \right] = -\frac{1}{3}
\]

\[
\therefore Y_{12} = \left( \frac{-1}{3} \right) \left( \frac{1}{7} \right) = \frac{-1}{21}
\]
17.6 Z and Y Parameters for Circuits with Dependent Sources

Again, I will use the current injection method to illustrate an alternative. Please refer to the author’s example for their methods described in Tables 17.5-3 and 17.5-4.

We will do example 17.6-1 (pg 772) but determine both Z (Example 17.6-1) and Y (Exercise 17.6-1, pg 773)

Where \( m = \frac{2}{3} \)
Inject 1 A at input with output open.

\[ I_1 = I_T \]

KCL involving node associated with \( V_x \)

\[ \frac{2}{3} V_2 + I = 1 \quad \text{also} \quad I = \frac{V_2}{3} \]

\[ \frac{2}{3} V_2 + \frac{V_2}{3} = 1 \quad \text{or} \quad V_2 = Z_{21} = 1 = \frac{V_2}{3} = \frac{1}{3} \]

Now, \( V_x = I(2 + 3) = \left( \frac{1}{3} \right) (5) = \frac{5}{3} \)

\( \therefore V_1 = (1)(4) + \frac{5}{3} = Z_{11} = \frac{17}{3} \Omega \)

Inject 1 A at output terminal with input open.
KCL at “+” terminal of $V_2$.

$$\frac{V_2}{3} + \frac{2}{3} V_2 = I_T = 1 \text{ or } V_2 = Z_{22} = \Omega$$

Then current

$$\frac{2}{3} V_2 = \frac{2}{3} A, \text{ and } V_1 = V_x = V_2 - (2)(2/3) = 1 - 4/3 = -1/3$$

$$\therefore Z_{12} = \Omega$$

$$Z = \begin{bmatrix} \end{bmatrix}$$

Exercises in Section 17-6
Exercise 17.6-1 (pg 773)

Find Y parameters

\[ I_1 = 1 \text{ A} \]

Since \( V_2 = 0 \) the Dependent Current source is 0 A

\[ I_i = 1 \text{ A} \]

Exercise 17.6-1 (cont’d)

\[ V_i = (4 + 2)(1) = 6 \]

\[ \therefore Y_{11} = \frac{I_1}{V_i} = \]

And -\( I_2 = 1 \text{ A} \) or \( I_2 = -1 \text{ A} \)

\[ \therefore Y_{21} = \frac{I_2}{V_i} = \]
Exercise 17.6-1 (cont’d)

Inject 1 A at output terminal with input shorted. (i.e., $V_1=0$

\[ V_x + \frac{2}{3} V_2 + \frac{V_x - V_2}{2} = 0 \]

\[ \frac{V_2}{3} + \frac{V_2 - V_x}{2} = 1 \]

\[ \frac{3}{4} V_x + \frac{1}{6} V_2 = 0 \]

\[ -\frac{1}{2} V_x + \frac{5}{6} V_2 = 1 \]
Solving for $V_X$ and $V_2$:

$$V_2 = \frac{18}{17} \text{ but } \frac{1}{V_2} = \frac{17}{18}$$

$$V_X = -\frac{4}{17}$$

$$-I_1 = \frac{V_X}{4} = \left(-\frac{4}{17}\right)\left(\frac{1}{4}\right) = -\frac{1}{17}$$

$$\therefore I_1 = \frac{1}{17} \text{ and } Y_{12} = \frac{I_1}{V_2} = \frac{1}{17}$$

Work Exercise 17.7-1
For Z & Y parameters
(pag 776)
17.7 Hybrid and Transmission Parameters

Hybrids are used principally in transistor circuits

**Hybrid h**

\[
V_1 = h_{11}I_1 + h_{12}V_2 \\
I_2 = h_{21}I_1 + h_{22}V_2
\]

**Hybrid g**

\[
I_1 = g_{11}V_1 + g_{12}I_2 \\
V_2 = g_{21}V_1 + g_{22}I_2
\]
Transmission Parameters are used to describe cable, fiber, and transmission lines

\[ V_1 = A V_2 - B I_2 \]
\[ I_1 = C V_2 - D I_2 \]

or in matrix form, as

\[
\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = T \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}
\]

where

\[
A = \left. \frac{V_1}{V_2} \right|_{V_2=0} \quad C = \left. \frac{I_2}{V_2} \right|_{V_2=0}
\]
\[
B = \left. \frac{V_1}{-I_2} \right|_{V_2=0} \quad D = \left. \frac{I_2}{-I_2} \right|_{V_2=0}
\]

