Partial-Fraction Expansion

A.1 INTRODUCTION
Partial Fraction Expansion and Continuous-Time Signals

Application of the method of partial fraction expansions and continuous-time LT. The impulse response h[n] of a linear time-invariant (LTI) system can be expressed as a convolution of the input signal x[n] and the impulse response h[n]. For the discrete-time system, the transfer function H(z) is defined as the ratio of the output Y(z) to the input X(z).

\[ H(z) = \frac{Y(z)}{X(z)} \]

The convolution in the time domain is equivalent to the multiplication in the frequency domain:

\[ x[n] * h[n] = \mathcal{F}^{-1} \left\{ X(z)H(z) \right\} \]

The output of the system is given by the convolution of the input and the impulse response:

\[ y[n] = x[n] * h[n] \]

where y[n] is the output signal.

In the continuous-time domain, the convolution theorem states that:

\[ x(t) * h(t) = \mathcal{F}^{-1} \left\{ X(j\omega)H(j\omega) \right\} \]

and the corresponding output is:

\[ y(t) = x(t) * h(t) \]

where y(t) is the continuous-time output signal.
\[
\frac{(x + a)(1 + a)}{x + a} = (x + a)
\]

This equation is valid for all values of \(x + a\) except where the expression is undefined, i.e., where \(x + a = 0\). In this case, the factor \(x + a\) would be in the denominator, making the expression undefined.

\[
\frac{(x + m)(1 + m)}{x + m} = (x + m)
\]

The factor \(x + m\) is also true for all values of \(x + m\) except where the expression is undefined, i.e., where \(x + m = 0\).

Example 1

\[
\frac{x + m}{1} + \frac{1}{x + m} = (x + m)
\]

The factor \(x + m\) is also true for all values of \(x + m\) except where the expression is undefined, i.e., where \(x + m = 0\).

Example 2

\[
\frac{x + m}{1} + \frac{1}{x + m} = (x + m)
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The factor \(x + m\) is also true for all values of \(x + m\) except where the expression is undefined, i.e., where \(x + m = 0\).

where \(a\) and \(b\) are constants.

\[
\frac{a}{x - a} + \frac{b}{x - a} = \frac{a + b}{x - a}
\]

This can be transformed into the quadratic equation:

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\frac{a}{x - a} + \frac{b}{x - a} = \frac{a + b}{x - a}
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In the context of discrete-time systems, the exponential representation of the difference equation is used to analyze and design control systems. The transfer function of a system can be expressed in terms of its zeros and poles. The roots of the characteristic polynomial determine the system's stability and transient response. For a given system, the output can be calculated in terms of its initial conditions and input. The partial-fraction expansion is used to simplify the transfer function, making it easier to analyze and design control systems.
Example A.3

\begin{align*}
\frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2} &= (\omega_x^2 + \omega_y^2 + 1)H
\end{align*}

Thus,

\begin{align*}
\ell &= \frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2}
\end{align*}

Example A.4

\begin{align*}
\frac{\omega_x^2 - 1}{\ell} + \frac{(\omega_x^2 - 1)}{\ell} - \frac{\omega_x^2 - 1}{\ell} &= (\omega_x^2 - 1)H
\end{align*}

Example A.5

\begin{align*}
\frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2} &= (\omega_x^2 + \omega_y^2 + 1)H
\end{align*}

Then,

\begin{align*}
\ell &= \frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2}
\end{align*}

Example A.6

\begin{align*}
\frac{\omega_x^2 - 1}{\ell} + \frac{(\omega_x^2 - 1)}{\ell} - \frac{\omega_x^2 - 1}{\ell} &= (\omega_x^2 - 1)H
\end{align*}

Example A.7

\begin{align*}
\frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2} &= (\omega_x^2 + \omega_y^2 + 1)H
\end{align*}

Then,

\begin{align*}
\ell &= \frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2}
\end{align*}

Example A.8

\begin{align*}
\frac{\omega_x^2 - 1}{\ell} + \frac{(\omega_x^2 - 1)}{\ell} - \frac{\omega_x^2 - 1}{\ell} &= (\omega_x^2 - 1)H
\end{align*}

Example A.9

\begin{align*}
\frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2} &= (\omega_x^2 + \omega_y^2 + 1)H
\end{align*}

Then,

\begin{align*}
\ell &= \frac{\omega_x^2 + \omega_y^2 + 1}{\omega_z^2}
\end{align*}
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