We'll begin today with a wrapup of the material on antennas. Then we'll continue with the next topic of amplifiers.

In the wrapup on antennas we'll consider some highlights of a few commonly used types of antennas.

**Dipoles**

Arrays of dipoles are commonly used for base
station antennas, so it's important to consider beam characteristics and input impedances for individual dipoles.

Figure 4.9 Radiation resistance, input resistance and directivity of a thin dipole with sinusoidal current distribution.

From Chpt. 4 of Antenna Theory - Analysis and Design, C. Balanis (J. Wiley, 1997)
Figure 4.6 - Elevation plane amplitude patterns for a thin dipole with sinusoidal current distribution ($l = \lambda/4, \lambda/2, 3\lambda/4, \lambda$).

(Also from Chpt. 4 of Balanis, same reference). For $l > \lambda$ multiple lobes form.
Note that these patterns are for far field radiation. Far field radiation is defined to occur when the maximum optical path length to the point of observation (usually at broadside to the antenna) differs from a parallel ray approximation by some prescribed value, typically $\frac{\pi}{8}$. 
This leads to an expression of \( R \approx \frac{2D^2}{\lambda} \), where \( \lambda \) is the operating wavelength and \( D \) is the maximum aperture dimension of the antenna. Most radar and telecommunications problems are far-
field in nature.

For antennas with multiple (usually identical) elements in an array the pattern multiplication principle is often valid. It is strictly valid where array elements radiate independently and where the maximum aperture dimension of the array satisfies the far-field criterion.
The multiplication principle states

$$E_{\text{total}} = E_{\text{single element at origin}} \times \text{array factor}$$

Example:

Broadside array with phase shifts to steer beam.

$$(\text{uniform spacing})$$

$$(\text{uniform spacing})$$
The array factor for this array takes the form:

\[ AF = a_0 + a_1 e^{j(kF' \hat{a}_r + \beta)} + a_2 e^{j2(kF' \hat{a}_r + \beta)} + a_3 e^{-j2(kF' \hat{a}_r + \beta)} \]

The progressive phase shift \( \beta \) (between adjacent elements) is adjusted to steer the beam and \( a_0, a_1, a_2 \) are used to shape the beam (primarily...
to control sidelobes. For a uniformly excited array $a_0 = a_1 = a_2 = 1$. For the array shown:

$$
\hat{a}_r = \delta \sin \Theta \cos \phi.
$$

With these simplifications we can write

$$
|AF| = \frac{\sin \left( \frac{5}{2} (k d \sin \Theta \cos \phi + \beta) \right)}{\sin \left( \frac{1}{2} (k d \sin \Theta \cos \phi + \beta) \right)}
$$

The element pattern for the $z$-directed dipole
The total pattern is then
\[ \bar{E} (\text{total}) = \hat{e}_\theta \ E_\theta \times |AF| \]

The intensity pattern is the quantity most often plotted. This is
\[ U = r^2 |\bar{W}av| \]
\[ \overline{\mathbf{w}}_{\text{av}} = \hat{\mathbf{a}}_r \mathbf{w}_{\text{av}} = \hat{\mathbf{a}}_r \frac{1}{2\pi} |E_{\text{total}}|^2 \]

**Monopole**

A dipole (that is center-fed) can be converted to a monopole as shown:

![Diagram showing conversion of a balanced dipole to an infinite ground plane with an unbalanced feed.]

The radiation pattern above the ground plane is identical to that of
the dipole (for the same region). The major difference is in terms of the input impedance. We have

\[ Z_{\text{in}} = \frac{1}{2} Z_{\text{in}} \]

For handheld instruments a monopole that is less than \( \lambda/4 \) in length is often used. The antenna in this case has low radiation resistance and
The radiation resistance of a short vertical whip over a perfect conducting ground plane is given by:

\[ R_r = 40\pi^2 \left( \frac{h}{\lambda} \right)^2 \]

where \( h \) is the antenna height and \( \lambda \) is the wavelength. The capacitance is given by

\[ C_a = \frac{24.2h}{\log(2h/a) - 0.7353} \]
where \( a \) is the whip diameter in m (\( h \) is also in m) and \( C_a \) is the capacitance in pF. Of course the values for \( R_r \) and \( C_a \) are modified by the size and material of the ground plane. Additional capacitance may also exist at the feedpoint depending on how the
antenna is constructed. 15

Loop antennas

These are infrequently used in communication and radar systems but are commonly used for direction finding. The text provides some information about these but they are not considered here.