Note: The reason \( I_2 \) is negative is because the Transmission model requires that \( I_2 \) be in the opposite direction of the two-port model.
Inverse Transmission Parameters

\[ V_2 = AV_1 - B I_1 \]
\[ I_2 = CV_1 - D I_1 \]

or in matrix form, as

\[
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
= \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_1 \\
-I_1
\end{bmatrix}
= T
\begin{bmatrix}
V_1 \\
-I_1
\end{bmatrix}
\]

\[
A' = \frac{V_2}{V_1} \bigg|_{I_1=0} \\
C' = \frac{I_2}{V_1} \bigg|_{I_1=0} \\
B' = \frac{V_2}{-I_1} \bigg|_{I_1=0} \\
D' = \frac{I_2}{-I_1} \bigg|_{I_1=0}
\]

Note: The reason \( I_1 \) is negative is because the Transmission model requires that \( I_1 \) be in the opposite direction of the two-port model.

Exercises in Section 17-7
Exercise 17.7-1 (pg 776)

Find the hybrid h parameter model of the circuit shown.

To compute \( h_{11} \) and \( h_{21} \), we inject a 1 A current at the input and short the output (\( V_2 = 0 \)).

Exercise 17.7-1 (cont’d)

KCL at the input:

\[
\frac{V_1}{1} + \frac{V_1}{9} = 1 \quad \text{or} \quad V_1 = \frac{9}{10} = 0.9
\]

\[
h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 
\]

\[
I_1 = I_T
\]

\[
V_1
\]

\[
+ 
\]

\[
- 
\]

\[
9 \Omega
\]

\[
1 \Omega
\]

\[
5i
\]

\[
i
\]

\[
V_2
\]

\[
+ 
\]

\[
- 
\]
Exercise 17.7-1 (cont’d)

The current $I = \frac{V_1}{1} = 0.9$

$\therefore 1.0 - I = 5 I - I_2$

$I_2 = 6I - 1.0 = (6)(0.9) - 1.0 = 4.4$

$\therefore h_{21} = \frac{I_2}{I_1} \bigg|_{V_2=0}$

Exercise 17.7-1 (cont’d)

Now, we inject 1 A at output and open the input.
Exercise 17.7-1 (cont’d)

KCL at output: \( I + 5I = 1 \) or \( I = \frac{1}{6} \) A

\[ \therefore V_2 = (1 + 9)I = \frac{10}{6} \]

\[ V_1 = (1)I = \frac{1}{6} \]

\[ h_{12} = \frac{V_1}{V_2} \bigg|_{I_2=0} = \frac{1/6}{10/6} = \]

\[ h_{22} = \frac{I_2}{V_1} \bigg|_{I_2=0} = \frac{1}{10/6} = \]

\[ H = \begin{bmatrix} \end{bmatrix} \]

Exercise 17.7-1 (cont’d)

Let’s continue the Exercise by computing the “g” parameters.
We find \( g_{11} \) and \( g_{21} \) by opening \( I_2 = 0 \)

\[ g_{11} = \frac{I_1}{V_1} \bigg|_{I_2=0} \quad g_{21} = \frac{V_2}{V_1} \bigg|_{I_2=0} \]

We inject 1 A current for \( I_1 \) and determine \( V_1 \) and determine \( V_2 \).
Exercise 17.7-1 (cont’d)

KCL: \( I_1 + 5I = I_1 = 1 \)
\[ I = 1/6 \text{A} \]
\[ \therefore V_1 = (1)I = 1/6 \]

\[ g_{11} = \frac{1}{V_1} = \quad \]

KVL: \((9)(5I) + V_2 = V_1\)
\[ V_2 = V_1 - 45I = 1/6 - 45/6 = -44/6 \]

\[ g_{21} = \frac{V_2}{V_1} = \frac{-44/6}{1/6} = \quad \]

Exercise 17.7-1 (cont’d)

