We have discussed the theory of multirate signal processing in class. This project will apply the multirate signal processing algorithm on image scaling problem. A discrete 1-D signal (vector/sequence) can be scaled by a factor of $\frac{L}{M}$, where $L$ and $M$ are positive integers with greatest common divisor 1, i.e. $M$ and $L$ are relatively prime, by using the following multirate linear system.

$$x[n] \xrightarrow{\uparrow L} H(z) \xrightarrow{\downarrow M} y[n]$$

The input image is first upsampled by $L$, scaling by a filter $H(z)$ and then downsampled by $M$. The goal is to obtain a scaled image with most of the details preserved and simultaneously minimize any artifact created by the scaling algorithm.

**Part I: Time-Domain Approach**

1. Write a Matlab function `scaling_filter.m` to design the impulse response of the scaling linear-phase filter $H(z)$ that should be used. The function should have the following input-output format:

   ```matlab
   function [h] = scaling_filter(L,M,N)
   
   L : Upsampling factor
   M : Downsampling factor
   N : Filter order (filter length = N+1)
   h : Impulse response of the filter $H(z)$
   ```

2. Write a Matlab function `scaler.m` to scale the size of an image (2-D signal/matrix) by a given ratio. The function should have the following input-output format:

   ```matlab
   function [image_out] = scaler(image_in,L,M,h)
   
   L : Upsampling factor
   M : Downsampling factor
   h : Impulse response of the filter $H(z)$
   image_in: Original input image
   image_out: Scaled output image
   ```
3. The file `zoneplate.mat` which can be downloaded from the class website at http://www-ee.uta.edu/online/oraintara/ee5350/Termprj.htm contains a $256 \times 256$ test image. Downsampling the zoneplate image by a factor of 3 in both horizontal and vertical direction. Do you see any aliasing/distortion? Now using your function `scaling_filter.m` to design a filter $h[n]$ which will be used to scale the image by a factor of $1/3$, i.e $L = 1$ and $M = 3$. Plot the impulse, magnitude and phase responses of the filter. Use your function ` scaler.m` to scale the zoneplate image. Display and compare the scaled image with the one without filtering.

4. Upsample the zoneplate image by a factor of 2 ($L = 2$ and $M = 1$) without any filtering. What is the effect of imaging distortion appearing in the upsampled image. Apply a scaling filter obtained by using the filter ` scaling_filter.m` to the upsampled image. Display and compare the results.

5. Design an FIR filter for scaling the image by a factor of 5/4 ($L = 5$ and $M = 4$). Plot the impulse, magnitude and phase responses of the filter. Use the filter to scale the zoneplate image by a factor of 5/4. Is there any major distortion? Explain.

6. Subtract the zoneplate image by its mean value and call the new image `zoneplate0`. Use the filter obtained in 5. to scale this image `zoneplate0` by a factor of 5/4. At the end, add the mean value of the original zoneplate image back to the scaled output of the `zoneplate0` image. Compare the results obtained from 5. and 6. How would you explain the results? If there is still residual artifact, how would you improve it further.

**Part II: Frequency-Domain Approach**

7. In this part, we will scale the image using a frequency-domain approach, i.e. the filter $H(z)$ is assumed to be ideal. This can be done by dividing the image into blocks of $N \times N$ pixels, and using the discrete Fourier transform (DFT) to calculate the frequency response of the signal. These blocks will be process independently. Repeat parts 3. to 6. by using frequency-domain approach. How are the results compared to that obtained from the time-domain approach. Do you notice any artifact?

8. How would you improve the quality of the reconstruction images obtained in part 7? Explain your solution and the algorithm, and give some simulation examples if you can.