Microstrip Patch Antennas

A variety of different
types have been designed. We'll limit the discussion to one of the most common ones, i.e., the simple rectangular patch.

![Diagram of a rectangular patch antenna with dimensions L, W, and Z.]

Feed line

(\text{The feed may also be through the ground plane using a coaxial probe.})
For the principal -E and principal -H planes, the field patterns are:

**Principal E-plane**

\[ \theta = 90^\circ, \quad 0^\circ \leq \phi \leq 90^\circ, \quad 2\pi \leq \phi \leq 360^\circ \]

\[ E\phi = -j \frac{2k\omega W_0 e^{-jk\pi r}}{\pi r} \times \left\{ \frac{\sin \left( \frac{k\alpha h}{2} \cos \phi \right)}{\frac{k\alpha h}{2} \cos \phi} \right\} \cos \left( \frac{k\alpha Le}{2} \sin \phi \right) \]

where \( Le = L + 2\Delta L \)

Due to fringing fields
\[ \frac{\Delta L}{h} = 0.412 \frac{(\varepsilon_{\text{reff}} + 0.3)(\frac{W}{h} + 0.264)}{(\varepsilon_{\text{reff}} - 0.258)(\frac{W}{h} + 0.8)} \]

\( h \) is the thickness of the circuit board substrate and \( \varepsilon_{\text{reff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[ 1 + 12 \frac{b}{w} \right]^{-\frac{1}{2}} \)

where \( \varepsilon_r \) is the relative permittivity of the circuit board substrate.
The principal H-plane pattern is found as
($\varphi = 0^\circ, 0^\circ \leq \theta \leq 180^\circ$)

$$E_\theta = -j k_0 W V_0 e^{-jk_0 r} \frac{\sin \theta}{\pi r}$$

$$X \left[ \frac{\sin \left( \frac{k_0 h}{2} \sin \theta \right)}{\frac{k_0 h}{2} \sin \theta} \frac{\sin \left( \frac{k_0 W \cos \theta}{2} \right)}{\frac{k_0 W}{2} \sin \theta} \right]$$

The input impedance is determined by inserting the feed as shown:

```
  L
  ↑
W0  ↓  W
  ↑
  Y0
```
\[ R_{in} \approx \frac{1}{2(G_1 + G_{12})} \cos^2 \left( \frac{\pi}{L} y_0 \right) \]

where

\[ G_1 = \frac{I_1}{120 \pi^2} \]

\[ I_1 = \int_0^{\pi} \left[ \sin \left( \frac{K_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta d\theta \]

\[ G_{12} = \frac{1}{120 \pi^2} \int_0^{\pi} \left[ \sin \left( \frac{K_0 W \cos \theta}{2} \right) \right]^2 \times J_0 (K_0 L \sin \theta) \sin^3 \theta d\theta \]

Select \( \omega_0 \) to achieve characteristic impedance
that matches to $R_{in}$. The characteristic impedance is given as:

$$Z_c = \begin{cases} \frac{w_0}{\sqrt{\varepsilon_{\text{eff}}}} \ln \left( \frac{8h}{w_0} + \frac{w_0}{4h} \right), & \frac{w_0}{h} \leq 1 \\ \frac{w_0}{h} + 1.393 \varepsilon_{\text{eff}} & \frac{w_0}{h} > 1 \end{cases}$$

$$+ 0.667 \ln \left( \frac{w_0}{h} + 1.444 \right) \left( \frac{w_0}{\lambda_0} \right)$$

For $\frac{w_0}{h} > 1$ the directivity, $D_0$, is about 8.2 dB for $W \ll \lambda_0$, and about $8 \left( \frac{W}{\lambda_0} \right)$ for $W \gg \lambda_0$. Trade-offs between efficiency
and bandwidth are shown in the following figure from Balanis (Chpt. 14, same reference as before).