We now inject 1 A current at the output terminal and short circuit the input terminals (i.e., \( V_1 = 0 \))

\[ I_1 \]
\[ V_1 = 0 \]
\[ 9 \Omega \]
\[ 1 \Omega \]
\[ 5i \]
\[ I_2 = I_T = 1 \text{A} \]

\[ I = 0 \text{ due to short circuit, thus “} 5I \text{” dependent current source is zero.} \]
Exercise 17.7-1 (cont’d)

\[ I_1 = -I_2 = -1A \]
\[ V_2 = (9)(I_2) = 9 \]
\[ g_{12} = \frac{I_1}{I_2} \bigg|_{V_1=0} = \quad = \]
\[ g_{22} = \frac{V_2}{I_2} \bigg|_{V_1=0} = \quad = \]
\[ \therefore g = \begin{bmatrix} \end{bmatrix} \]

Let’s continue our Exercise by computing the T and T’ parameters

\[ A = \frac{V_1}{I_2} \bigg|_{I_1=0} \quad B = \frac{V_1}{I_2} \bigg|_{V_1=0} \]
\[ C = \frac{I_1}{V_2} \bigg|_{I_1=0} \quad D = \frac{I_2}{V_2} \bigg|_{V_1=0} \]

We inject 1.0 A current at the input terminal open circuit the output
Exercise 17.7-1 (cont’d)

From previous slide where we calculated “g” parameters:

\[ V_1 = \frac{1}{6}, \; V_2 = -\frac{44}{6} \]

\[ A = \frac{V_1}{V_2} = \frac{\frac{1}{6}}{-\frac{44}{6}} = \text{Open Circuit Voltage Ratio} \]

\[ C = \frac{I_1}{V_2} = -\frac{\frac{6}{44}}{} = \text{Open Circuit transfer admittance} \]

Exercise 17.7-1 (cont’d)

The B and D parameters are determined by leaving the 1.0 A current injected at the input terminals and shorting the output terminals.

From previous slides:

\[ V_1 = 0.9, \; I_2 = 4.4 \]

\[ B = -\frac{V_1}{I_2} = -\frac{0.9}{4.4} = \text{Short circuit terms for impedance} \]

\[ D = -\frac{I_1}{I_2} = \frac{1}{4.4} = \text{Short circuit current ratio} \]
Exercise 17.7-1 (cont’d)

For the inverse transmission, T’ parameters, inject 1 A current at the output terminal

\[ A' = \left. \frac{V_2}{V_1} \right|_{I_i = 0} \quad B' = -\left. \frac{V_2}{I_1} \right|_{V_i = 0} \]

\[ C' = \left. \frac{I_2}{V_1} \right|_{I_i = 0} \quad D' = -\left. \frac{I_2}{I_1} \right|_{V_i = 0} \]

see previous slide

\[ V_1 = \frac{1}{6}, \quad V_2 = \frac{10}{6} \]

Exercise 17.7-1 (cont’d)

\[ A' = \frac{V_2}{V_1} = \frac{10/6}{1/6} = \begin{cases} 10/6 = 1/6 \quad \text{open circuit inverse voltage ratio} \\ \end{cases} \]

\[ C' = \frac{I_2}{V_1} = \frac{1}{1/6} = \begin{cases} 1/6 \quad \text{open circuit admittance transfer} \\ \end{cases} \]

We now short circuit the input terminal \( V_1 = 0 \) see previous slide

\[ I_i = -1 \, A, \quad V_2 = 9 \]
Exercise 17.7-1 (cont’d)

\[ B' = -\frac{V_2}{I_1} = -\frac{9}{-1} = \] Short circuit impedance transfer

\[ D' = -\frac{I_2}{I_1} = \frac{-1}{-1} = \] Short circuit inverse current ratio
### Table 17.8-1 Parameter Relationships between Two-Port Networks

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>Z₁₁</td>
<td>Z₁₂</td>
<td>Z₂₁</td>
<td>Z₂₂</td>
<td>ΔZ</td>
<td>ΔY</td>
<td>ΔY</td>
</tr>
<tr>
<td>Y</td>
<td>Y₁₁</td>
<td>Y₁₂</td>
<td>Y₂₁</td>
<td>Y₂₂</td>
<td>( \frac{h₁₁}{h₁₁} )</td>
<td>( \frac{h₁₂}{h₂₁} )</td>
<td>( \frac{\Delta h}{\Delta h} )</td>
</tr>
<tr>
<td>H</td>
<td>H₁₁</td>
<td>H₁₂</td>
<td>H₂₁</td>
<td>H₂₂</td>
<td>( \frac{\Delta h}{\Delta h} )</td>
<td>( \frac{\Delta h}{\Delta h} )</td>
<td>( \frac{\Delta h}{\Delta h} )</td>
</tr>
<tr>
<td>G</td>
<td>G₁₁</td>
<td>G₁₂</td>
<td>G₂₁</td>
<td>G₂₂</td>
<td>( \frac{\Delta h}{\Delta h} )</td>
<td>( \frac{\Delta h}{\Delta h} )</td>
<td>( \frac{\Delta h}{\Delta h} )</td>
</tr>
</tbody>
</table>

\[ \Delta Z = Z₁₁Z₂₂ - Z₁₂Z₂₁, \Delta Y = Y₁₁Y₂₂ - Y₁₂Y₂₁, \Delta h = h₁₁h₂₂ - h₁₂h₂₁, \Delta T = AD - BC. \]

---

1 | EE 2446 CIRCUIT ANALYSIS II

2 | Introduction to Electrical Circuits

6th edition

Dorf and Svoboda

Chapter 17, Section 8
Relationships between Two-Port Parameters

Provided parameters exist for \( Z, Y, h, g, T \) and \( T' \) they can be related to one another (see table 17.8-1, pg 777)

\[
\begin{align*}
\text{Z\&Y} & \\
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} &= Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{ and} \\
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} &= Z^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \\
\therefore Y &= Z^{-1} \text{ and } Z = Y^{-1}
\end{align*}
\]

\[
\begin{align*}
\text{Y \& H} & \\
 h_{11} &= \frac{V_1}{I_1} \bigg|_{V_2=0} \\
 h_{21} &= \frac{I_2}{I_1} \bigg|_{V_2=0} \\
 h_{12} &= \frac{V_1}{V_2} \bigg|_{I_1=0} \\
 h_{22} &= \frac{I_2}{V_2} \bigg|_{I_1=0}
\end{align*}
\]
Relationships between Two-Port Parameters

**Y & H cont’d**

\[ h_{11} = \frac{1}{\frac{I_1}{V_1}} \bigg|_{V_2 = 0} = \frac{1}{Y_{11}} \]

\[ h_{21} = \frac{I_2}{I_1} \bigg|_{V_2 = 0} = \frac{I_2}{V_1} \bigg|_{I_1 = 0} = \frac{Y_{21}}{Y_{11}} \]

\[ h_{22} = \frac{I_2}{V_2} \bigg|_{I_2 = 0} = \frac{1}{V_2} \bigg|_{I_2 = 0} = \frac{1}{Z_{22}} \]

But \[ Z_{22} = \frac{Y_{11}}{\Delta Y} \]

where \[ \Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} \]

\[ h_{22} = \frac{\Delta Y}{Y_{11}} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}} \]
Relationships between Two-Port Parameters

**Y & H cont'd**

\[
 h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \frac{Z_{12}}{Z_{22}} = -\frac{Y_{12}}{Y_{11}/\Delta Y}
\]

\[
 \therefore h_{12} = -\frac{Y_{12}}{Y_{11}}
\]

---

**Z**

\[
 Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}
\]

\[
 Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}
\]

\[
 Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}
\]

\[
 Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}
\]

**Y**

\[
 Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}
\]

\[
 Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}
\]

\[
 Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}
\]

\[
 Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}
\]

\[
 Z = Y^{-1} \text{ or } Y = Z^{-1}
\]
\[ h_{11} = \frac{V_1}{I_1} | V_2 = 0 \]
\[ h_{21} = \frac{I_2}{I_1} | V_2 = 0 \]
\[ h_{12} = \frac{V_1}{V_2} | I_1 = 0 \]
\[ h_{22} = \frac{I_2}{V_2} | I_1 = 0 \]

\[ g_{11} = \frac{I_1}{V_1} | I_2 = 0 \]
\[ g_{21} = \frac{V_2}{V_1} | I_2 = 0 \]
\[ g_{12} = \frac{I_1}{I_2} | V_1 = 0 \]
\[ g_{22} = \frac{V_2}{I_2} | V_1 = 0 \]

\[ h = g^{-1} \text{ or } g = h^{-1} \]