Reflector antennas

These are commonly used for spacecraft,
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i.e., satellite communications, as well as for many radar applications. We won't discuss these antennas in detail but the following rules of thumb can be used for initial designs. (These expressions are also useful for general cases where the antenna is large in both dimensions relative to λ.).
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For beamwidth in a particular plane,

$$\theta_{\text{HPBW}} = \frac{b \lambda}{d} \quad \text{where}$$

$d$ is the dimension of the antenna in that plane and $b$ is a constant that varies from about 0.88 to 2 depending on the amplitude weighting across the aperture. 1.4 may be considered a typical value.
\[ D_0 = \frac{4\pi}{(\Theta_{HPRW1})_\text{rad} \cdot (\Theta_{HPRW2})_\text{rad}} \]

where \( \Theta_{HPRW1} \) and \( \Theta_{HPRW2} \) are the half-power beamwidths in two orthogonal planes. 0.8 may be considered a typical efficiency. Much more could be said about antennas but we need to move on.
Amplifiers

Let's consider first rf amplifiers since these are the first in the receiver line up. Then we'll consider if amplifiers. (Power amplifiers will be considered separately at a later time). At frequencies near and within the GHz range, a single
transistor amplifier stage is frequently used for rf amplification. A gain of 10–20 dB is normally adequate for this stage since its purpose is largely to establish the noise figure of the receiver. The $IP_3$ should be high enough to avoid interferences which are passed by the preselect filter, but the $IP_3$ requirement is not
as high as for if amplifying stages. A number of different types of devices are used, including SiBJTs (useful up to nearly 2 GHz), GaAs FETs (useful to above 18 GHz), HEMTs (useful to SiGe above 18 GHz), and SiHBTs (useful to above 10 GHz). Using modern technology, GaAs FETs can be made smaller than other
technologies. Also above 1 GHz GaAs FETs have better noise and IM distortion than bipolar transistors. However, GaAs FETs and HBTs are often viewed as low-yield, high-power and high cost options. Nonetheless the technologies are frequently and continuously changing. Research on SiGe devices
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appears to be very active at the current time. The current status of HEMTs is not clear.

Basic design considerations

The basic idea is to achieve the desired gain by designing
appropriate matching circuits. For optimum gain we design the matching circuit so that $\Gamma_s = \Gamma_{in}^*$ and $\Gamma_L = \Gamma_{out}^*$. $\Gamma_{in}$ and $\Gamma_{out}$ are determined in terms of the s-parameters for the device (usually specified for the device at particular bias levels and particular frequencies). Note that designing at the
optimum gain point is not always the best. Stability of the circuit and bandwidth need to be considered in evaluating the overall result. For narrow-bandwidth amplifiers, optimization for maximum gain is usually easier, but stability of the circuit has to be evaluated. For example, is the circuit unconditionally stable or is it unstable for some
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Generator and/or load 33 impedances? A stabilizing element can be added if necessary.

For narrow-bandwidth designs optimization for maximum gain or minimum noise figure involves the use of 'matching' circles on a Smith Chart. These circles provide the following information:
For a unilateral device, i.e., $S_{12} \approx 0$, we have

$$G_T = \frac{1 - |\Gamma_3|^2}{1 - S_{11} \Gamma_3^*} \frac{|S_{21}|^2}{|1 - S_{22} \Gamma_2|^2}$$

Transducer gain

For noise matching we have

$$F = F_{\text{min}} + \frac{R_N}{G_S} |Y_S - Y_{\text{opt}}|^2$$

where $Y_S = G_S + jB_S$ source admittance

$Y_{\text{opt}}$ = optimum source admittance that results in minimum noise figure.
$F_{\text{min}} =$ minimum noise figure of transistor, attained when $Y_s = Y_{\text{opt}}$

$R_N =$ equivalent noise resistance of transistor

$G_s =$ real part of source admittance

Notice that

$Y_s = \frac{1}{Z_0} \frac{1 - \Gamma_s}{1 + \Gamma_s}$

$Y_{\text{opt}} = \frac{1}{Z_0} \frac{1 - \Gamma_{\text{opt}}}{1 + \Gamma_{\text{opt}}}$