\[ T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \]
\[ A = \frac{V_1}{V_2} | I_2 = 0 \]
\[ C = \frac{I_1}{V_2} | I_2 = 0 \]
\[ B = \frac{V_1}{-I_2} | V_2 = 0 \]
\[ D = \frac{I_1}{-I_2} | V_2 = 0 \]

\[ T' = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \]
\[ A' = \frac{V_2}{V_1} | I_1 = 0 \]
\[ C' = \frac{I_2}{V_1} | I_1 = 0 \]
\[ B' = \frac{V_2}{-I_1} | V_1 = 0 \]
\[ D' = \frac{I_2}{-I_1} | V_1 = 0 \]

\[ T = T^{-1} \text{ or } T' = T'^{-1} \]
Exercises in Section 17-8

Exercise 17.8-1 (pg 778)

Determine $Z$ when $Y = \begin{bmatrix} 2/15 & -1/5 \\ -1/10 & 2/5 \end{bmatrix}$

\[
Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{2/5}{(2/15)(2/5)-(1/10)(1/5)} = \frac{2/5}{1/30} = \Omega
\]

\[
Z_{12} = -\frac{Y_{12}}{\Delta Y} = -\frac{-(-1/5)}{1/30} = \Omega
\]

\[
Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{-(-1/10)}{1/30} = \Omega
\]

\[
Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{2/15}{1/30} = \Omega
\]
Exercise 17.8-2 (pg 778)

Determine the T-parameters from the Y-parameters of exercise 17.8-1.

\[ A = \frac{V_1}{V_2}|_{I_2=0} = \frac{V_1/I_1}{V_2/I_1}|_{I_2=0} = \frac{Z_{11}}{Z_{21}} \]

But \( Z_{11} = \frac{Y_{22}}{\Delta Y} \) and \( Z_{21} = -\frac{Y_{21}}{\Delta Y} \)

\[ \therefore A = \frac{Y_{22}}{\Delta Y} = -\frac{Y_{21}}{\Delta Y} = \frac{2}{5} \]

But \( A = \frac{2}{5} = -\frac{1}{10} \)

Exercise 17.8-2 (cont’d)

\[ C = \frac{I_1}{V_2}|_{I_2=0} = \frac{1}{V_2/I_2}|_{I_2=0} = \frac{1}{Z_{21}} \]

But \( Z_{21} = -\frac{Y_{21}}{\Delta Y} \)

\[ \therefore C = -\frac{\Delta Y}{Y_{21}} = 1/30 = -1/10 \]

\[ B = \frac{V_1}{-I_2}|_{I_2=0} = -\frac{1}{I_2/V_2}|_{I_2=0} = -\frac{1}{Y_{21}} = -\frac{1}{-1/10} \]

\[ D = \frac{I_1}{-I_2}|_{I_2=0} = -\frac{I_1/V_1}{I_2/V_2}|_{I_2=0} = -\frac{Y_{11}}{Y_{21}} = -\frac{2/15}{-1/10} \]
17.9 Interconnection of two-port Networks

A.1: Parallel Connection (use Y-parameters)
A.2: Series Connection (use Z-parameters)
B: Cascade Connection (use T-parameters)

Parallel Case (Y).
\[ I_1 = I_{1a} + I_{1b} \quad I_2 = I_{2a} + I_{2b} \]
\[ V_1 = V_{1a} = V_{1b} \quad V_2 = V_{2a} = V_{2b} \]
\[ \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = Y_1 \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = Y_1 \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \]
\[ \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = Y_2 \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = Y_2 \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \]
\[ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = (Y_1 + Y_2) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \]
\[ \therefore \ Y = Y_1 + Y_2 \]

**Series Case (Z)**
\[
\begin{align*}
I_1 &= I_{1a} = I_{1b} \quad I_2 = I_{2a} = I_{2b} \\
V_1 &= V_{1a} + V_{1b} \quad V_2 = V_{2a} + V_{2b} \\
\left[ \begin{array}{c} V_{1a} \\ V_{2a} \end{array} \right] &= Z_1 \left[ \begin{array}{c} I_{1a} \\ I_{2a} \end{array} \right] = Z_1 \left[ \begin{array}{c} I_1 \\ I_2 \end{array} \right] \\
\left[ \begin{array}{c} V_{1b} \\ V_{2b} \end{array} \right] &= Z_2 \left[ \begin{array}{c} I_{1b} \\ I_{2b} \end{array} \right] = Z_2 \left[ \begin{array}{c} I_1 \\ I_2 \end{array} \right] \\
\left[ \begin{array}{c} V_1 \\ V_2 \end{array} \right] &= \left[ \begin{array}{c} V_{1a} \\ V_{2a} \end{array} \right] + \left[ \begin{array}{c} V_{1b} \\ V_{2b} \end{array} \right] = (Z_1 + Z_2) \left[ \begin{array}{c} I_1 \\ I_2 \end{array} \right] \\
\therefore Z &= Z_1 + Z_2
\end{align*}
\]

Cascade Case (T)
Note that:

\[
\begin{bmatrix}
V_{1a} \\
I_{1a}
\end{bmatrix} = T_1 \begin{bmatrix}
V_{2a} \\
-I_{2a}
\end{bmatrix}, \quad \begin{bmatrix}
V_{2a} \\
-I_{2a}
\end{bmatrix} = \begin{bmatrix}
V_{1b} \\
I_{1b}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
V_{1a} \\
I_{1a}
\end{bmatrix}, \quad \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix} = \begin{bmatrix}
V_{2b} \\
-I_{2b}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = T_1 \begin{bmatrix}
V_{2a} \\
-I_{2a}
\end{bmatrix} = T_1 \begin{bmatrix}
V_{1b} \\
I_{1b}
\end{bmatrix} = T_1 T_2 \begin{bmatrix}
V_{2b} \\
-I_{2b}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = T_1 T_2 \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix} = T \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix}
\]

Where \( T = T_1 T_2 \)

---

**Exercises in Section 17-9**
Exercise 17.9-1 (pg 781)

Determine the total transmission parameters of the cascade connection of the three-port networks showing in Figure E17.9-1.

Exercise 17.9-1 (cont’d)

\[ A_a = \frac{V_{1a}}{V_{2a}} \bigg|_{I_{2a}=0} \]

(note that \( I_{1a} \) and \( I_{2a} \) are zero)

\[ C_a = \frac{I_{1a}}{V_{2a}} \bigg|_{I_{2a}=0} \]
Exercise 17.9-1 (cont’d)

\[ B_a = \frac{V_{1a}}{-I_{2a}} \bigg|_{V_{2a}=0} = \Omega \]

\[ D_a = \frac{I_{1a}}{-I_{2a}} \bigg|_{V_{2a}=0} = \]

\[ \therefore T_a = \begin{bmatrix} \end{bmatrix} \]

Exercise 17.9-1 (cont’d)

\[ A_b = \frac{V_{1b}}{V_{2b}} \bigg|_{I_{2b}=0} = \]

\[ C_b = \frac{I_{1b}}{V_{2b}} \bigg|_{I_{2b}=0} = \]

\[ N_b \]
Exercise 17.9-1 (cont’d)

\[ B_b = \frac{V_{1b}}{-I_{2b}} \bigg|_{V_{2b}=0} = \]

\[ D_{2b} = \frac{I_{1b}}{-I_{2b}} \bigg|_{V_{2b}=0} = \]

\[ T_b = \begin{bmatrix} \end{bmatrix} \]

Exercise 17.9-1 (cont’d)

\[ N_c \]

\[ A_c = \frac{V_{1c}}{V_{2c}} \bigg|_{I_{2c}=0} = B_c = \frac{V_{1c}}{-I_{2c}} \bigg|_{I_{2c}=0} = \Omega \]

\[ C_c = \frac{I_{1c}}{V_{2c}} \bigg|_{I_{2c}=0} = D_c = \frac{I_{1c}}{-I_{2c}} \bigg|_{I_{2c}=0} = \therefore T_c = \begin{bmatrix} \end{bmatrix} \]
Exercise 17.9-1 (cont’d)

\[
T = T_a T_b T_c = \begin{bmatrix}
1 & 12 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
1/6 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 3 \\
0 & 1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
3 & 12 \\
1/6 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 3 \\
0 & 1
\end{bmatrix}
= \begin{bmatrix}
3 & 9+12 \\
1/6 & 1/2+1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
3 & 21 \\
1/6 & 3/2
\end{bmatrix}
\]

*Or* \(A = 3, \ B = 21, \ C = 1/6, \ D = 3/2\)